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Research Paper

Effect of Network-Induced Delay on Stability in Networked Control System.

Engineering

KEYWORDS : network-induced delay; networked control system; NCS; asymptotic stability.

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ABSTRACT

In this paper, effect of network-induced delay on stability in networked control systems has been discussed. Four conditions of network-induced delay have been considered. In first condition, delay is less than the sampling period. Second, delay is equal to the sampling period. Third, delay is longer than the sampling period and fourth, delay is integer multiple of sampling period. A full state feedback controller has been included in closed-loop system. The problem of stability has been described in two ways, first, in finding the roots of the characteristics equation and second, as a response to the initial conditions of the networked closed-loop system. This work has been illustrated with the help of a numerical example.

1 Introduction

Networked control systems (NCSs) are feedback control systems wherein the control loops are closed through a real-time network [1]. A major trend in the modern control community is evolving in the form of the integration of control, computation, communication and networking [2]. As a result of this two main areas of research came into existence: control of networks and control over networks. Control of networks includes congestion control, routing control, data caching and power management. Control over networks deals with the control problems through data networks. Remote data transfers and data exchange among users, reduced complexity in wiring connections and ease in maintenance are the advantages of networked control systems [3].

In recent past, designing of the controller for the NCS has gain attention. Several methods of controller design have been proposed for NCS with considering different assumptions [4]. Basically, three control approaches have been adopted while designing a controller. First approach is a scheduling approach that guarantees network quality of service (QoS). Control approach is related with the guaranteeing the quality of performance (QoP). Third approach considers both of the quality of service and the quality of performance [6]. Disadvantage of all the three approaches is that they require sufficiently large memory so as to handle the past information [5]. Most of the recent design approaches for the systems with time delay are widely applicable for the closed-loop network control systems [7], [8].

Some important issues discussed in this paragraph must be addressed [9], [1]. With reference to Figure 1, sensor-to-controller delay and controller-to-actuator delay are the network-induced delay. It occurs while exchanging data among devices connected to the shared network. This will be either constant or time varying and can degrade the performance of control systems. A control system without considering the delay can even destabilize the system.

In this paper, effect of network-induced delay on stability in networked control system has been described. Both the roots of the characteristics equation and the responses to initial conditions of the closed-loop system have been adopted for stability analysis. MATLAB/Simulink has been used for the simulations.

2 Modeling of Network Control System

In this modeling of networked control system, we consider the data transmission delay caused by scheduling and transmission time. The NCS model considering network-induced delay is shown in Figure 1. The model consists of a continuous plant controller. There are two delays due to network: the sensor-to-controller delay (τ_{sc}) and the controller-to-actuator delay (τ_{ca}). The computational delay of controller can be absorbed into τ_{ca} [12]. The sensor-to-controller delay can be lumped together with controller-to-actuator delay such as $\tau = \tau_{sc} + \tau_{ca}$ for analysis.

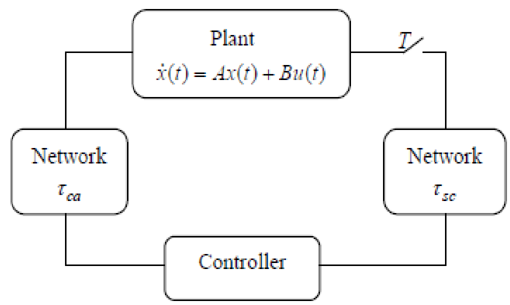

Figure 1: NCS model with networked-induced delay.

2.1 Modeling of NCS when Time Delay is less than the Sampling Period

A continuous-time state-space system with network-induced delay is described by [11]:

$$\dot{x} = Ax(t) + Bu(t-\tau) \tag{1}$$

$$y = Cx(t) \tag{2}$$

$$u(t) = -Kx(t) \tag{3}$$

where, time delay τ is less than the sampling period T and . $t = kT$ for $k = 0,1,2\ldots$. The above system is infinite dimensional. So the control input $u(t)$ is sampled and fed to a zero order hold so that

$$u(t) = u(kT) \quad \text{for } kT \le t \le kT + T \tag{4}$$

The solution of the equation (1) is:

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-h)} Bu(h-\tau)dh \tag{5}$$

where, $x(0)$ are initial states. We get the following equations for.

$$x(kT) = e^{AkT}x(0) + \int_0^{kT} e^{A(kT-h)} Bu(h-\tau)dh \tag{6}$$

$$x(kT+T) = e^{A(kT+T)}x(0) + \int_0^{kT+T} e^{A(kT+T-h)} Bu(h-\tau)dh$$

After multiplying equation (6) by e^{AT} and subtracting it from equation (7), we get

$$x(kT+T) = e^{AT}x(kT) + \int_{kT}^{kT+T} e^{A(kT+T-h)} Bu(h-\tau)dh \tag{8}$$

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Delayed control signal $u(t-\tau)$ is piecewise constant because $u(t)$ is piecewise constant between two consecutive sampling instants [13]. So delayed signal will change between the sampling instants. Now we can write the equation (8) in the following way:

$$\int_{kT}^{kT+\tau} e^{A(kT+T-h)} Bu(h-\tau)dh + \int_{kT+\tau}^{kT+T} e^{A(kT+T-h)} Bu(h-\tau)dh \tag{9}$$

where,

$$u(h-\tau) = \begin{cases} u(kT-T) & kT \le h \le kT+\tau \\ u(kT) & kT+\tau \le h \le kT+T \end{cases} \tag{10}$$

Thus

$$x(kT+T) = e^{AT}x(kT) + \left(\int_{kT}^{kT+\tau} e^{A(kT+T-h)} Bdh\right)u(kT-T) + \left(\int_{kT+\tau}^{kT+T} e^{A(kT+T-h)} Bdh\right)u(kT) \tag{11}$$

Let $l = kT+T-h$, then we get

$$x(kT+T) = e^{AT}x(kT) + \left(\int_{T-\tau}^{T} e^{Al} Bdl\right)u(kT-T) + \left(\int_{0}^{T-\tau} e^{Al} Bdl\right)u(kT) \tag{12}$$

From equation (3), closed loop state-space model:

$$\begin{bmatrix} x(kT+T) \\ u(kT) \end{bmatrix} = \begin{bmatrix} e^{AT} - K\left(\int_0^{T-\tau} e^{Al} Bdl\right) & \left(\int_{T-\tau}^{T} e^{Al} Bdl\right) \\ -K & 0 \end{bmatrix} \begin{bmatrix} x(kT) \\ u(kT-T) \end{bmatrix} \tag{13}$$

The closed-loop system represented by equation (13) is said to be asymptotically stable if the roots of the characteristics equation lie inside the unit circle of the z plane. The characteristics equation is:

$$|z - \Phi| = 0 \tag{14}$$

where;

$$\Phi = \begin{bmatrix} e^{AT} - K\left(\int_0^{T-\tau} e^{Al} Bdl\right) & \left(\int_{T-\tau}^{T} e^{Al} Bdl\right) \\ -K & 0 \end{bmatrix}$$

2.2 Modeling of NCS when Time Delay is equal to the Sampling Period

In this case, the network-induced delay is equal to the sampling period T i.e.

$$\tau = T \tag{15}$$

Now, equation (8) can be written as:

$$x(kT+T) = e^{AT}x(kT) + \int_0^T e^{A(T-h)} Bu(kT-T)dh \tag{16}$$

Let $l = T-h$, then we get:

$$x(kT+T) = e^{AT}x(kT) + \left(\int_0^T e^{Al}dl\right)Bu(kT-T) \tag{17}$$

From equation (3), the closed loop state-space model of networked control system when time delay is equal to the sampling period:

$$\begin{bmatrix} x(kT+T) \\ u(kT) \end{bmatrix} = \begin{bmatrix} e^{AT} & \left(\int_0^T e^{Al}dl\right)B \\ -K & 0 \end{bmatrix} \begin{bmatrix} x(kT) \\ u(kT-T) \end{bmatrix} \tag{18}$$

Equation (18) is said to be asymptotically stable, if the roots of characteristics equation lie inside the unit circle of z plane. The characteristics equation is:

$$|z - \tilde{\Phi}| = 0 \tag{19}$$

where,

$$\tilde{\Phi} = \begin{bmatrix} e^{AT} & \left(\int_0^T e^{Al}dl\right)B \\ -K & 0 \end{bmatrix}$$

2.3 Modeling of NCS when Time Delay is longer than the Sampling Period

In this case, the network-induced delay is longer than sampling period T i.e.

$$\tau = dT + \tau' \tag{20}$$

where, d is an integer and τ' is less than the sampling period T. Now, equation (8) can be modified:

$$x(kT+T) = e^{AT}x(kT) + \int_{kT-dT}^{kT-dT+T} e^{A(kT-dT+T-h)} Bu(h-\tau)dh \tag{21}$$

where,

$$u(h-\tau) = \begin{cases} u(kT-dT-T) & kT-dT \le h \le kT-dT+\tau' \\ u(kT-dT) & kT-dT+\tau' \le h \le kT-dT+T \end{cases} \tag{22}$$

Thus

$$x(kT+T) = e^{AT}x(kT) + \left(\int_{kT-dT}^{kT-dT+\tau'} e^{A(kT-dT+T-h)} Bdh\right)u(kT-dT-T) + \left(\int_{kT-dT+\tau'}^{kT-dT+T} e^{A(kT-dT+T-h)} Bdh\right)u(kT-dT) \tag{23}$$

Let $l = kT-dT+T-h$, then we get:

$$x(kT+T) = e^{AT}x(kT) + \left(\int_{T-\tau'}^{T} e^{Al} Bdl\right)u(kT-dT-T) + \left(\int_{0}^{T-\tau'} e^{Al} Bdl\right)u(kT-dT) \tag{24}$$

From equation (3), the closed loop state-space model of networked control system when time delay is longer than sampling period:

$$\begin{bmatrix} x(kT+T) \\ u(kT-dT) \\ \vdots \\ u(kT-T) \\ u(kT) \end{bmatrix} = \begin{bmatrix} e^{AT} & \left(\int_{T-\tau'}^{T} e^{Al}dl\right)B & \left(\int_0^{T-\tau'} e^{Al}dl\right)B & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & I \\ -K & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x(kT) \\ u(kT-dT-T) \\ \vdots \\ u(kT-2T) \\ u(kT-T) \end{bmatrix} \tag{25}$$

Equation (25) is said to be asymptotically stable, if the roots of characteristics equation lie inside the unit circle of z plane. The characteristics equation is:

$$|zI - \Psi| = 0 \tag{26}$$

where,

$$\Psi = \begin{bmatrix} e^{AT} & \left(\int_{T-\tau}^{T} e^{Al} dl\right) B & \left(\int_{0}^{T-\tau} e^{Al} dl\right) B & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & I \\ -K & 0 & 0 & \cdots & 0 \end{bmatrix}$$

2.4 Modeling of NCS when Time Delay is integer multiple of the Sampling Period

In this case, the network-induced delay is integer multiple of the sampling period T i.e.

$$\tau = dT \tag{27}$$

where, d is an integer.

Now, equation (8) can be modified:

$$x(kT+T) = e^{AT} x(kT) + \int_{kT-dT}^{kT-dT+T} e^{A(kT-dT+T-h)} Bu(kT-dT)dh \tag{28}$$

Let $l = kT - dT + T - h$, then we get:

$$x(kT+T) = e^{AT} x(kT) + \left(\int_0^T e^{Al} dl\right) Bu(kT-dT) \tag{29}$$

From equation (3), the closed loop state-space model of networked control system when time delay is integer multiple of the sampling period:

$$\begin{bmatrix} x(kT+T) \\ u(kT-dT) \\ \vdots \\ u(kT-T) \\ u(kT) \end{bmatrix} = \begin{bmatrix} e^{AT} & 0 & \left(\int_0^T e^{Al} dl\right) B & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & I \\ -K & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x(kT) \\ u(kT-dT-T) \\ \vdots \\ u(kT-2T) \\ u(kT-T) \end{bmatrix} \tag{30}$$

Equation (30) is said to be asymptotically stable, if the roots of characteristics equation lie inside the unit circle of z plane. The characteristics equation is:

$$|zI - \tilde{\Psi}| = 0 \tag{31}$$

where,

$$\tilde{\Psi} = \begin{bmatrix} e^{AT} & 0 & \left(\int_0^T e^{Al} dl\right) B & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & I \\ -K & 0 & 0 & \cdots & 0 \end{bmatrix}$$

3 Simulation

Consider the system [10]:

$$\dot{x}(t) = Ax(t) + Bu(t-\tau)$$
$$y(t) = Cx(t)$$
$$u(t) = -Kx(t)$$

where,

$$A = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

This system is sampled with period $T = 0.2s$ and τ is the network-induced delay. We use the pole-placement technique to the design of the system. The desired closed-loop poles for this system are $s = -1.8 \pm j2.4$. For this case, the state-feedback gain matrix K can be calculated as:

$$K = [29.6 \quad 3.6]$$

Let

$$G = e^{AT}, \quad H = \int_0^T e^{Al} dl\, B$$

The discrete-time open-loop system:

$$x(kT+T) = Gx(kT) + Hu(kT) = \begin{bmatrix} 1.4411 & 0.2286 \\ 4.7096 & 1.4411 \end{bmatrix} x(kT) + \begin{bmatrix} 0.0214 \\ 0.2286 \end{bmatrix} u(kT)$$

For this system, the characteristics equation is. $|zI - G| = 0$ The roots are 2.4787 and 0.4034. One root is not lying within unit circle of z plane of Figure 2 (a). So, this system is not stable as shown in Figure 3 (a).

The discrete-time closed-loop system:

- Without considering network-induced delay

The closed-loop system:

$$x(kT+T) = [G - HK]x(kT) = \begin{bmatrix} 0.8073 & 0.1515 \\ -2.0576 & 0.6180 \end{bmatrix}$$

The characteristics equation is $|zI - (G - HK)| = 0$. The roots are $0.7127 + j0.5503$ and $0.7127 - j0.5503$. Both the roots are lying within unit circle of z plane of Figure 2 (b). Thus, the closed-loop control system is asymptotically stable as shown in Figure 3 (b).

- Considering network-induced delay $0 \le (\tau = 0.1s) < T$ From equation (13), the closed-loop system:

$$\begin{bmatrix} x(kT+T) \\ u(kT) \end{bmatrix} = \begin{bmatrix} 0.9180 & -0.2944 & 0.0163 \\ 4.1865 & 0.9180 & 0.1252 \\ -29.6000 & -3.6000 & 0 \end{bmatrix} \begin{bmatrix} x(kT) \\ u(kT-T) \end{bmatrix}$$

For this networked control system, the roots of the characteristics equation (14) are $0.5287 + j1.3807$, $0.5287 + j1.3807$, and 0.7786.

Two roots are not lying within unit circle of z plane of Figure 2 (c). Thus, the closed-loop system considering network-induced delay is not stable as shown in Figure 3 (c).

Considering network-induced delay $\tau = T$

From equation (18), the closed-loop system:

$$\begin{bmatrix} x(kT+T) \\ u(kT) \end{bmatrix} = \begin{bmatrix} 1.4411 & 0.2286 & 0.0214 \\ 4.7096 & 1.4411 & 0.2286 \\ -29.6000 & -3.6000 & 0 \end{bmatrix} \begin{bmatrix} x(kT) \\ u(kT-T) \end{bmatrix}$$

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For this NCS, the roots of the characteristics equation (19) are 0.0853, $1.3984 + j0.5125$, and $1.3984 - j0.5125$. Two roots are not lying within unit circle of z plane of Figure 2 (d). Thus, the closed-loop system is not stable as shown in Figure 3 (d).

Considering network-induced delay $\tau = dT + \tau' = 0.3\,s$

Here, $d = 1$, and $\tau' = 0.1\ s$.

From equation (25), the networked closed-loop control system:

$$\begin{bmatrix} x(kT+T) \\ u(kT-T) \\ u(kT) \end{bmatrix} = \begin{bmatrix} 1.4411 & 0.2286 & 0.0163 & 0.0051 \\ 4.7096 & 1.4411 & 0.1252 & 0.1035 \\ 0 & 0 & 0 & 1.0000 \\ -29.6000 & -3.6000 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(kT) \\ u(kT-2T) \\ u(kT-T) \end{bmatrix}$$

For this NCS, the roots of the characteristics equation (26) are -0.4670, $0.1914, 1.6798$, and 1.4780. Two roots are not lying within unit circle of z plane of Figure 2 (e). Thus, the closed-loop system is not stable as shown in Figure 3 (e).

- Considering network-induced delay $\tau = dT + \tau' = 0.3\,s$ Here, $d = 1$ and $\tau' = 0.1\,s$ From equation (30), the networked closed-loop control system:

$$\begin{bmatrix} x(kT+T) \\ u(kT-2T) \\ u(kT-T) \\ u(kT) \end{bmatrix} = \begin{bmatrix} 1.4411 & 0.2286 & 0 & 0.0214 & 0 \\ 4.7096 & 1.4411 & 0 & 0.2286 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 \\ -29.6000 & -3.6000 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(kT) \\ u(kT-3T) \\ u(kT-2T) \\ u(kT-T) \end{bmatrix}$$

For this NCS, the roots of the characteristics equation (31) are 0, 2.0919, 1.2537, 0.1230, and -0.5865. Two roots are not lying within unit circle of z plane of Figure 2 (f). Thus, the closed-loop system is not stable as shown in Figure 3 (f).

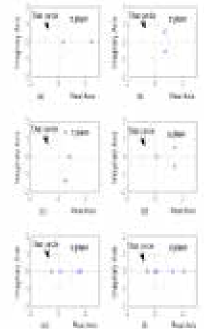

Figure 2: Roots of characteristics Equation:

(a) $|zI - G| = 0$; (b) $|zI - (G - HK)| = 0$; (c) $|zI - \Phi| = 0$; (d) $|zI - \tilde{\Phi}| = 0$; (e) $|zI - \Psi| = 0$; (f) $|zI - \tilde{\Psi}| = 0$.

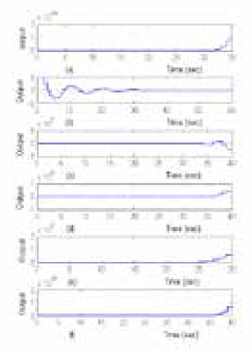

Figure 3: Response to initial conditions $x(0) = [5;5]$: (a) Open-loop control system (b) Closed-loop control system without considering network-induced delay; (c) Closed-loop control system considering the network-induced delay $0 \le (\tau = 0.1s) < T$; (d) Closed-loop control system considering the network-induced delay $\tau = T$; (e) Closed-loop control system considering the network-induced delay $\tau = dT + \tau' = 0.3s$; (f) Closed-loop control system considering the network-induced delay. $\tau = dT = 0.4\,s$

4 Conclusion

In this paper, the effect of network-induced delay on stability in network control system has been described. Four conditions of network-induced delay have been considered. Full state feedback networked control system has been designed with the help of pole-placement technique. The response of close-loop NCS considering network-induced delay is unstable compared to the system considering no delay. The close-loop systems become more unstable as delay increases. From Figure 2 and Figure 3, it has been confirmed that an NCS considering delay is an open-loop system. Because, the network-induced delay is affecting the performance of the system, so a suitable time delay compensation technique must be adopted.

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