Research Paper

Volume : 2 | Issue : 12 | December 2013 • ISSN No 2277 - 8179

Uncertainty modeling : A case study in cost-benefit analysis

Economics

KEYWORDS : Cost-Benefit analysis, Fuzzy set, Probability distribution, Uncertainty modeling

Tazid Ali	Department of Mathematics, Dibrugarh University, Assam, India
Rubab F. Nomani	Department of Economics, DHSK College, Dibrugarh, Assam India

ABSTRACT

Certainty is generally elusive in science. Models used to replicate real life problems are imperfect. Input parameters are usually tainted with uncertainty. So decisions about the future must be made in the absence of certainty. Ignoring uncertainty in scientific calculations often leads to underestimate or overestimation of results. Hence proper modeling of uncertainty is an important aspect in science. In this paper we have attempted to study the effect of uncertainty in cost-benefit analysis by modeling the input parameters as uncertain quantities. We have examined uncertainty propagation in a hypothetical case study of smoking ban.

1. Introduction:
In science there are two views regarding uncertainty – the traditional view and the modern view. The traditional view is that uncertainty is undesirable and should be avoided by all possible means. Science should strive for certainty in all its manifestations (precision, specificity, sharpness,, consistency, etc.) and hence uncertainty (imprecision, nonspecificity, vagueness, inconsistency, etc.) is regarded as unscientific. However the modern view is that uncertainty is an important factor and it should not be ignored or avoided. Ignoring uncertainty may lead to underestimation or overestimation of any result.

The perception of uncertainty may increase confidence in the validity of a measurement result. In real life situations, it is needed to identify and quantify uncertainty involved for reliable estimation of measurement result. Without acquiring knowledge of uncertainty of such a measure, it is impossible to judge the fitness of the value as a basis for making decisions relating to economics, social sciences, health, safety, commerce or scientific excellence. Uncertainty can be described as a portion of the measurement result beyond which we are not sure of its true value.

Uncertainty pertaining to the portion of a system can be modeled. Modeling is the work of making simple description of a system or a phenomenon of reality that can be used for the purpose of reproducing, simplifying, analyzing and understanding it. Uncertainty modeling is an important enhancement to study and investigate the maximum utility of systems models

When we go for modeling of uncertainty it is required to identify the factors which influence the system and regarding those parameters which are tainted with uncertainty . If there is a possibility of obtaining inaccurate result due to lack of knowledge or insufficient data, then uncertainty modeling is a way for making adequate predictions. For this we should study the availability of information regarding precision or imprecision, sharpness or vagueness, specific or nonspecific, consistence or inconsistence etc. associated with the parameters. If the input parameters of a model are uncertain the resultant parameter(s) of the model is also uncertain .i.e., uncertainty is inherent in a model consisting of uncertain parameters. The purpose of uncertainty modeling is to assign appropriate mathematical framework of real world information based on different kinds of uncertainties involved. In real life applications, uncertainties are present in different forms.

Uncertainties can be broadly classified into two categories:
Aleatory uncertainty: It is due to the randomness or variability in nature. This uncertainty cannot be reduced, so it is also called irreducible. Aleatory uncertainty is usually referred as variability in literature. Probability theory is usually preferred to study this kind of uncertainty.

Epistemic uncertainty: This uncertainty occurs due to lack of knowledge or information, unavailability of data, imprecision in measurement, vagueness in expression etc.. Further investigation or more information helps to reduce this type of uncertainty if not completely eliminate it. Such uncertainties can be handled by the analyst based on his intuition and empirical knowledge.

In a mathematical model if the input parameters are tainted with uncertainty then these uncertainties will be carried over to the output parameter. This is called uncertainty propagation. For this first the uncertain input parameters must be modeled. Modeling of uncertainty will depend on the nature of parameter and the availability of data. If a parameter exhibits randomness and sufficient information is available in this regard then the parameter is modeled by a probability distribution. If a parameter is deterministic but its exact value is not known, then it can be modeled by a fuzzy number (possibility distribution) [10]. There are other tools for modeling an uncertain parameter like p-box, body of evidence etc. when the gaps in our knowledge involve both aleatory and epistemic uncertainty. Once the uncertain input parameters are modeled, the uncertainty should be propagated to the output parameters. Again for this there are different methods available like Monte-Carlo, probability bounds analysis, Dempster-Shafer theory [5] etc. What is important in uncertainty propagation is to model the input parameters correctly and to choose the right method of propagation. In the next section we shall apply uncertainty modeling and propagation in a technique called cost-benefit analysis, which is widely used in economics as well as other fields.

2. Cost Benefit Analysis:
Cost benefit analysis (CBA) [13, 15] is a technique for assessing the **monetary social costs and benefits** of, say a capital investment project **over a given time period.** The profit of a project equals the difference between its benefits and costs. Costs and benefits accrue over time. To express this flow of cost and benefits into some simple measure two alternative criteria are followed: (i) Present value criterion, and (ii) internal rate of return criterion. In this paper we will deal with the first criterion. Individuals normally prefer to enjoy the benefits now rather than later – so the value of future benefits has to be discounted. The value of the project in terms of present consumption can be expressed as

$$PV = B_o - C_o + (B_1 - C_1)/(1 + v) + (B_2 - C_2)/(1 + v)^2 + \dots + (B_n - C_n)/(1 + v)^n$$

Where PV is the present value of the net benefits accruing from the project over its lifetime

B indicates benefits, C indicates costs, N is the project's economic life, and v is the time preference (discount rate) which is (usually) assumed to be uniform.

In Cost benefit Analysis, benefits are defined as increases in human well being (utility) and costs are defined as reductions in human wellbeing. For a policy or project to qualify on cost-benefit grounds, its social benefits must exceed its social costs. According to the present value criterion, all projects having positive values are to be undertaken as this would maximize the social welfare.

3. Uncertainty in cost benefit analysis:
Uncertainties arise at various points in cost benefit analysis. In many situations, the information regarding costs/benefits is not certainly known (imprecisely known or sufficient data may not be available). Future consequences of policies and projects are far from predictable (The expectation about the frequency and severity caused by earthquake on which analysis of earthquake disaster management are based may or may not be realized). Values can be assigned to outputs only with some, often unknown degrees of error. Discount rates can at best be imperfectly measured i.e., future level of discount rates is uncertain, short term and long term rates will differ in the capital market. Moreover intangible costs/benefits do not have market value. Their valuation is purely a subjective matter. Consequently, we often cannot plausibly assign a single present value to a project. Instead, we can assign a range of possible values, each corresponding to different assumptions about the discount rate and the values of inputs. Uncertainty does not render cost benefit analysis redundant; in fact, the information about the possible affects (present values) helps in policy and decision making under uncertainty environment.

In the case study that follows we have incorporated uncertainty in the input costs/benefits and illustrated the uncertainty propagation to the output.

4. A hypothetical case study:
We have attempted to study the uncertainties in CBA of a hypothetical case of national smoking ban for one year. We want to investigate and judge different types of uncertainty involved with various heads of benefit and cost in the above case. Benefits and costs may be real and pecuniary. Real benefits and costs may be direct or indirect, tangible or intangible, final or intermediate. All possible benefits and costs should be allowed for cost benefit analysis. However for convenience we have considered four heads of benefits: three tangible benefits and one intangible benefit; and four heads of costs : three tangible costs and one intangible cost.

Costs:
(C_1) **Production losses to Cigarette Company:** It is a tangible cost which can be measured at source of production. However for some reason or the other the companies may be reluctant to reveal the true losses. So information should be collected from indirect sources. This may give us an interval where there is high probability that the value will lie but at the same time there are chances that the value may spill over to the right and / or left of the interval. Hence this parameter can be expressed as a trapezoidal fuzzy number.

(C_2) **Loss of satisfaction by smokers:** It is an intangible cost which cannot be measured in money term. If smoking is banned in an area the satisfaction of the smokers will suffer due to loss of satisfaction. We will have to quantify this loss of satisfaction. For this we may ask smokers how much extra they are willing to pay if cigarette is made available during banned period. This quantity will vary from individual to individual. So it will be a random parameter and hence it can be represented by a probability distribution. This will represent the loss in satisfaction of a single individual. Suppose P is the total population and assume that 35 to 40 percent of the total population are non-smokers.

Then total loss in satisfaction can be expressed as

[35, 40]/100 X P X (cdf of the probability distribution)

(C_3) Loss to the Treasury: It is a tangible cost which can be estimated from concerned department. We assume that this value lies in some interval.

(C4) **Loss in profits in Cafes and Restaurants**: It is a tangible cost. Smoking ban may reduce customer in cafes and Restaurant leading to loss of profits. Exact estimation may not be possible. It may give us an interval with some value in between as most likely. So it can be expressed as a triangular fuzzy number.

Benefits:
(B_1) **Reduction in costs of healthcare:** It is a tangible benefit. If smoking is banned for a year in a particular area, the possibility of diseases caused by smoking may be reduced. This will lead to benefit. If data is collected for some previous years, the costs of healthcare will vary from year to year. This amount may lie within a certain range. So in this case we can model the parameter by an interval.

(B_2) **Reduction in damages (due to fire):** It is a tangible benefit. Nobody can deny the possibility of devastation by fire due to cigarette smoking. If smoking is banned, the costs of damages due to fire might be reduced. This reduced amount of cost is not fixed. If someone investigates this reduced cost for a number of years, the possibility is that this amount of cost may be different in its estimation. One can think that the value may be somewhere within a range which is taken from a set of observed data. This expected value may be more close to a value within this range. For the sake of convenience we represent this benefit by a triangular fuzzy number.

(B_3) **Gain in satisfaction by nonsmokers:** It is an intangible benefit which cannot be measured in money term. For this one can make survey by collecting feedback (gain in satisfaction) from the non-smokers of the population. The nonsmokers will be asked to assign some value to their gain in satisfaction. This quantity will vary from individual to individual. So it will be a random parameter and hence it can be represented by a probability distribution. This will represent the gain in satisfaction of a single individual. Suppose P is the total population and assume that 60 to 65 percent of the total populations P are non-smokers.

Then total gain in satisfaction will be give by

[60, 65]/100 X P X (cdf of the probability distribution)

(B_4) Savings from smoking cessation. It is a tangible benefit. If smoking is banned the amount spent for buying cigarettes will be saved. No of cigarettes consumed by people varies from individual to individual. So savings from it will vary from person to person. It will be a random parameter and so can be represented by probability distribution. The distribution will represent the savings of an individual. Since 35 to 40 persons are smokers, the total benefit will be given by

[35, 40] /100 X P X (cdf of the probability distribution)

For simplicity we assume that the total population is 100 and 35 to 40 persons are smokers and 60 to 65 persons are non-smokers. We take the following values as inputs (in lakhs, say):

B1= [1.2, 1.8]

B2= [1, 1.1, 1.8]

B3= [60, 65]/100*100* lognormal (0.026, 0.005)

B4= [35, 40]*100*100* normal (0.065, 0.003)

C1= [1, 1.3, 1.4, 1.6]

C2= [35, 40]/100*100* lognormal (0.064, 0.001)

C3= [0.58, 0.7]

C4= [1.3, 1.6, 1.7]

Then PV (Present value) = $\Sigma B_i - \Sigma C_i$

The uncertainty is propagated to the output parameter (i.e., PV)

Volume : 2 | Issue : 12 | December 2013 • ISSN No 2277 - 8179

by hybrid method [6] and computation is done using software Ramas Risk Calc [4].

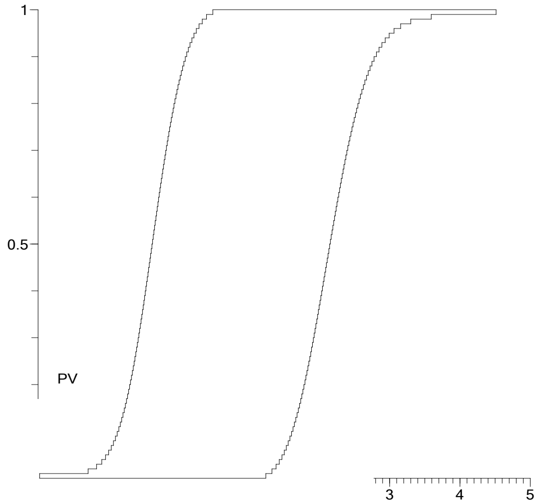

Fig. 9 PV representation under uncertain environment

It is found that the value of PV varies in the interval **[-1.72257,3.88903]**. It has two cdfs which gives the upper and lower probability of PV attaining any value in the range.

If PV is positive smoking should be banned otherwise not. We will determine the probability that the PV is greater than 0. In fact we will get two probability values here: one corresponding to the upper cdf and one corresponding to the lower cdf.

From the graph it is clear that for the value of PV to be less than or equal to zero there is an upper probability of **0.5** and a lower probability of zero. Hence we conclude that for PV to be positive there is an upper probability of (**1 - 0.5**) i.e., **0.5** and a lower probability of one. Now if uncertainty is ignored, that is average or mean values are only considered (midpoint for intervals, core for triangular fuzzy number, mean for probability distribution, midpoint of core for trapezoidal fuzzy number) the value of PV comes out to be **0.425**. However under uncertain environment, for the PV to be equal to or less than **0.425**, there is only an upper probability of 0**.9**. In other words PV is positive with a probability of **1- 0.9** i.e., **0.1** only Now decision makers may decide upon their policy of smoking ban based on the above findings.

5. Conclusion

Uncertainty is encountered in many real life situations. We tried to express a real life problem by a model. But no model is a perfect representation of the reality. Again in many cases the input parameters cannot be precisely known. This uncertainty in the input parameters is propagated to the output or result through the model. Here we have attempted to illustrate this fact by incorporating uncertainty in the input parameters of cost-benefit analysis. The results show that PV does not have a fixed value but a range of values with different probabilities. Moreover we have seen that if uncertainty is avoided in the analysis then the PV comes out to be 0**.425** which is an overestimation in the sense that under uncertain environment the (upper) probability is only **0.1**

REFERENCE

[1] Ferson, S.(2002), RAMAS Risk Calc 4.0 Software : Risk Assessment with Uncertain Numbers, Applied Biomathematics, Setauket, New York. | [2]Ferson,S.,Kreinovich,V.,Ginzburg,L.,Mayers,D.S.,Sentz,K.(2003),Constructing Probabilities Boxes and Dempster-Shafer Structures, Sandia National Laboratories. | [3] Gilboa, I.,Postlewaite, A.W., Schmeidler, D. (2008), Probability and Uncertainty in Economic | Modeling, Journal of Economic Perspective-Volume 22, Number3,pp. 173-188. | [4] Klir, G. J.,Yuan, B.(2009), Fuzzy sets and Fuzzy Logic: Theory and Application, PHI Learning Private Ltd,M-97,Connaught Circus, New Delhi-11001. | [5] Misra, S.K., Puri, V.K.,(1991), Development and Planning: Theory and Practice, Fifth Edition, Himalaya Publishing House, Bombay- Delhi-Nagpur. | [6] Musgrave, R. A., Musgrave, P. B.(1989), Public Finance in Theory and Practice, Fifth Edition, Mc Graw-Hill Book Company.