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Research Paper

Properties of Anti- Fuzzy Kernel and Anti-Fuzzy Subsemiautomata

Mathematics

KEYWORDS : Antifuzzy subgroup; Antifuzzy kernel; Antifuzzy, subsemiautomata; Fuzzy subgroup

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ABSTRACT

The concept of fuzzy set was first introduced by Zadeh, in 1965. Rosenfeld introduced fuzzy subgroup in the year 1971. Biswas defined antifuzzy subgroups in the year 1988. In this paper we define antifuzzy kernel and antifuzzy subsemiautomata and proved some of its properties.

Introduction

Wee developed Fuzzy automata theory in the year 1967. Das defined fuzzy kernel and fuzzy subsemiautomata and proved some of its properties in the year 1997. Using Biswas definition of antifuzzy subgroup we have defined antifuzzy kernel and antifuzzy subsemiautomata and proved some of its results in this paper.

Preliminaries

2.1 Definition:

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $A: X \rightarrow [0,1]$ and $A(x)$ is interpreted as the degree of membership of element xx in fuzzy set A for each $x \in Xx \in X$.

2.2 Definition:

A fuzzy subset $\lambda\lambda$ of a group GG is a fuzzy subgroup of GG if for all $x, y \in Gx, y \in G$.

$$\lambda(x * y) \geq \lambda(x) \wedge \lambda(y)$$

$$\lambda(x^{-1}) \geq \lambda(x)\lambda(x^{-1}) \geq \lambda(x)$$

2.3 Definition:

A fuzzy subset λ of a group G is a fuzzy normal subgroup of G if $\lambda(x * y * x^{-1}) \geq \lambda(y)$ for all $x, y \in G$

2.4 Definition:

Let G be a group. A fuzzy subset μ of G is called an antifuzzy subgroup of G if for all $x, y \in G$

$$\mu(xy) \leq \max\{\mu(x), \mu(y)\}$$

$$\mu(x^{-1}) \leq \mu(x)\mu(x^{-1}) \leq \mu(x)$$

2.5 Definition:

A antifuzzy subgroup $\mu\mu$ of group GG is called an antifuzzy normal subgroup of GG if

$\mu(xy) = \mu(yx)$ for all $x, y \in G$

2.6 Definition:

A fuzzy semiautomaton over a finite group $(Q, *)$ is a triple (Q, X, μ), where XX is a finite set and μ is a fuzzy subset of Q x X x Q

Note: Throughout this paper $S = (Q, X, \mu)$ denotes a fuzzy semiautomaton over a finite group $(Q, *)$ with identity element e.

Main Results:

***Definition* 3.1:**

A fuzzy subset λ of Q is called an antifuzzy kernel of SS if

λ is a antifuzzy normal subgroup of

Q.

$\lambda(p * r^{-1}) \leq \mu(q * k, x, p) \vee \mu(q, x, r) \vee \lambda(k)$ for all $p, q, k, r \in Q,\ x \in X$

Definition 3.2:

A fuzzy subset λ of Q is called an antifuzzy subsemiautomaton of SS if the following conditions hold

$\lambda\lambda$ is a antifuzzy subgroup of Q.

$\lambda(p) \leq \mu(q, x, p) \vee \lambda(q)$ for all $p, q \in Q,\ x \in X$

Proposition 3.3:

Let λ be a antifuzzy kernel of S. Then λ is a antifuzzy subsemiautomaton of S if and only if $\lambda(p) \leq \mu(e, x, p) \vee \lambda(e)$ for all $p \in Q,\ x \in X$

Proof: Let us assume that the given condition is satisfied.

Then for all $p, q, r \in Q,\ x \in X$ we have

$$\lambda(p) \leq \lambda(p * r^{-1} * r)$$

$$\leq \lambda(p * r^{-1}) \vee \lambda(r)$$

$$\leq \mu(q, x, p) \vee \mu(e, x, r) \vee \lambda(q) \vee \lambda(r)$$

$$\leq \mu(q, x, p) \vee \mu(e, x, r) \vee \lambda(e) \vee \lambda(q)$$

$$\leq \mu(q, x, p) \vee \lambda(q)$$

Thus λ is a fuzzy semiautomaton.

The converse is immediate.

Proposition 3.4:

Let λ be a antifuzzy kernel of S and $\nu\nu$ be a antifuzzy subsemiautomaton of S. Then

$\lambda * \nu$ is a antifuzzy subsemiautomaton of SS.

Proof: Given λ is a antifuzzy normal subgroup and $\nu\nu$ is a antifuzzy subgroup of Q, it follows that $\lambda * \nu$ is a fuzzy subgroup of Q and $\lambda * \nu = \nu * \mu$

$$(\lambda * \nu)(p) \leq \lambda(p * r^{-1}) \vee \lambda(r)$$

$$\leq \big(\mu(a * b, x, p) \vee \mu(a, x, r) \vee \lambda(b)\big)$$

$$\vee \big(\mu(a, x, r) \vee \nu(a)\big)$$

$$= \mu(a * b, x, p) \vee \lambda(b) \vee \nu(a)$$

for all $a, b, p \in Q,\ x \in X$

Thus for all $p, q \in Q,\ x \in X$

$$(\lambda * \nu)(p) \leq \wedge\{\mu(a * b, x, p) \vee \lambda(b)$$

$$\vee \nu(a) / a, b \in Q,\ a * b = q\}$$

$$= \mu(q, x, p) \vee = \mu(q, x, p) \vee$$

$$(\wedge\{\lambda(b) \vee \nu(a) / a, b \in Q,\ a * b = q\})$$

$$= \mu(q,x,p) \vee (\nu * \lambda)(q)$$
$$= \mu(q,x,p) \vee (\lambda * \nu)(q)$$

Hence $\lambda * \nu$ is a antifuzzy subsemiautomaton of SS.

Proposition 3.5:
If λ and ν are antifuzzy kernels of SS then $\lambda * \nu$ is a antifuzzy kernel of S

Proof: $\lambda * \nu$ is a antifuzzy normal subgroup of Q, since λ and ν are antifuzzy normal

subgroup of Q.Now

$$(\lambda * \nu)(p * r^{-1}) \leq \lambda(p * q^{-1}) \vee \nu(q * r^{-1})$$

$$\leq \big(\mu(a * b * c, x, p) \vee \mu(a * b, x, q) \vee \lambda(c)\big) \vee\vee$$

$$\big(\mu(a * b, x, p) \vee \mu(a, x, r) \vee \nu(b)\big)$$
$$= \mu(a * b * c, x, p) \vee \mu(a, x, r) \vee \lambda(c) \vee \lambda(b)$$

for all $a, b, c, p, r \in Q$ and $x \in X$

Thus for all $p, q, r, k \in Q$ and $x \in X$

$$(\lambda * \nu)(p * r^{-1}) \leq \wedge\{\mu(q * b * c, x, p) \vee \mu(q, x, r) \vee \lambda(c) \vee \lambda(b) / b, c \in Q;\ b * c = k\}$$
$$= \big(\mu(q * k, x, p) \vee \mu(q, x, r)\big) \vee \big(\wedge\{\lambda(c) \vee \nu(b) / b, c \in Q;\ b * c = k\}\big)$$
$$= \mu(q * k, x, p) \vee \mu(q, x, r) \vee (\lambda * \nu)(k)$$

Thus $\lambda * \nu$ is a antifuzzy kernel of S.

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