

# An optimal method to find critical path in fuzzy project networks



## Mathematics

**KEYWORDS :** Critical Path; Fuzzy PERT; Path float; Triangular Fuzzy numbers

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### ABSTRACT

*In this paper a new approach to determine the critical path in a fuzzy project network based on the maximum total float of the path is proposed. The advantage of the proposed method is explained through a numerical example and compared with existing methods. The comparisons reveal that the proposed approach is an improved method over existing methods and more efficient in finding the critical path.*

### Introduction

In recent years, the use of project management in industries and service organizations has been greatly expanded. In several situations, the activity times of projects are imprecise. Hence, it is difficult to estimate activity duration time exactly. These situations can be handled by fuzzy set theory by representing the imprecise and vague activity times by fuzzy numbers [1]. Several researchers have combined fuzzy set theory with PERT (Project Evaluation and Review Technique) to find time estimates in project planning and control problems under different conditions. Dubois et al. [2] extended the fuzzy arithmetic operational model to compute the latest starting time of each activity in a project network. Liang and Han [3] presented an algorithm to perform fuzzy critical path analysis for project network problem. Zilenski [4] noted that the backward recursion fails to compute the sets of possible values of the latest starting times and floats of activities and extended some results for interval numbers. Nasution[5] proposed interactive fuzzy subtraction to compute total floats and to find critical paths. Yakchali et al. [6] analyzed possibly critical path in network with imprecise activity time duration and time lags, represented by means of interval or fuzzy. C T Chen et al. [7] proposed criticality degree of activities and paths to find fuzzy critical paths. Shankar et al. [8] proposed a new approach for finding the fuzzy total float of each activity. Kumar et al. [10] proposed a new method to find the exact optimal solution of fuzzy critical path problems. Bonnal et al. [9] in their review on fuzzy PERT noted that all the methodologies proposed to reduce these inaccuracy of the fuzzy PERT are cumbersome and of little interest for practitioners, while the methodologies based on the fuzzy extension of the crisp approach are certainly the most significant ones.

This study focuses on proposing a fuzzy PERT method by means of maximum slack of the path to analyze critical path. The advantage of considering maximum slack of path is that it considers both the core and spreads of the fuzzy number in determining the critical path. The efficiency of the proposed method over the existing straight forward methods [3,7] is analyzed.

### Existing methods

In this subsection some previous methods [3,7] are reviewed.

If the fuzzy float of activity  $i$  is  $(a_i, b_i, c_i)$ , then Chen et al. [7] defined the criticality of the activity as

$$\text{Critical Degree of activity } i (CD_i) = \begin{cases} 1 & b_i \leq 0 \\ \frac{-a_i}{b_i - a_i} & a_i < 0 < b_i \\ 0 & a_i \geq 0 \end{cases}$$

The criticality of path  $\pi(P_k) = \min_{i \in P_k} \{CD_i\}$  where  $P_k$  is the  $k^{\text{th}}$  path in the network and the path with maximum criticality is critical path.

Liang et al. [3] proposed the fuzzy path completion time as the sum of the total floats of each activity in the path and applied the decision maker's risk attitude index to find the critical path.

### Proposed Approach

In this section, a fuzzy extension of crisp approach is presented to overcome the draw backs of the existing straight forward approach and criticality degree methods. The steps of the proposed method to calculate the fuzzy critical path is as follows:

#### Step 1 : Construct the fuzzy project network

**Step 2: Calculate the fuzzy earliest event times  $E_j$  using,**  
 $E_j = \max\{E_i \oplus t_j / j = 2, 3, \dots, n\}$  where  $t_j$  is the fuzzy activity time of

activity (i,j) and  $E_1 = (0,0,0)$

**Step 3 :** Calculate fuzzy latest times  $E_i$ 's using,

$$E_i = \min\{E_j \ominus t_j / i < j, j = n - 1, n - 2, \dots, 1\},$$

where  $E_n = E_n$

**Step 4 :** Calculate the total float of each activity (i,j) using

$$F_j = E_j \ominus (E_i \oplus t_j)'$$

$1 \leq i < j \leq n$  and denote  $F_j = (a_j, b_j, c_j)$

**Step 5:** Find all possible paths  $(P_k)$  in the project network and calculate the maximum

total float of the path  $\max(F(P_k))$ , defined as

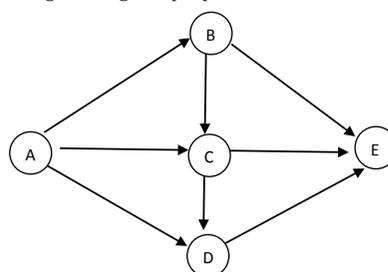
$$\max(F(P_k)) = \{(\max\{a_j\}, \max\{b_j\}, \max\{c_j\}) / \forall (i,j) \in P_k \text{ and } F_j = (a_j, b_j, c_j)\}$$

**Step 6:** Find  $\text{Min}_{P_k \in S} \{\max(F(P_k))\}$

where S denotes the set of all paths in the project network and the corresponding path is critical path.

### Numerical Example

The efficiency of proposed approach over the existing methods is shown using a numerical example. The fuzzy network structure is shown in Fig.1 and activity times are presented in Table I. Fuzzy total floats and criticality degree values of each activity are calculated and presented in Table I. The possible paths and their corresponding values obtained using existing and proposed approach is presented in Table II. The fuzzy critical paths obtained using existing and proposed method is shown in Table III.



**Fig. 1: Fuzzy network**

**Table I: Activities and their activity times**

Activity	Activity Time	Total Float	CD
A-B	(0.6,3,5.4)	(-29.6,0,29.6)	1
A-C	(0.2,1,1.8)	(-11.6,10,31.6)	0.537
A-D	(0.4,2,3.6)	(-7.6,12,31.6)	0.3878
B-E	(1,9,17)	(-19.2,6,31.2)	0.7619
B-C	(1.6,8,14.4)	(-29.6,0,29.6)	1
C-E	(1,7,13)	(-29.6,0,29.6)	1
C-D	(0.6,3,5.4)	(-29.2,0,29.2)	1
D-E	(0.8,4,7.2)	(-29.2,0,29.2)	1

**Table II: Comparison of paths**

Paths	Fuzzy Completion Path [3]	Proposed maximum fuzzy total flow	Criticality of path [7]
A-B-E	(-48.8,6,60.8)	(-19.2,6,31.2)	0.7619
A-B-C-D-E	(-117.6,0,117.6)	(-29.2,0,29.6)	1
A-B-C-E	(-88.8,0,88.8)	(-29.6,0,29.6)	1
A-C-E	(-41.2,10,61.2)	(-11.6,10,31.6)	0.537
A-D-E	(-36.8,12,60.8)	(-7.6,12,31.6)	0.3878
A-C-D-E	(-70,10,90)	(-11.6,10,31.6)	0.537

**Table III: Critical paths of the existing and proposed methods**

Existing Methods				Proposed method	
Chen's method		Liang's method			
Fuzzy Critical Path	Fuzzy project completion	Fuzzy Critical Path	Fuzzy project completion	Fuzzy Critical Path	Fuzzy project completion
A-B-C-D-E	(3.6,18,32.8)	A-B-C-D-E	(3.6,18,32.8)	A-B-C-E	(3.2,18,32.8)
A-B-C-E	(3.2,18,32.8)	A-B-C-E	(3.2,18,32.8)		

**Result Discussion and Conclusion**

From the results shown in Table III, it is understood that, the fuzzy project completion time obtained by the existing methods [3,7] is different corresponding to different fuzzy critical paths, where as it is unique by using the proposed method. Hence the proposed approach determines the critical path efficiently over the existing methods which enable it to be applied at a wide range of project networks.

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