Research Paper

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Study on Flow Past an Exponentially Accelerated Vertical Plate with Uniform Heat Flux

Mathematics

KEYWORDS : exponential, accelerated vertical plate, heat flux

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ABSTRACT

An analysis is performed to study the flow of an incompressible viscous fluid past an exponentially accelerated infinite vertical plate with uniform heat flux. The dimensionless governing equations are solved using Laplace-transform technique. The velocity and temperature are studied for different physical parameters like thermal Grashof number, time and an accelerating parameter a. It is observed that velocity increases with increasing values of a or Gr.

Nomenclature

A - constant

a' - accelerating parameter

a - dimensionless accelerating parameter

C_p - specific heat at constant pressure

g - acceleration due to gravity

Gr - thermal Grashof number

k - thermal conductivity of the fluid

Pr - Prandtl number

p - pressure

T - temperature of the fluid near the plate

T_w - temperature of the plate

T_∞ - temperature of the fluid far away from the plate

t' - time

t - dimensionless time

u' - velocity of the fluid in the x-direction

u_0 - velocity of the plate

u - dimensionless velocity

y - coordinate axis normal to the plate

y' - dimensionless coordinate axis normal to the plate

Greek symbols

α - thermal diffusivity

β - volumetric coefficient of thermal expansion

μ - coefficient of viscosity

ν - kinematic viscosity

ρ - density

τ - dimensionless skin-friction

θ - dimensionless temperature

η - similarity parameter

erfc - complementary error function

1. Introduction

The problem of unsteady free convection fluid flows are encountered in many industrial problems such as filtration processes, the drying of porous materials in textile industries and the saturation of porous materials by chemicals, nuclear reactors, spacecraft design and solar energy collectors. A few representative fields of interest in which combined heat and mass transfer plays an important role, are design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and pollution of the environment.

Sakiadis [1,2] studied the growth of the two dimensional velocity boundary layer over a continuously moving horizontal plate emerging from a wide slot at uniform velocity. Soundalgekar [3] was the first to present an exact solution for the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the usual Laplace transform technique and the effects of heating or cooling of the plate on the flow field were discussed through Gr. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [4]. The Skin-friction for accelerated vertical plate has been studied analytically by Hossian and Shayo [5].

The object of the present paper is to study the flow past an exponentially accelerated infinite vertical plate with uniform heat flux. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2. Mathematical Formulation

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with uniform heat flux is considered. The x - axis is taken along the plate in the vertical direction and the y - axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature in a stationary condition. At time t >0, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a t)$ in its own plane and the temperature of the plate is raised to T_w. At the same time, the heat is supplied from the plate to the fluid at uniform rate. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\beta\left(T - T_\infty\right) + \nu \frac{\partial^2 u'}{\partial y'^2} \qquad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} \qquad (2)$$

With the following initial and boundary conditions

u' = 0   T = T$_¥$ for all $y', t' \le 0$

t' >0: u' = u_0 exp (at),   T = T_w at y' = 0   (3)

u' ® 0,   T® T$_¥$ as y' ®¥

On introducing the following non-dimensional quantities

$$u = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y' u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad (4)$$

$$Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, \quad \Pr = \frac{\mu C_p}{k}, \quad a = \frac{a'\nu}{u_0^2}$$

in equations (1) to (3), leads to

$$\frac{\partial u}{\partial t} = Gr\,\theta + \frac{\partial^2 u}{\partial y^2} \qquad (5)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \qquad (6)$$

The initial and boundary conditions in a non-dimensional form are

u = 0, q = 0 for all y, t £ 0

t > 0: u = exp (at) $\frac{\partial \theta}{\partial y} = -1$ at y = 0 (7)

u ® 0, q ® 0 for all y ® ¥

The dimensionless governing equations (5) and (6), subject to the boundary conditions (7), are solved by the usual Laplace-transform technique and the solutions are derived as follows :

$$\theta = 2\sqrt{t}\left[\frac{\exp(-\eta^2 a)}{\sqrt{\pi}} - \eta\sqrt{a}\,\mathrm{erfc}(\eta\sqrt{a})\right] \qquad (8)$$

$$u = \frac{e^{at}}{2}\left[e^{-2\eta\sqrt{at}}\,\mathrm{erfc}(\eta - \sqrt{at}) + e^{2\eta\sqrt{at}}\,\mathrm{erfc}(\eta + \sqrt{at})\right] + \frac{Gr\,t}{(\Pr - 1)}\left[\begin{array}{l}(1+2\eta^2)\,\mathrm{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2) \\ -(1+2\eta^2\Pr)\,\mathrm{erfc}(\eta\sqrt{\Pr}) + \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2\Pr)\end{array}\right] \qquad (9)$$

where $\eta = \frac{y}{2\sqrt{t}}$

3. Results and Discussion

For physical understanding of the problem numerical computations are carried out for different parameters a, Gr and t upon the nature of the flow and transport. The values of Prandtl number Pr are chosen such that they represent air (Pr=0.71) and water (Pr=7.0). The numerical values of the velocity and temperature are computed for different physical parameters like a, Prandtl number, thermal Grashof number and time.

The variations of velocity profiles for different values of time (t = 0.2, 0.4, 0.6), a=0.5, Gr=5 and Pr = 0.71 are shown in Fig.1. It is clear that the velocity increases with increasing values of the time.

The effects of velocity for different values of (a = 0.2, 0.5, 0.8), Gr=5, Pr = 0.71 at t=0.2 are studied and presented in Fig.2. It is observed that the velocity increases with increasing values of a.

The velocity profiles are shown for different thermal Grashof number (Gr = 2, 5, 10) are shown in the Fig.3. It is observed that velocity increases with increasing values of Gr.

The temperature profiles are calculated for different values of the Prandtl number Pr = 0.71 and Pr=7.0 are shown in Fig.4. It is observed that temperature increases in the presence of air than in water.

From the velocity field, the effect of heat transfer on the skin-friction is studied and is given in dimensionless form as

$$\tau = -\left(\frac{du}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{du}{d\eta}\right)_{\eta=0} \qquad (10)$$

$$\tau = \frac{1}{\sqrt{\pi t}}\left[e^{at}\left(1 + \sqrt{\pi a t}\,\mathrm{erf}\sqrt{at}\right) - \frac{Gr\,t}{(\sqrt{\Pr}+1)}\right]$$

Hence, from the equations (9) and (10), wall shear stress is as follows.

The numerical values of t are presented in Table 1. It is observed that from this Table, skin-friction increases with increasing values of the accelerating parameter. and time. It is also observed that the skin-friction decreases with increasing Grashof number.

Table 1: Values of the non-dimensional skin-friction

Gr	a	t	Pr=0.71	Pr=7.0
2	2	0.2	2.2366	2.3720
5	2	0.2	1.8258	2.1644
10	2	0.2	1.1411	1.8183
2	2	0.4	3.6885	3.3880
2	0	0.2	-0.2738	-0.1384
2	5	0.2	7.7381	7.8736

4. Conclusions

The theoretical solution of flow past an exponentially accelerated infinite vertical plate with uniform heat flux is considered. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal Grashof number, a and t are studied graphically. It is observed that the velocity increases with increasing values of Gr, a and t.

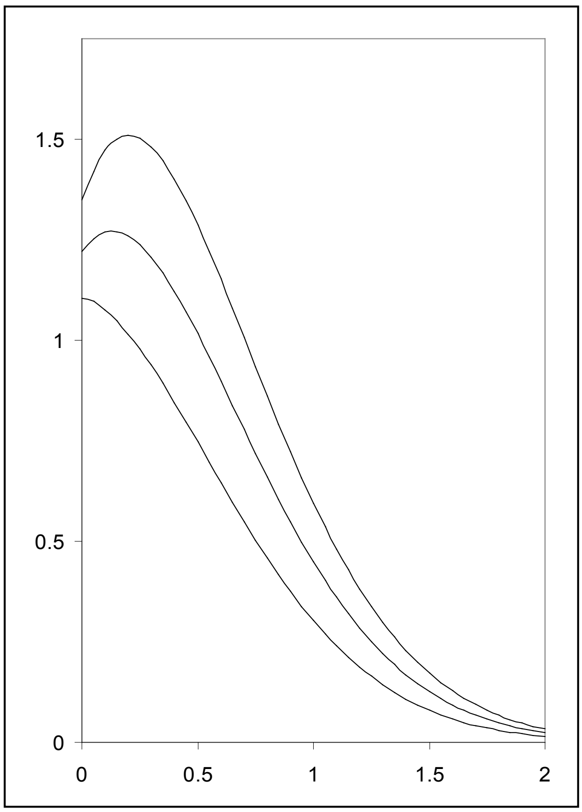

Figure 1: Velocity profiles for different t

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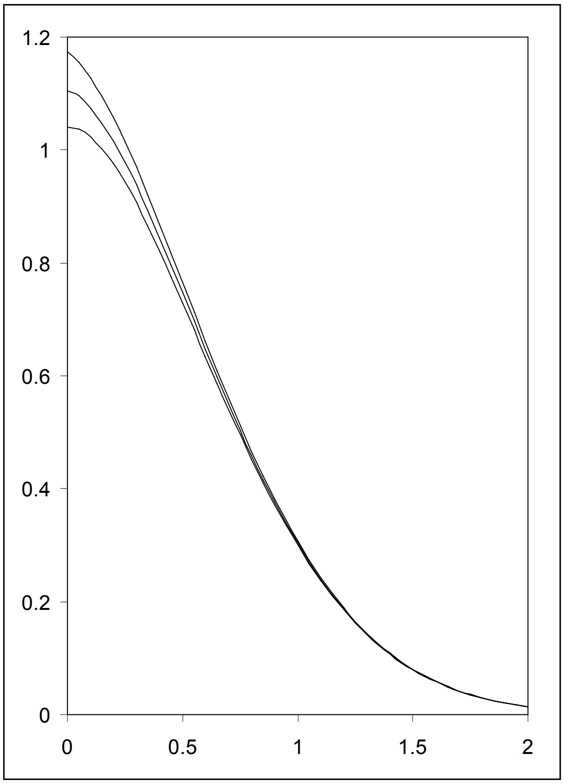

Figure 2: Velocity profiles for different a

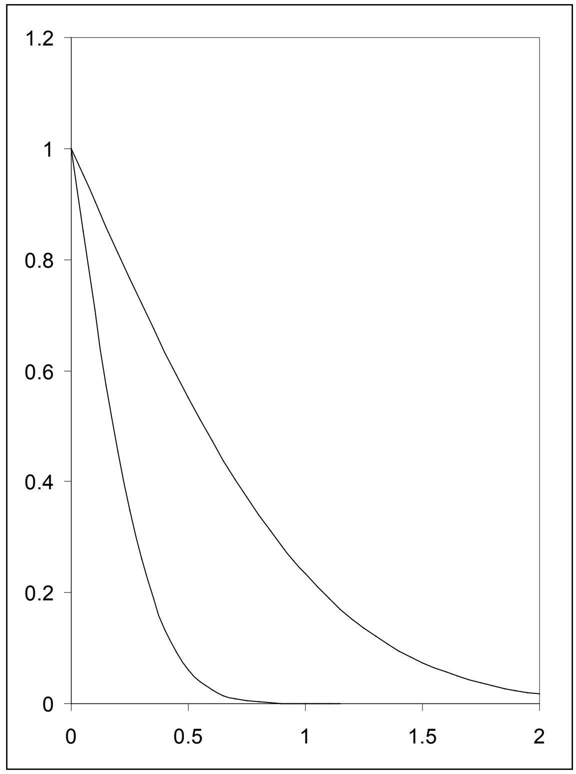

Figure 4: Temperature profiles for different Pr

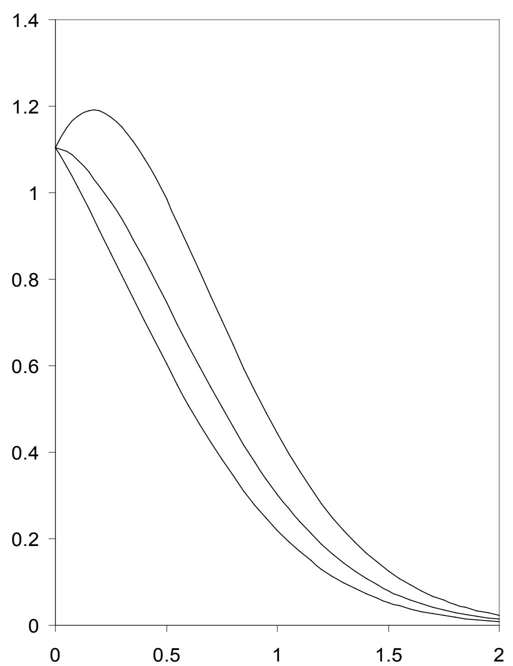

Figure 3: Velocity profiles for different Gr

REFERENCE

[1] B.C SAKIADIS, Boundary layer | behaviour on continuous solid surfaces, | AICHE Journal, 7 (1961), 26-28. | | [2] B.C SAKIADIS, Boundary layer | behaviour on continuous solid surfaces | II, AICHE Journal, 7 (1961), 221-225. | | [3] V.M. SOUNDALGEKAR, Free | convection effects on stokes problem | for an infinite vertical plate, Journal of | Heat transfer, 99 (1977), 449-501. | | [4] A.K.SINGH and NAVEEN KUMAR, | Free convection flow past an | exponentially accelerated vertical plate, | Astrophysics and Space science, 98 | (1984), 245-258. | | [5] M.A. HOSSIAN and L.K. SHAYO, | The Skin friction in the unsteady free | convection flow past an accelerated | plate, Astrophysics and Space science, | 125 (1986), 315-324. | | | |