

Computational Method Based on Distribution in Fuzzy Time Series Forecasting



Statistics

KEYWORDS : Fuzzy set, Fuzzy Time Series, Root Mean Square Prediction Error

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ABSTRACT

In the past decades many forecasting methods has been developed for fuzzy time series. Fuzzy time series is a dynamic method for forecasting with linguistic values. It has been widely used for forecasting the time series data. This study mainly focuses on forecasting the terrorist victims and to enhance the forecasting accuracy based on statistical distribution. The performance of forecasting accuracy is evaluated by Root Mean Square Prediction Error with other existing methods.

INTRODUCTION

Fuzzy set theory is a generalization of the set theory. It is an extension of standard crisp set theory. It can be defined as a mathematical formalism that enables us to eliminate indefiniteness and deal with incomplete, inaccurate information of both qualitative and quantitative by nature. The fuzzy set theory, advanced by L.Zadeh (1965) one of the well known representatives of modern applied mathematics is by excluding any definite description of the task offers such as solution scheme of the problem that a subjective reasoning and evaluation plays a principal role in evaluating indefinite, unclear fact. The use of fuzzy verbal notions in every day speech (much, more, little, small, many, a number of etc) enable us to give a qualitative description of the problem which must be tackled and take account of its indefinite nature as well as attain the description of the factors that cannot be described qualitatively.

Forecasting is an important role in our daily life such as economic, stock index, population growth, weather, etc to make a forecast with 100% accuracy may be impossible, but we can try our best to reduce the forecasting errors. Fuzzy set theory was originally developed to handle problems in the existing statistical time series methods could not effectively analyze time series with small amount of data and historical data. Song and Chissom (1993, 1996) proposed a first order time invariant model and time variant model of fuzzy time series. In the recent years, some methods have been developed on fuzzy time series to make prediction in many research areas, such as university enrollments, weather forecast, stock price forecast, staff scheduling, business and production planning and multi stage management decision analysis, terrorism forecasting etc. Fuzzy set theory is a generalization of the set theory. It is an extension of standard crisp set theory. It can be defined as a mathematical formalism that enables us to eliminate indefiniteness and deal with incomplete, inaccurate information of both qualitative and quantitative by nature. The fuzzy set theory was developed by Zadeh (1965) one of the well known representatives of modern applied mathematics is by excluding any definite description of the task offers such as solution scheme of the problem that a subjective reasoning and evaluation plays a principal role in evaluating indefinite, unclear fact. The use of fuzzy set in every day speech (much, more, little, small, many, a number of etc) enable us to give a qualitative description of the problem which must be tackled and take account of its indefinite nature as well as attain the description of the factors that cannot be described qualitatively.

The rest of the paper is organized into six sections. The first section is introduction about fuzzy time series, second section is discuss the related works, third section is basic concepts of fuzzy time series, section four is description of the proposed method, section five is computational results and discussion,

and then the final section is conclusion.

RELATED WORKS

Some of the researchers has been presented in fuzzy time series to forecasting the university enrollment and enhanced the forecasting accuracy to make a forecast with 100% accuracy may be impossible, but we can try our best to reduce the forecasting errors. Fuzzy set theory was originally developed to handle problems in the existing statistical time series methods could not effectively analyze time series with small amount of data and historical data. Song and Chissom (1993, 1996) was firstly introduced the concept of fuzzy time series based on the historical enrolments of University of Alabama. Song and Chissom (1994) presented the time invariant fuzzy time series model and time variant fuzzy time series model based on the fuzzy set theory of enrolment data. Song et al.,(1994,1996,2003) were existing some forecasting methods. Huarng (2001) pointed out that the length of intervals affect forecasting results in fuzzy time series, and he had proposed the distribution based length method and the average based length method for handling the forecasting problems. He proposed the Length of intervals affects forecasting results in fuzzy time series using distribution- and average-based length approaches. Chen et al., (1996, 2004) used the higher order fuzzy time series model to forecast the enrollments of the University of Alabama and temperature prediction.

Egrioglu (2009) proposed a hybrid approach in order to analyze seasonal fuzzy time series. It is based on partial high order bivariate fuzzy time series forecasting. The order of this model is determined by utilizing Box-Jenkins method. Li et al., (2012) proposed a new computational intelligence approach to the problem of time series forecasting, using a neuro-fuzzy system (NFS) with auto-regressive integrated moving average (ARIMA) models and a novel hybrid learning method. Song (2003) proposed sample autocorrelation functions on fuzzy time series for model selection. This idea is to select a number of different data sets from each fuzzy set and calculate the sample autocorrelation function for each data set. Singh (2007) proposed an improved and versatile method of forecasting based on the concept fuzzy time series forecasting. The developed model has been presented in a form of simple computational algorithms. It utilizes various difference parameters being implemented on current state for forecasting the next state values to accommodate the possible vagueness in the data in a better way and making it a robust method. Huarng (2005) proposed a Type 2 fuzzy time series model. Type 2 model, extra observations are used to enrich or to refine the fuzzy relationships obtained from Type 1 models and then to improve forecasting performance.

Cheng (2008) proposed a new fuzzy time-series model which incorporates the adaptive expectation model into forecasting

processes to modify forecasting errors. Liu et al., (2010) proposed an improved fuzzy time series forecasting method that can effectively deal with seasonal time series. This method can determine appropriate length interval. Moreover, a systematic search algorithm is used to find the best window base. It can be provide decision analysts with more precise forecasted values. Liu (2009) developed an integrated fuzzy time series forecasting system in which the forecasted value will be a trapezoidal fuzzy number instead of a single-point value. Furthermore, this system can effectively deal with stationary, trend, and seasonal time series and increase the forecasting accuracy. Wong et al.,(2010) proposed Traditional Time Series Method (ARIMA model and Vector ARMA model) and Fuzzy Time Series Method (Two-factor model, Heuristic model, and Markov model) for the forecasting problem. Reuter et al., (2010) presented an artificial neural network for modeling and forecasting of fuzzy time series. Analysis and forecasting of time series with fuzzy data may be carried out with the aid of artificial neural networks. Hwang et al. (2001) presented a method for forecast the enrollment of the University of Alabama. Jilani et al.,(2008) developed a simple time Quantile based time series forecasting method. (Melike et al. (2005) presented the first order fuzzy time series to predict the wheat production. Shy-Ming Chen et al., (2004, 2006) presented first order and time method for fuzzy time series. Hao-Tien Liu (2007) presented an improved fuzzy time series forecasting method to forecast using trapezoidal membership function. Sullivan et al. proposed the Morkov model, which is a used linguistic label with probability distribution to forecast the enrollments. Li-Wei Lee et al. (2006) proposed the two-factor high-order fuzzy time series to predict temperature and the Taiwan Futures Exchange (TAIFEX). Narges Shafaei Bajestani et al., (2011) proposed high-order type2 fuzzy time series model to improve the forecasting performance. Tiffany Hui-Kuang Yu et al.,(2008) introduced Bivariate time series model to reduce the forecasting error. Wangren Qiu et al., (2011) used Ensemble technique to improve the forecasting accuracy based on fuzzy time series model. Aladag et al. (2008) presented the feed forward neural networks to define fuzzy relations in higher order fuzzy time series. Chi chen wang (2011) compared the two forecasting method on ARIMA time series method and fuzzy time series method based on the amount of Taiwan export. Young Keun Bang et al. (2011) presented the hierarchical classification technique consists of two levels, called upper clusters and lower fuzzy sets, using the cross-correlation clustering algorithm combined with the k-means clustering algorithm, and it was able to improve classification accuracy.

FUZZY TIME SERIES

Song and Chissom defined their fuzzy time series by mean of discrete fuzzy sets. The discrete fuzzy sets can be defined as follows: Let U be the universe of discourse, when $U = \{u_1, u_2, \dots, u_n\}$. A fuzzy set A_i of U is defined by

$$\tilde{A}_i = \mu_{\tilde{A}_i}(\mu_1)/u_1 + \mu_{\tilde{A}_i}(\mu_2)/u_2 + \dots + \mu_{\tilde{A}_i}(\mu_n)/u_n$$

Where $\mu_{\tilde{A}_i}$ is the membership function of \tilde{A}_i ,

$$\mu_{\tilde{A}_i} : U \rightarrow [0, 1]. \mu_{\tilde{A}_i}(u_i)$$

denotes the membership

value of u_i in $\tilde{A}_i, \mu_{\tilde{A}_i}(u_i)$

Definition 1:

Let $Y(t), (t = \dots, 0, 1, 2, \dots)$ is subset of R. let $Y(t)$ be the universe of discourse defined by the fuzzy set $\mu_i(t)$. if $F(t)$ consists of $\mu_i(t)$ ($i=1,2,3,\dots$), $F(t)$ is called a fuzzy time series on $Y(t)$.

Definition 2:

If there exists a fuzzy relationship $R(t-1,t)$, such that $F(t) = F(t-1) \circ R(t-1,t)$, where \circ is an arithmetic operator, then $F(t)$ is said to be caused by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ can be denoted by $F(t-1) \rightarrow F(t)$.

Definition 3:

Suppose $F(t)$ is calculated by $F(t-1)$ only, and $F(t) = F(t-1) \circ R(t-1, t)$. For any t , if $R(t-1, t)$ is independent of t , then $F(t)$ is considered a time invariant fuzzy time series. Otherwise, $F(t)$ is time – variant.

Definition 4:

Suppose $F(t-1) = A_i$ and $F(t) = A_j$, a fuzzy logical relationship can be defined as $A_i \rightarrow A_j$. Where A_i and A_j are called the left-hand side and the right hand side of the fuzzy logical relationship, respectively.

PROPOSED MODEL FOR FUZZY TIME SERIES

The proposed method can be described as follows:

Step 1: Define the universe of discourse U. find the maximum D_{max} and the minimum D_{min} from the historical data. The universe of discourse is defined as

$$U = [D_{min} - t_\alpha(n) \frac{s}{\sqrt{n}}, D_{max} + t_\alpha(n) \frac{s}{\sqrt{n}}]$$

Where $t_\alpha(n)$ is the 95% level of significance of t- distribution with n degrees of freedom.

Step 2: Determine the appropriate length of interval l. the length of interval l is computed by

(i) Calculate first difference between the A_{vt-1} and A_{vt} , the computed the average difference, shown as follows:

$$Avg_Diff = \frac{\sum_{i=1}^n (D_{i-1} - D_i)}{n - 1}$$

(ii) Take one half of the average difference as the length.

Step 3: Define the fuzzy number.

Step 4: Establish the fuzzy logical relationship groups.

Step 5: Calculated the forecasted outputs. The forecasted values are calculated by the following rules:

Rule 1: If the fuzzy logical relationship group of A_i is empty i.e, $A_i \rightarrow \phi$, then the value of forecast is middle value (m_i) of the interval u_i .

Rule 2: If the fuzzy logical relationship group of A_i is one- to-one, i.e., $A_i \rightarrow A_j$, then the value of forecast is middle value (m_j) of the interval u_j .

Rule 3: If the fuzzy logical relationship group of A_i is one-to-many, i.e., $A_i \rightarrow A_{j1}, A_{j2}, \dots, A_{jn}$. Then the value of forecast is the average of $m_{j1}, m_{j2}, \dots, m_{jn}$, the middle values of $u_{j1}, u_{j2}, \dots, u_{jn}$, respectively.

Step 6: Calculate the forecasting error using Root Mean Square Prediction Error (RMSPE).

$$RMSPE = \sqrt{\frac{1}{n} \sum_{t=1}^n \left(\frac{forecasted\ value - Actual\ Value}{Actual\ Value} \right)^2}$$

COMPUTATIONAL RESULTS

Step 1: Define the universe of discourse and intervals. The Universe of discourse U is defined as follows:

$$U = [D_{min} - t_\alpha(n) \frac{s}{\sqrt{n}}, D_{max} + t_\alpha(n) \frac{s}{\sqrt{n}}]$$

where $D_{min} = 13055$ and $D_{max} = 19337$ is the minimum and maximum value of historical data set, $t_{0.05}(22) = 2.07, s = 1816.49$.

The Universe of discourse U is

$$U = [D_{\min} - t_{\alpha}(n) \frac{s}{\sqrt{n}}, D_{\max} + t_{\alpha}(n) \frac{s}{\sqrt{n}}]$$

$$= \left[13055 - 2.07 \left(\frac{1816.49}{4.6904} \right), 19337 + 2.07 \left(\frac{1816.49}{4.6904} \right) \right]$$

$$U = [12253.33, 20138.67]$$

Step 2: Define fuzzy numbers. The number of interval can be computed by

$$\text{Fuzzy number} = \frac{20138.67 - 12253.33}{300} = 26$$

Thus there are 21 intervals, which are $u_1=[12253.33, 12553.33]$, $u_2 = [12553.33, 12853.33]$, $u_3 = [12853.33, 13153.33]$,..., $u_{26}=[20053.33, 20353.33]$.

Step 3: Fuzzify the enrollments of University of Alabama. For example the enrollment in the year 1971 is 13055, which is located at the range of $u_1 = [13753.33, 14053.33]$. Thus corresponding fuzzy number of year 1971 is assigned as A_{i_1} , table 1 shows the fuzzy number of the enrollments.

Step 4: To define the fuzzy logical relationship groups

Step 5: Calculated the forecasted outputs.

Table 1: Forecasted value of enrollments

Year	Enrollments (A_{v_i})	Fuzzy Set	Chen Method	Lee and Chou Method	Proposed Method
1971	13055	-	-	-	-
1972	13563	A_4	14000	14025	13739
1973	13867	A_5	14000	14568	14639
1974	14696	A_7	15500	14568	15539
1975	15460	A_{10}	16000	15654	15557
1976	15311	A_9	16000	15654	15539
1977	15603	A_{10}	16000	15654	15557
1978	15861	A_{11}	16000	15654	16739
1979	16807	A_{14}	16833	16197	17639
1980	16919	A_{15}	16833	17283	16439
1981	16388	A_{13}	16833	17283	15539
1982	15433	A_{10}	16833	16197	15557
1983	15497	A_{10}	16000	15654	15557
1984	15145	A_9	16000	15654	15539
1985	15163	A_9	16000	15654	15539
1986	15984	A_{11}	16000	15654	16739
1987	16859	A_{14}	16000	16197	17639
1988	18150	A_{19}	16833	17283	19139
1989	18970	A_{21}	19000	18369	19439
1990	19328	A_{22}	19000	19454	19289
1991	19337	A_{22}	19000	19454	19289
1992	18876	A_{21}	-	-	19439
RMSPE			4.0516 %	3.1113%	0.03609 %

Step 6: Calculate the forecasting error using Root Mean Square Prediction Error (RMSPE).

$$RMSPE = 0.03609 \%$$

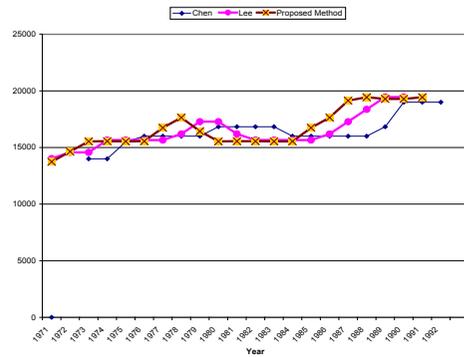


Figure 1: Actual and forecasted value of enrollment data set

CASE STUDY ON TERRORIST ATTACKS DATA IN INDIA

This information contain the victims of terrorist attacks in India during the period of 1994 to 2012 is represented in table 2. The victim of terrorist attack is predicted by fuzzy time series method, and then compared the forecasting accuracy with other methods. The forecasting steps are following as:

Step 1: define the universe of discourse and intervals. The Universe of discourse U is defined as follows:

$$U = [D_{\min} - t_{\alpha}(n) \frac{s}{\sqrt{n}}, D_{\max} + t_{\alpha}(n) \frac{s}{\sqrt{n}}]$$

$$= \left[760 - 2.0 \left(\frac{1276.8}{4.8} \right), 5839 + 2.0 \left(\frac{1276.8}{4.8} \right) \right]$$

$$U = [148, 6451]$$

where $A_{\min} = 760$, $A_{\max} = 5839$ are the minimum and maximum value of victims data. $t_{0.05}(19) = 2.09$, $s = 1276.38$. The Universe of discourse U is

Step 2: Define fuzzy numbers.

$$\text{fuzzy numbers} = \frac{6451 - 148}{200} = 3$$

There are 31 intervals, which are $u_1 = [148, 348]$, $u_2 = [348, 548]$, $u_3 = [548, 748]$, ..., $u_{26} = [6148, 6348]$, $u_{32} = [6348, 6548]$.

Step 3: Fuzzify the victims of terrorist attacks.

Step 4: To define the fuzzy logical relationship groups

Step 5: Calculated the forecasted outputs.

Table 2: Fuzzy forecasted values

Year	Victims (A_j)	Fuzzy set	Forecasted
1994	4032	-	-
1995	3875	A_{19}	4381.33
1996	4181	A_{21}	4048
1997	4115	A_{20}	3748
1998	3764	A_{19}	4381.33
1999	3754	A_{19}	4381.33
2000	4975	A_{25}	5848
2001	5839	A_{29}	4048
2002	3973	A_{20}	3748

2003	3702	A ₁₈	2648
2004	2642	A ₁₃	2448
2005	2519	A ₁₂	2848
2006	2775	A ₁₄	2648
2007	2615	A ₁₃	2448
2008	2613	A ₁₃	2448
2009	2231	A ₁₁	1848
2010	1880	A ₉	1648
2011	1073	A ₈	848
2012	760	A ₄	848

CONCLUSION

In this paper we have proposed distribution based fuzzy time series model for forecasting the victims of terrorist attacks and enhances the forecasting accuracy. The forecasting accuracy is measured by Root Mean Square Prediction Error (RMSPE). The RMSPE of our method is 0.03609 %, which is better than Chen method (4.0516%) and Lee method (3.1113 %). Table 10 shows that the forecasted value of terrorist victims is slowly decreased for every year are concluded.

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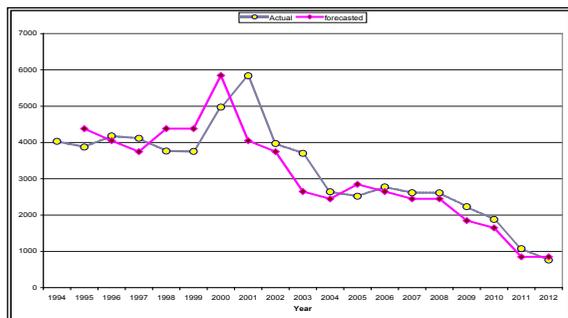


Figure 2: Actual and Forecast values

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