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# Anti S-Fuzzy Subhemirings of a Hemiring

Mathematics

**KEYWORDS :** fuzzy set, anti S-fuzzy subhemiring, pseudo anti S-fuzzy coset.

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## ABSTRACT

*In this paper, we made an attempt to study the algebraic nature of an anti S-fuzzy subhemiring of a hemiring .*

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**INTRODUCTION:**

There are many concepts of universal algebras generalizing an associative ring ( *R ; + ; .* ). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras ( *R ; + ; .* ) share the same properties as a ring except that ( *R ; +* ) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. ( b+c ) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh[12], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti fuzzy left h-ideals in hemiring was introduced by Akram.M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[6]. In this paper, we introduce the some Theorems in anti S-fuzzy subhemiring of a hemiring.

**1.PRELIMINARIES:**

**1.1 Definition:** A S-norm is a binary operation S: [0, 1]´[0, 1] ® [0, 1] satisfying the following requirements;

(i) 0 S x = x, 1 S x = 1 (boundary condition)

(ii) x S y = y S x (commutativity)

(iii) x S ( y S z) = ( x S y) S z (associativity)

(iv) if x £ y and w £ z, then x S w £ y S z ( monotonicity).

**1.2 Definition:** Let X be a non-empty set. A **fuzzy subset** A of X is a function A : X ® [0, 1].

**1.3 Definition:** Let ( R , + , . ) be a hemiring. A fuzzy subset A of R is said to be an anti S-fuzzy subhemiring(anti fuzzy subhemiring with respect to S-norm) of R if it satisfies the following conditions:

(i) $m_A(x+y) \leq S( m_A(x), m_A(y) )$,

(ii) $m_A(xy) \leq S( m_A(x), m_A(y) )$, for all x and y in R.

**1.4 Definition:** Let A and B be fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by A´B, is defined as A´B ={á (x, y ), $m_{A´B}$(x,y) ñ / for all x in G and y in H }, where $m_{A´B}(x, y) = \max \{ m_A(x), m_B(y) \}$.

**1.5 Definition:** Let A be a fuzzy subset in a set S, the anti-strongest fuzzy relation on S, that is a fuzzy relation on A is V given by $m_V(x, y) = \max \{ m_A(x), m_A(y) \}$, for all x and y in S.

**1.6 Definition:** Let ( R, +, . ) and ( R|, +, . ) be any two hemirings. Let f : R → R| be any function and A be an anti S-fuzzy subhemiring in R, V be an anti S-fuzzy subhemiring in f (R) = R|, defined by $m_V(y) = \inf_{x \in f^{-1}(y)} m_A(x)$, for all x in R and y in R|. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

**1.7 Definition:** Let A be an anti S-fuzzy subhemiring of a hemiring ( R, +, . ) and a in R. Then the pseudo anti S-fuzzy coset $(aA)^p$ is defined by

$( (am_A)^p )(x) = p(a)m_A(x)$, for every x in R and for some p in P.

2. PROPERTIES OF ANTI S-FUZZY SUBHEMIRING OF A HEMIRING

**2.1 Theorem:** Union of any two anti S-fuzzy subhemiring of a hemiring R is an anti S-fuzzy subhemiring of R.

**Proof:** Let A and B be any two anti S-fuzzy subhemirings of a hemiring R and x and y in R. Let A = {( x, $m_A(x)$) / xÎR} and B = {( x, $m_B(x)$ ) / xÎR} and also let C = AÈB = { ( x, $m_C(x)$ ) / xÎR}, where max { $m_A(x)$, $m_B(x)$ } = $m_C(x)$. Now, $m_C(x+y) \leq$ max { S( $m_A(x)$, $m_A(y)$ ), S( $m_B(x)$, $m_B(y)$ ) } £ S( $m_C(x)$, $m_C(y)$ ). Therefore, $m_C(x+y) \leq S( m_C(x), m_C(y) )$, for all x and y in R. And, $m_C(xy) \leq$ max { S( $m_A(x)$, $m_A(y)$ ), S( $m_B(x)$, $m_B(y)$ ) } £ S ( $m_C(x)$, $m_C(y)$ ). Therefore, $m_C(xy) \leq S (m_C(x), m_C(y) )$, for all x and y in R. Therefore C is an anti S-fuzzy subhemiring of a hemiring R.

**2.2 Theorem:** The union of a family of anti S-fuzzy subhemirings of hemiring R is an anti S-fuzzy subhemiring of R.

**Proof:** It is trivial.

**2.3 Theorem:** If A and B are any two anti S-fuzzy subhemirings of the hemirings $R_1$ and $R_2$ respectively, then anti-product A´B is an anti S-fuzzy subhemiring of $R_1$´$R_2$.

**Proof:** Let A and B be two anti S-fuzzy subhemirings of the hemirings $R_1$ and $R_2$ respectively. Let $x_1$ and $x_2$ be in $R_1$, $y_1$ and $y_2$ be in $R_2$. Then ($x_1$, $y_1$) and ($x_2$, $y_2$) are in $R_1$´$R_2$. Now, $m_{A´B}$[ ($x_1$, $y_1$) + ($x_2$, $y_2$) ] ≤ max { S( $m_A(x_1)$, $m_A(x_2)$ ), S( $m_B(y_1)$, $m_B(y_2)$ ) } £ S ( $m_{A´B}$ ($x_1$, $y_1$), $m_{AxB}$ ($x_2$, $y_2$) ). Therefore, $m_{A´B}$ [($x_1$, $y_1$) + ($x_2$, $y_2$) ] ≤ S( $m_{A´B}$($x_1$, $y_1$), $m_{A´B}$ ($x_2$, $y_2$) ). Also, $m_{A´B}$[($x_1$, $y_1$)($x_2$, $y_2$)] ≤ max{ S( $m_A(x_1)$, $m_A(x_2)$ ), S( $m_B(y_1)$, $m_B(y_2)$ ) }£ S( $m_{AxB}$ ($x_1$, $y_1$), $m_{AxB}$ ($x_2$, $y_2$) ). Therefore, $m_{A´B}$[($x_1$, $y_1$)($x_2$, $y_2$)] ≤ S( $m_{AxB}$($x_1$, $y_1$), $m_{AxB}$($x_2$, $y_2$) ). Hence A´B is an anti S-fuzzy subhemiring of hemiring of $R_1$´$R_2$.

**2.4 Theorem:** Let A be a fuzzy subset of a hemiring R and V be the anti-strongest fuzzy relation of R. Then A is an anti S-fuzzy subhemiring of R if and only if V is an anti S-fuzzy subhemiring of R´R.

**Proof:** Suppose that A is an anti S-fuzzy subhemiring of a hemiring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R´R. We have, $m_V(x+y) = \max\{m_A(x_1+y_1), m_A(x_2+y_2)\} \le \max\{S(m_A(x_1), m_A(y_1)), S(m_A(x_2), m_A(y_2))\}$ £ $S(m_V(x_1, x_2), m_V(y_1, y_2)) = S(m_V(x), m_V(y))$. Therefore, $m_V(x+y) \le S(m_V(x), m_V(y))$, for all x and y in R´R. And, $m_V(xy) = \max\{m_A(x_1y_1), m_A(x_2y_2)\} \le \max\{S(m_A(x_1), m_A(y_1)), S(m_A(x_2), m_A(y_2))\}$ £ $S(m_V(x_1, x_2), m_V(y_1, y_2)) = S(m_V(x), m_V(y))$. Therefore, $m_V(xy) \le S(m_V(x), m_V(y))$, for all x and y in R´R. This proves that V is an anti S-fuzzy subhemiring of R´R. Conversely assume that V is an anti S-fuzzy subhemiring of R´R, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R´R, we have $\max\{m_A(x_1+y_1), m_A(x_2+y_2)\} = m_V(x+y) \le S(m_V(x), m_V(y)) = S(m_V(x_1, x_2), m_V(y_1, y_2)) = S(\max\{(m_A(x_1), m_A(x_2)\}, \max\{m_A(y_1), m_A(y_2)\}$. If $x_2 = 0$, $y_2 = 0$, we get, $m_A(x_1+y_1) \le S(m_A(x_1), m_A(y_1))$, for all $x_1$ and $y_1$ in R. And, $\max\{m_A(x_1y_1), m_A(x_2y_2)\} = m_V(xy) \le S(m_V(x), m_V(y)) = S(m_V(x_1, x_2), m_V(y_1, y_2)) = S(\max\{m_A(x_1), m_A(x_2)\}, \max\{m_A(y_1), m_A(y_2)\}$. If $x_2 = 0$, $y_2 = 0$, we get $m_A(x_1y_1) \le S(m_A(x_1), m_A(y_1))$, for all $x_1$ and $y_1$ in R. Therefore A is an anti S-fuzzy subhemiring of R.

**2.5 Theorem:** A is an anti S-fuzzy subhemiring of a hemiring ( R, +, . ) if and only if $m_A(x+y) \le S(m_A(x), m_A(y))$, $m_A(xy) \le S(m_A(x), m_A(y))$, for all x and y in R.

**Proof:** It is trivial.

**2.6 Theorem:** If A is an anti S-fuzzy subhemiring of a hemiring ( R, +, . ), then H = {x / xÎR: $m_A(x) = 0$} is either empty or is a subhemiring of R.

**Proof:** It is trivial.

**2.7 Theorem:** Let A be an anti S-fuzzy subhemiring of a hemiring ( R, +, . ). If $m_A(x+y) = 1$, then either $m_A(x) = 1$ or $m_A(y) = 1$, for all x and y in R.

**Proof:** It is trivial.

**2.8 Theorem:** Let A be an anti S-fuzzy subhemiring of a hemiring ( R, +, . ), then the pseudo anti S-fuzzy coset $(aA)^p$ is an anti S-fuzzy subhemiring of a hemiring R, for every a in R.

**Proof:** Let A be an anti S-fuzzy subhemiring of a hemiring R. For every x and y in R, we have, $((am_A)^p)(x+y) \le p(a)S(m_A(x), m_A(y))$ £ $S(p(a)m_A(x), p(a)m_A(y)) = S(((am_A)^p)(x), ((am_A)^p)(y))$. Therefore, $((am_A)^p)(x+y) \le S((am_A)^p)(x), ((am_A)^p)(y))$. Now, $((am_A)^p)(xy) \le p(a)S(m_A(x), m_A(y))$ £ $S(p(a)m_A(x), p(a)m_A(y)) = S(((am_A)^p)(x), ((am_A)^p)(y))$. Therefore, $((am_A)^p)(xy) \le S(((am_A)^p)(x), ((am_A)^p)(y))$. Hence $(aA)^p$ is an anti S-fuzzy subhemiring of a hemiring R.

**2.9 Theorem:** Let ( R, + , . ) and ( $R^I$, +, . ) be any two hemirings. The homomorphic image of an anti S-fuzzy subhemiring of R is an anti S-fuzzy subhemiring of $R^I$.

**Proof:** Let f : R ® $R^I$ be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let V = f(A), where A is an anti S-fuzzy subhemiring of R. Now, for f(x), f(y) in $R^I$, $m_v(f(x)+f(y)) \le m_A(x+y) \le S(m_A(x), m_A(y))$, which implies that $m_v(f(x)+f(y)) \le S(m_v(f(x)), m_v(f(y)))$. Again, $m_v(f(x)f(y)) \le m_A(xy) \le S(m_A(x), m_A(y))$, which implies that $m_v(f(x)f(y)) \le S(m_v(f(x)), m_v(f(y)))$. Hence V is an anti S-fuzzy subhemiring of $R^I$.

**2.10 Theorem:** Let ( R, +, .) and ( $R^I$, +, .) be any two hemirings. The homomorphic preimage of an anti S-fuzzy subhemiring of $R^I$ is an anti S-fuzzy subhemiring of R.

**Proof:** Let V = f(A), where V is an anti S-fuzzy subhemiring of $R^I$. Let x and y in R. Then, $m_A(x+y) = m_v(f(x+y)) \le S(m_v(f(x)), m_v(f(y))) = S(m_A(x), m_A(y))$, which implies that $m_A(x+y) \le S(m_A(x), m_A(y))$. Again, $m_A(xy) = m_v(f(xy)) \le S(m_v(f(x)), m_v(f(y))) = S(m_A(x), m_A(y))$ which implies that $m_A(xy) \le S(m_A(x), m_A(y))$. Hence A is an anti S-fuzzy subhemiring of R.

**2.11 Theorem:** Let ( R, +, . ) and ( $R^I$, +, . ) be any two hemirings. The anti-homomorphic image of an anti S-fuzzy subhemiring of R is an anti S-fuzzy subhemiring of $R^I$.

**Proof:** Let f : R ® $R^I$ be an anti-homomorphism. Then, $f(x+y) = f(y)+f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R. Let V = f(A), where A is an anti S-fuzzy subhemiring of R. Now, for f(x), f(y) in $R^I$, $m_v(f(x)+f(y)) \le m_A(y+x) \le S(m_A(y), m_A(x)) = S(m_A(x), m_A(y))$ which implies that $m_v(f(x)+f(y)) \le S(m_v(f(x)), m_v(f(y)))$. Again, $m_v(f(x)f(y)) \le m_A(yx) \le S(m_A(y), m_A(x)) = S(m_A(x), m_A(y))$ which implies that $m_v(f(x)f(y)) \le S(m_v(f(x)), m_v(f(y)))$. Hence V is an anti S-fuzzy subhemiring of $R^I$.

**2.12 Theorem:** Let ( R, + , . ) and ( $R^I$, +, . ) be any two hemirings. The anti-homomorphic preimage of an anti S-fuzzy subhemiring of $R^I$ is an anti S-fuzzy subhemiring of R.

**Proof:** Let V = f(A), where V is an anti S-fuzzy subhemiring of $R^I$. Let x and y in R.

Then, $m_A(x+y) = m_v(f(x+y)) \le S(m_v(f(y)), m_v(f(x))) = S(m_A(x), m_A(y))$ which implies that $m_A(x+y) \le S(m_A(x), m_A(y))$. Again, $m_A(xy) = m_v(f(xy)) \le S(m_v(f(y)), m_v(f(x))) = S(m_A(x), m_A(y))$ which implies that $m_A(xy) \le S(m_A(x), m_A(y))$. Hence A is an anti S-fuzzy subhemiring of R.

**In the following Theorem $\circ$ is the composition operation of functions:**

**2.13 Theorem:** Let A be an anti S-fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then $A\circ f$ is an anti S-fuzzy subhemiring of R.

**Proof:** Let x and y in R.Then we have, $(m_A\circ f)(x+y) = m_A(f(x)+f(y)) \le S(m_A(f(x)), m_A(f(y))) \le S((m_A\circ f)(x), (m_A\circ f)(y))$ which implies that $(m_A\circ f)(x+y) \le S((m_A\circ f)(x), (m_A\circ f)(y))$. And, $(m_A\circ f)(xy) = m_A(f(x)f(y)) \le S(m_A(f(x)), m_A(f(y))) \le S((m_A\circ f)(x), (m_A\circ f)(y))$ which implies that $(m_A\circ f)(xy) \le S((m_A\circ f)(x), (m_A\circ f)(y))$. Therefore $A\circ f$ is an anti S-fuzzy subhemiring of a hemiring R.

**2.14 Theorem:** Let A be an anti S-fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then $A\circ f$ is an anti S-fuzzy subhemiring of R.

**Proof:** Let x and y in R. Then we have, $(m_A\circ f)(x+y) = m_A(f(y)+f(x)) \le S(m_A(f(x)), m_A(f(y))) \le S((m_A\circ f)(x), (m_A\circ f)(y))$, which implies that $(m_A\circ f)(x+y) \le S((m_A\circ f)(x), (m_A\circ f)(y))$. And $(m_A\circ f)(xy) = m_A(f(y)f(x)) \le S(m_A(f(x)), m_A(f(y))) \le S((m_A\circ f)(x), (m_A\circ f)(y))$, which implies that $(m_A\circ f)(xy) \le S((m_A\circ f)(x), (m_A\circ f)(y))$. Therefore $A\circ f$ is an anti S-fuzzy subhemiring of a hemiring R.

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