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Research Paper

Transfer Function Modeling for Global Warming

Statistics

KEYWORDS : impulse response weights, transfer function, ARIMA, global warming, forecasting.

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ABSTRACT

Time series analysis is a powerful tool to determine dynamic models aiming at defining and controlling most relevant variables of a system. Transfer Function model is one of the popular techniques in the time series modeling for forecasting. When there is an output series which is influenced by an input series, the objective of the Transfer Function modeling is to identify the role of input series in determining the variable of interest. In this paper, the Transfer Function model is fitted to the Global Warming entities such as atmospheric temperature and atmospheric CO2 emissions. The Transfer Function model has been used to identify a model and estimate parameters for forecasting of atmospheric temperature.

1. INTRODUCTION

In forecasting and analysis of time series data, it is well demonstrated that Autoregressive Integrated Moving Average (ARIMA), intervention, and transfer function models are very effective in handling practical applications. Modeling and forecasting of multi-variable time series is to employ transfer function models. Transfer function models according to Box and Jenkins (1976) can be regarded as extensions of classical regression models, and are useful in many applications.

Univariate model uses a single dependent or output variable as a function of its own history and previous errors. Transfer function model, on the other hand, may have single or multiple inputs that may possibly affect the system. The dynamic characteristics of a system are fully understood explicitly only through a transfer function model. The dynamic nature of the transfer function relationship lies in its ability to account for the instantaneous and lagged effects of an input variable on the output variable.

Naill and Momani (2009) used the Box-Jenkins methodology to build ARIMA model for monthly rainfall data for long term series and recently Chen et al., (2009) have applied seasonal ARIMA model for forecasting of inbound air travel arrivals to Taiwan. Nogales et al., (2006) have proposed transfer function model to predict electricity prices based on both past electricity prices and demands and discussed the rationale to build it.

In this paper, the Transfer Function model is fitted to the atmospheric temperature which is mostly influenced by atmospheric CO_2 emissions. Atmospheric CO_2 emissions values (ppm - parts per million) are recorded by the Mauna Loa Observatory, Hawaii, USA. The data covers from January, 1974 to May, 2013 (473 months). The Mauna Loa atmospheric CO_2 measurements constitute the longest continuous record of atmospheric CO_2 concentrations available in the world. The global land-ocean atmospheric temperature indexes (in 0.01 degree celsius) are collected from NASA for 473 months (January, 1974 to May, 2013). The dynamic relationship between the atmospheric CO_2 emissions and the atmospheric temperature is studied through the Transfer Function model. Finally, the fitted Transfer Function model can also be used for forecasting.

2. TRANSFER FUNCTION MODEL

2.1. Single Input Transfer Function Models

Assume that x_t and y_t are both stationary time series. Here, the output series y_t and the input series x_t are related through a linear filter as

$$y_t = v(B)x_t + n_t$$

where $v(B) = \sum_{j=-\infty}^{\infty} v_j B^j$

According to Box, Jenkins and Reinsel (1994), v(B) is referred to as the transfer function of filter and the above said equation is called the transfer function model and n_t is the noise series of the system that is independent of the input series x_t. The coefficients v_j in the transfer function model are often called the impulse response weights or impulse response function and it does not depend on time. The TFM is said to be causal if $v_j = 0$, for $j<0$ (ie) the system does not respond to input series until they have been actually applied to the system. It is said to be stable if the sequence of these impulse response weight is absolutely summable.

The purpose of transfer function modeling are to identify and estimate the transfer function v(B) and the noise model for n_t based on the available information of the input series x_t and the output series y_t .

The rational form of the transfer function v(B) is due to alleviate the difficulties which are

$$v(B) = \frac{\omega(B)B^b}{\delta(B)}$$

that the information on x_t and y_t is finite and the transfer function v(B) may contain an infinite number of coefficients.

where $\omega(B) = \omega_o - \omega_1 B - \ldots - \omega_s B^s$

$$\delta(B) = 1 - \delta_1 B - \ldots - \delta_r B^r$$

b - delay parameter representing the actual time lag that elapses before the impulse of the input variable produces an effect on the output variable.

r -decay parameter which represents the patterned changes in the slope of the function.

s -the order of the regression designates the number of the lags for unpatterned spikes in the transfer function.

The order of s, r and b and their relationship to impulse response weights v_j is defined by

$$\delta(B)v(B) = \omega(B)B^b$$

(or)

$$v_j - \delta_1 v_{j-1} - \delta_2 v_{j-2} - \ldots - \delta_r v_{j-r} = \begin{cases} -\omega_{j-b}, & j=b+1\ldots b+s \\ 0, & j>b+s \end{cases}$$

with $v_b = \omega_0$ and $v_j = 0$ for $j<b$.

2.2. The Cross-Correlation Function

CCF is a useful measure of strength and direction of correlation between two random variables. The Cross-Correlation function

(CCF) is defined as

$$\rho_{xy}(k) = \frac{\gamma_{xy}(k)}{\sigma_x \sigma_y} \qquad \text{for } k = 0, \pm 1, \pm 2, \ldots$$

where σ_x and σ_y are the standard deviations x_t and y_t.

2.3. Box - Jenkins Methodology

The main stages of the Box-Jenkins methodology for analyzing and modeling time series include model identification, model parameters estimation and diagnostic checking for the appropriate model. The first step of the process is model identification in which the data is examined to check for the most appropriate class of ARIMA processes through selecting the order of the consecutive and seasonal differencing required to make series stationary as well as specifying the order of the regular and seasonal autoregressive and moving average polynomials necessary to adequately represent the time series model.

The general form of ARIMA(p,d,q)(P,D,Q)$_s$ model describing the current value y_t of a time series by its own past is:

$$\varphi_p(B)\varphi_P(B^s)(1-B)^d(1-B^s)^D y_t = \theta_q(B)\Theta_Q(B^s)e_t$$

where

$$\varphi_p(B) = 1-\varphi_1 B-\varphi_2 B^2-\ldots-\varphi_p B^p$$

$$\phi_P(B) = 1-\phi_1 B^s-\phi_2 B^{2s}-\ldots-\phi_P B^{Ps}$$

y_t = the current value of the time series examined

B = the backward shift operator

$BY_t = Y_{t-1}$ and $B^{12}Y_t = Y_{t-12}$

$(1-B)^d$ = d^{th} order nonseasonal difference

$(1-B^s)^D$ =seasonal difference of order D

$$\theta_q(B) = 1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q$$

$$\Theta_Q(B) = 1-\Theta_1 B^s-\Theta_2 B^{2s}-\ldots-\Theta_Q B^{Qs}$$

The next part of this step is to identify the values of p and q, which are the AR(p) and MA(q) components for both seasonal and non-seasonal series using autocorrelation function (ACF) and partial ACF (PACF). For model selection, there is one goodness of fit statistic which is Schwarz Bayesian Information Criterion (BIC). The model one whose BIC value is minimum is to be considered as a suitable model. After choosing the most appropriate model, the model parameters are estimated by using the maximum likelihood estimation. In diagnostic checking, the residuals from the fitted model shall be examined against adequacy. This is usually done by correlation analysis through the residual ACF plots and the goodness-of-fit test by means of Chi-square statistics χ^2. If the residuals are correlated, then the model should be refined by choosing the appropriate model as said above. Otherwise, the autocorrelations are white noise and the model is adequate to represent our time series.

2.4. Identification of Transfer Function Models

2.4.1. Method of prewhitening

This filter is an inverse transformation which turns the input series into white noise. Suppose that x_t, follows a seasonal ARIMA model which is fitted by using Box-Jenkins methodology described in section 2.3. The fitted seasonal ARIMA model for x_t is as follows

$$\varphi_p(B)\varphi_P(B^s)(1-B)^d(1-B^s)^D x_t = \theta_q(B)\Theta_Q(B^s)\alpha_t$$

where α_t is white noise with variance σ^2_α then

$$\alpha_t = \theta_q(B)^{-1}\Theta_Q(B^s)^{-1}\varphi_p(B)\varphi_P(B^s)(1-B)^d(1-B^s)^D x_t$$

The filtered output series y_t using above prewhitening model to generate the series

$$\beta_t = \theta_q(B)^{-1}\Theta_Q(B^s)^{-1}\varphi_p(B)\varphi_P(B^s)(1-B)^d(1-B^s)^D y_t$$

The sample Cross–correlation Function, $\hat{\rho}_{\alpha\beta}(k)$ between α_t and β_t to estimate v_k

$$\hat{v}_k = \frac{\hat{\sigma}_\beta}{\hat{\sigma}_\alpha}\hat{\rho}_{\alpha\beta}(k)$$

This significance of the CCF and its equivalent $\hat{v}_k$ can be tested by comparing it with its standard error $(n-k)^{\frac{-1}{2}}$.

2.4.2. Specification of the order r and s

Identify b, $\delta(B) = (1-\delta_1 B-\ldots-\delta_r B^r)$ and

$\omega(B) = (\omega_0-\omega_1 B-\ldots-\omega_s B^s)$ by matching the pattern of $\hat{v}_k$ with the known theoretical pattern of the $\hat{v}_k$. Once b, r and s are chosen preliminary estimate $\hat{\omega}_j$ and δ_j can be found from their relationship with $\hat{v}_k$ as shown in equation.

The preliminary estimate of the transfer function v(B) is defined as

$$\hat{v}(B) = \frac{\hat{\omega}(B)}{\hat{\delta}(B)}B^b$$

2.4.3. Model the Noise

The estimated noise can be obtained as

$$\hat{n}_t = y_t - \frac{\hat{\omega}(B)}{\hat{\delta}(B)}x_{t-b}$$

To model the estimated noise, observe its ACF and PACF and determine the order of the ARIMA model

$$\varphi_p(B)(1-B)^d n_t = \theta_q(B)\varepsilon_t$$

2.4.4. Overall Model Fitting

Steps 2.4.1 through 2.4.3 provide the transfer noise model

$$y_t = \frac{\omega(B)}{\delta(B)}x_{t-b} + \frac{\theta_q(B)}{\varphi_p(B)(1-B)^d}\varepsilon_t$$

Model selection criteria such as AIC and BIC can be used to pick the 'best' model among competing models.

2.5. Estimation, Diagnostic checking and Forecasting

The estimation of the parameter can be obtained by using classical approach like ordinary least square or Maximum likelihood estimation.

After the model has been identified and its parameter estimated it is necessary to check the model adequacy before forecasting. The assumption is that the noise ε_t is white noise require the examination of the residuals $\hat{\varepsilon}_t$ through ACF and PACF which should not show any patterns. If the cross-correlation function between $\hat{\varepsilon}_t$ and $\hat{x}_t$ should show no patterns and lie within their two standard errors $2/\sqrt{(n-k)}$. If the cross-correlation test fails there is no need to check the autocorrelation for $\hat{\varepsilon}_t$. One should reidentify the transfer function v(B) and repeat the iterative model building steps until a satisfactory model is obtained.

Once a transfer function model is found to be adequate it can be used to improve the forecast of the output series y_t by using the past history of both the output series y_t and the associate input series x_t. That is particularly true if the input series is a leading indicator.

3. RESULTS AND DISCUSSIONS

A strong relationship has been identified between atmospheric temperature and present and past atmospheric CO_2 emissions. Such lagged atmospheric CO_2 emissions effects on atmospheric temperature may result delayed response. A transfer function model using atmospheric temperature, y_t (as an output series) and atmospheric CO_2 emissions, x_t (as an input series) which are known as global warming entities was taken here to model the global warming.

In order to get the input and output series as prewhitening, it can be modeled as seasonal ARIMA $(1,1,1)(0,1,1)_{12}$ model. The prewhitened input series is

$$\alpha_t = \frac{(1+0.174B)(1-B)(1-B^{12})}{(1-0.557B)(1-0.932B^{12})} x_t$$

The filtered output series is obtained as

$$\beta_t = \frac{(1-0.006B)(1-B)(1-B^{12})}{(1-0.561B)(1-0.916B^{12})} y_t$$

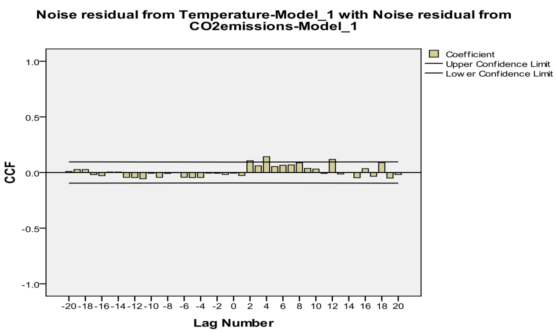

Figure 3.1: Sample Cross-Correlation Function of prewhitened input series and filtered output series

Based on sample CCF from Figure 3.1, it suggests a transfer function with the orders b=2, r=2 and s=2 as

$$v(B)x_t = \frac{(\omega_0 - \omega_1 B - \omega_2 B^2)B^2}{(1-\delta_1 B - \delta_2 B^2)} x_t$$

Once a tentative transfer function is obtained, the estimated noise series is modeled as AR(1) which is given by

$$(1-\varphi_1 B)n_t = \varepsilon_t$$

From Table 3.1, the fitted model becomes

$$y_t = \frac{(0.089B^2 - 2.707B^2 - 3.422B^4)}{(1+0.592B + 0.782B^2)} x_{t-2} + \frac{1}{(1+0.002B)} \varepsilon_t$$

Final adoption of a proposed model requires study of the residuals which does not show any pattern from Figure 3.2. Specifically, one has to check whether $\acute{a}_t$ are indeed white noises and are independent of the input series x_t and hence independent of the prewhitened input series. Figure 3.3 illustrates the CCF between noise residual of atmospheric temperature (ε_t) and the atmospheric CO_2 emissions (x_t) which does not reveal any pattern that is the series are independent. Using the fitted model, it can be used to obtain the forecast of the atmospheric temperature by using the past values of both atmospheric CO_2 emissions and atmospheric temperature.

Table 3.1: Model parameters of Transfer Function Model for atmospheric temperature

Transfer Function Model parameters					
		Estimate	SE	t	sig
AR	Lag 1	0.006	0.085	0.065	0.948
MA	Lag 1	0.561	0.071	7.926	0.000
MA, Seasonal	Lag 1	0.916	0.030	30.107	0.000
Numerator	Lag 0	0.089	1.814	0.049	0.961
	Lag 1	2.707	1.725	1.569	0.117
	Lag 2	3.422	2.061	1.661	0.097
Denominator	Lag 1	-0.592	0.142	-4.174	0.000
	Lag 2	-0.782	0.133	-5.886	0.000
AR , Noise	Lag 1	-0.002	0.047	-0.033	0.974

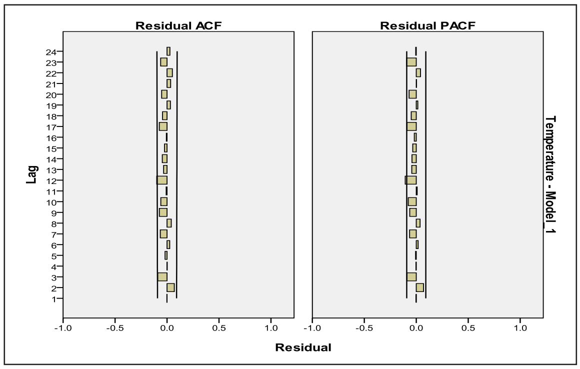

Figure 3.2: Residual ACF and PACF of proposed transfer function model

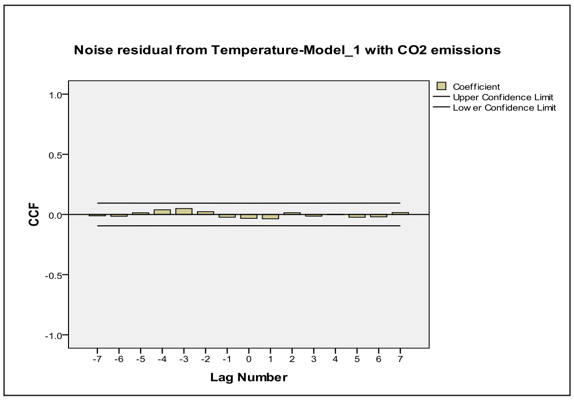

Figure 3.3: CCF of noise residual of atmospheric temperature and atmospheric CO2 emissions

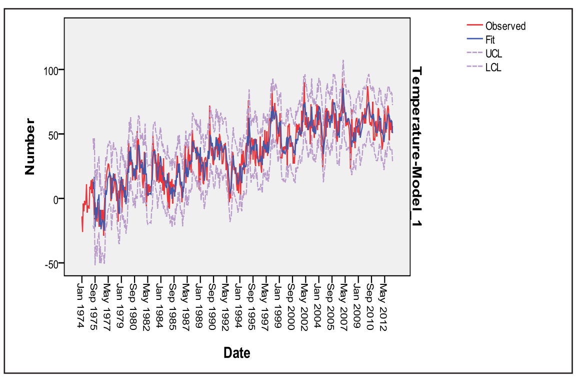

Figure 3.4: Forecasted graph of atmospheric temperature

4. CONCLUSIONS

In this paper, the identified transfer function model for atmospheric temperature is ARIMA $(1,1,1)(0,1,1)_{12}$ with the orders of b=2, s=2 and r=2 in the presence of atmospheric CO_2 emissions. This paper analyses exhaustively a transfer function model to forecast atmospheric temperature. A strong relationship has been demonstrated between current atmospheric CO_2 emissions and present and past atmospheric temperature. From a detailed analysis of the numerical results, it can be concluded that the quality of predictions using the proposed technique is considerably good when output series are influenced by input series.

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