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Heat and Mass Transfer Effects on Unsteady Free Convection Boundary Layer Flow past an Impulsively Started Vertical Surface with Newtonian Heating

Mathematics

KEYWORDS : Newtonian heating, Heat and Mass transfer, unsteady flow and free convection.

K. V. S. Raju	Department of Mathematics, KORM college of Engineering, Kadapa, A.P, India
T. Sudhakar Reddy	Department of Mathematics, Global college of Engineering and Technology, Kadapa, A.P, India
M. C. Raju	Department of Humanities and Sciences, AITS, Rajampet, A.P, India
S. Venkataramana	Department of Mathematics, Sri Venkateswara University, Tirupati, A.P, India
S. V. K. Varma	Department of Mathematics, Sri Venkateswara University, Tirupati, A.P, India

ABSTRACT

In this paper we have studied heat and mass transfer effects on unsteady free convection boundary layer flow past an impulsively started vertical surface with Newtonian heating, where the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature, and which is usually termed as conjugate convective flow. The equations governing the flow are studied in the closed form by using the Laplace transform technique. The effects of various physical parameters are studied through graphs and the expression for skin friction also derived and discussed.

1. Introduction:

In nature, there exist flow which are caused not only by the temperature differences but also the concentration differences. The rate of heat transfer is affected by these mass transfer differences especially in industries.The transport process exists in which heat and mass transfer simultaneously take place that results the combine buoyancy effect the thermal diffusion, the phenomenon of heat and mass transfer frequently occurs in chemically processed industries, distribution of temperature and moisture over agricultural fields, dispersion of fog and environment pollution and polymer production. Free convection flow of coupled heat and mass transfer flow occurs due to the temperature and concentration differences in the fluid as a result of driving forces. For example in atmospheric flows, thermal convection resulting from heating of the earth by sunlight is affected differences in water vapor concentration. The study of boundary layer flows over that surfaces have been investigated by some researchers ,since Soundalgekar [1] who was the first to study the flow of viscous fluid past an impulsively started infinite isothermal vertical plate by using Laplace transform technique. Chamkha et al. [2] analyzed the effect of buoyancy forces on the flow and heat transfer over a heated vertical or inclined surface which moves with non-uniform velocity in an ambient fluid. The effect of heat and mass transfer on free convection flow near an infinite vertical plate embedded in porous medium which moves with time dependent velocity in a viscous, electrically conducting in-compressible fluid , under the influence of uniform magnetic field applied normal to the plate was studied by Das and Jana [3]. Combined heat and mass transfer effects on MHD free convection flow past an oscollating plate embedded in porous medium was considered by Chaudhary and Jain [4].

It is known that heat transfer is concerned with the exchange of thermal energy from one physical system to another. Merkin [5] pointed out Newtonian heating (NH), where the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature, and which is usually termed conjugate convective flow. Recently, Newtonian heating conditions have been used by researchers in view of their practical applications in several engineering devices, for instance in a heat exchanger where the conduction in a solid tube wall is greatly influenced by the convection in the fluid flowing over it. Free convection flow above a nearly horizontal surface in a porous medium subject to Newtonian heating has been studied by Lesnic et al. [6]. Unsteady free convection flow past an impulsively started vertical surface in the presence of Newtonian heating was addressed by Chaudhary and Jain [7]. Forced convection boundary layer flow at a forward stagnation point with Newtonian heating was studied by Salleh et al. [8]. Recently, Niu et al. [9] analyzed the stability of thermal convection of an Oldroyd-B fluid in a porous medium with Newtonian heating. Very recently Hayat et al. [10] investigated Newtonian heating and magnetohydrynamic effects in flow of a Jeffery fluid over a radially stretching surface. An exact solution to the unsteady free-convection boundary-layer flow past an impulsively started vertical surface with Newtonian heating was presented by Chaudhary and Jain [11]. Mebine and Adigio [12] considered an unsteady free convection flow with thermal radiation past a vertical porous plate with Newtonian heating. Besides that Narahari and Ishak [13] have studied radiation effects on free convection flow a moving vertical plate with Newtonian heating. Das et al. [14] have analyzed radiation effects on unsteady free convection flow past a vertical plate with Newtonian heating. The unsteady free convective flow past a vertical porous plate with Newtonian heating has been studied by Sankar kumar et al. [15].

2. Nomenclature:

C^* : Specifies concentration in the Fluid C^*_∞ : Ambient concentration
C : Dimensionless concentration C_p : Specifies heat at constant pressure
T^* : Temperature fluid near the plate T^*_∞ : Ambient Temperature
T : Dimensionless temperature Gm : Modified Grashoff number
Gr : Thermal Grashoff number h :Heat transfer coefficient
g : Gravitation due to acceleration D : Chemical molecular Diffusivity
Nu : Nusselt number Pr : Prandtl number
Sc :Schmidt number Sh : Non dimensional Sherwood number
t^* : Time t :Dimensionless time

u* : Dimensional Velocity in x* - direction
u : Non dimensional velocity
x^* : Cartesian Co-ordinate along the plate
y : Dimensionless Co-ordinate along the plate
y^* : Cartesian Co-ordinate axis normal to the plate
y : Dimensionless Co-ordinate axis normal to the plate

Greek symbols

ν : Kinematic viscosity μ : Dynamic viscosity
θ : Dimensionless temperature ρ : fluid density
βT : Coefficient of volume expansion
κ : Thermal conductivity of the fluid
βc : Coefficient of volume expansion with concentration
τ : Non-dimensional skin friction
erf : Error function
erfc : Complementary error function

3. Formulation of the Problem:

We have considered the unsteady flow of an incompressible and electrically conducting viscous fluid past an impulsively started infinite vertical plate with Newtonian heating is considered. The $x^* x^*$-axis is taken along the plate in the vertically upward direction and the $y^* y^*$-axis is taken to be normal to the plate. Initially the plate and the fluid are at the same temperature $T^*_\infty T^*_\infty$ in the stationary condition with concentration level $C^*_\infty C^*_\infty$ at all points. At time $t^* > 0 t^* > 0$ the plate is exponentially accelerated with a velocity $u = U_c u = U_c$ in its own plane and the plate temperature is raised linearly with time t and the level of concentration near the plate is raised to $C^*_w C^*_w$. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. Using the above assumptions and the usual Bossinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T (T^* - T^*_\infty) + g\beta_C (C^* - C^*_\infty) \tag{1}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_P} \frac{\partial^2 T^*}{\partial y^{*2}} \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \tag{3}$$

The initial and boundary conditions are

$$u^* = 0, T^* = T^*_\infty, C^* = C^*_\infty \qquad for\ \ all\ \ y^*, t^* \le 0 \tag{4a}$$

$$u^* = U_c, \frac{\partial T^*}{\partial y^*} = -\frac{h}{k} T^*, C^* = C^*_w \qquad at\ \ y^* = 0, t^* > 0 \tag{4b}$$

$$u^* \to 0, T^* \to T^*_\infty, C^* \to C^*_\infty \qquad as\ \ y \to \infty, t^* > 0 \tag{4c}$$

To reduce the above equations into non-dimensional form, we introduce the following dimensionaless variables and parameters

$$u = \frac{u^*}{U_c},\ y = \frac{U_c y^*}{\nu},\ \ \theta = \frac{T^* - T^*_\infty}{T^*_\infty},\ t = \frac{t^* U_c^{\,2}}{\nu},\ \ C = \frac{C^* - C^*_\infty}{U_c \frac{C^*_\infty}{\cong} \frac{h\nu}{k}},\ \mathrm{P_r} = \frac{\mu c_p}{\kappa},$$

$$G_r = \frac{\nu g \beta_T T^*_\infty}{U_c^3},\ G_m = \frac{\nu g \beta_C C^*_\infty}{U_c^3},\quad S_c = \frac{\nu}{D},\ U_c = \frac{h\nu}{k} \tag{5}$$

Substituting the above quantities (5) in equations (1) to (3) leads to the following non dimensional equations:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_c C \tag{6}$$

$$\mathrm{P_r} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$S_C \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} \tag{8}$$

The corresponding initial and boundary conditions in non dimensional form are

$$u = 0, \theta = 0, C = 0 \qquad \text{for all } \ y^* \ge 0, t^* \le 0 \tag{9a}$$

$$u = 1, \partial\theta / \partial y = -(1 + \theta), C = 1 \ \ \text{at } \ y = 0,\ t^* > 0 \tag{9b}$$

$$u \to 0, \theta \to 0, C \to 0 \ \ \text{as } \ y \to \infty, t^* > 0 \tag{9c}$$

All the physical parameters are defined in the nomenclature.

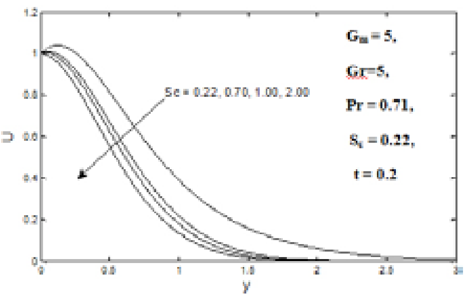

Figure.1 Effect of Sc on velocity

4. Solution of the Problem:

We solve the governing equations in exact form by the Laplace transform technique. The Laplace transforms of the equations (6) to (8) and the boundary conditions (9) are given by

$$\frac{d^2 \bar{\theta}}{dy^2} - (s\,\mathrm{P_r})\bar{\theta} = 0 \tag{10}$$

$$\frac{d^2 \bar{C}}{dy^2} - (sS_c)\bar{C} = 0 \tag{11}$$

$$\frac{d^2 \bar{u}}{dy^2} - s\bar{u} = -G_r \bar{\theta} - G_m \bar{C} \tag{12}$$

The corresponding boundary conditions are

$$\bar{u} = \frac{1}{s}, \frac{\partial \bar{\theta}}{\partial y} = -\left(\frac{1}{s} + \bar{\theta}\right), \bar{C} = \frac{1}{s} \quad at\ \ y = 0\ \ t < 0 \tag{13}$$

$$\bar{u} = 0, \bar{\theta} = 0, \bar{C} = 0 \qquad at\ \ y \to \infty\ \ t > 0$$

The general solution of the present problem for the temperature $\theta(y,t)$, the velocity $u(y,t)$ and the concentration $C(y,t)$ for $t > 0$ are given by

$Case\ 1:\quad S_c \ne 1$

$$u(y,t) = erfc(\eta) - a_1 \Pr \begin{pmatrix} erfc\left(\eta\sqrt{\Pr}\right) - erfc(\eta) - e^{\left(\frac{t}{\Pr} - y\right)} erfc\left(\eta\sqrt{\Pr} - \sqrt{\frac{t}{\Pr}}\right) + \\ e^{\left(\frac{t}{\Pr} - \frac{y}{\sqrt{\Pr}}\right)} erfc\left(\eta - \sqrt{\frac{t}{\Pr}}\right) \end{pmatrix} +$$

$$a_1\sqrt{\Pr}\left(2\sqrt{\frac{t}{\pi}} e^{\left(-\frac{y^2 \Pr}{4t}\right)} - y\sqrt{\Pr}\ erfc\left(\eta\sqrt{\Pr}\right) + y\ \ erfc(\eta) - 2\sqrt{\frac{t}{\pi}} e^{\left(-\frac{y^2}{4t}\right)}\right) +$$

$$a_1\left(\left(\frac{y^2 \Pr}{2} + t\right) erfc\left(\eta\sqrt{\Pr}\right) - y\sqrt{\Pr}\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2 \Pr}{4t}\right)} + y\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2}{4t}\right)} - \left(\frac{y^2}{2} + t\right) erfc(\eta)\right) -$$

$$a_2\left(\left(\frac{y^2 S_c}{2} + t\right) erfc\left(\eta\sqrt{S_c}\right) - y\sqrt{Sc}\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2 S_c}{4t}\right)} + y\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2}{4t}\right)} - \left(\frac{y^2}{2} + t\right) erfc(\eta)\right)$$

(14)

$Case\ 2: S_c = 1$

$$u(y,t) = erfc(\eta) - a_1 \Pr \begin{pmatrix} erfc\left(\eta\sqrt{\Pr}\right) - erfc(\eta) - e^{\left(\frac{t}{\Pr} - y\right)} erfc\left(\eta\sqrt{\Pr} - \sqrt{\frac{t}{\Pr}}\right) + \\ e^{\left(\frac{t}{\Pr} - \frac{y}{\sqrt{\Pr}}\right)} erfc\left(\eta - \sqrt{\frac{t}{\Pr}}\right) \end{pmatrix}$$

$$+ a_1\sqrt{\Pr}\left(2\sqrt{\frac{t}{\pi}} e^{\left(-\frac{y^2 \Pr}{4t}\right)} - y\sqrt{\Pr}\ erfc\left(\eta\sqrt{\Pr}\right) + y\ \ erfc(\eta) - 2\sqrt{\frac{t}{\pi}} e^{\left(-\frac{y^2}{4t}\right)}\right) +$$

$$a_1\left(\left(\frac{y^2 \Pr}{2} + t\right) erfc\left(\eta\sqrt{\Pr}\right) - y\sqrt{\Pr}\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2 \Pr}{4t}\right)} + y\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2}{4t}\right)} - \left(\frac{y^2}{2} + t\right) erfc(\eta)\right)$$

$$-\frac{G_m y}{2}\left(2\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2}{4t}\right)} - y\sqrt{\frac{t}{\pi}} erfc(\eta)\right) \tag{15}$$

$$\theta(y,t) = e^{\left(\frac{t}{\Pr} - y\right)} erfc\left(\eta\sqrt{\Pr} - \sqrt{\frac{t}{\Pr}}\right) - erfc\left(\eta\sqrt{\mathrm{P_r}}\right) \tag{16}$$

$$C(y,t) = erfc\left(\eta\sqrt{S_c}\right)$$

$$b = \frac{-1}{\sqrt{\Pr}},\ a_1 = \frac{bG_r}{\Pr - 1},\ a_2 = -\frac{G_m}{S_C - 1},\ \eta = \frac{y}{2\sqrt{t}} \tag{17}$$

Where

5. Skin friction

Knowing the velocity field, we now study the effect of t, Pr, F etc. on the skin friction. In the non dimensional form, it is given by

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$$\tau = \frac{\tau'}{\rho U_c^2} = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{\sqrt{\pi t}} + \frac{G_r\sqrt{P_r}}{\sqrt{P_r}+1}\left[\left(1+2\sqrt{\frac{t}{P_r\pi}}\right) - e^{\frac{t}{P_r}}\left(1+erf\left(\frac{t}{P_r\pi}\right)\right)\right] - \frac{G_m}{S_c-1}\left(2\left(\sqrt{\frac{S_c t}{\pi}} - \sqrt{\frac{t}{\pi}}\right)\right) \quad (18)$$

6. Results and discussion:

Numerical evaluation of the analytical results reported in the previous section was performed and a representative set of results is reported graphically through figures 1-4. These results are obtained to illustrate the influence of the various physical parameters like Schmidt number Sc on velocity, Prandtl number Pr on temperature, Schmidt number Sc on concentration and Skin friction.

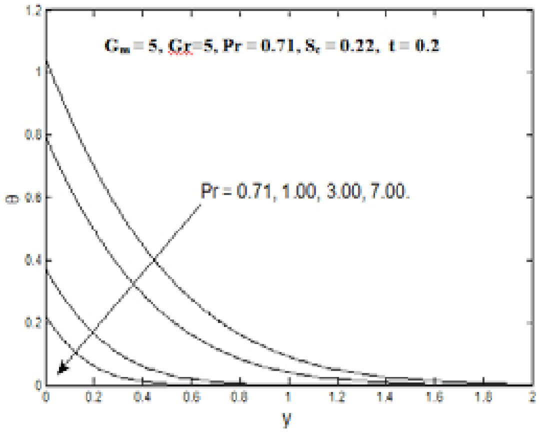

Figure.2. Effect of Pr on Temperature

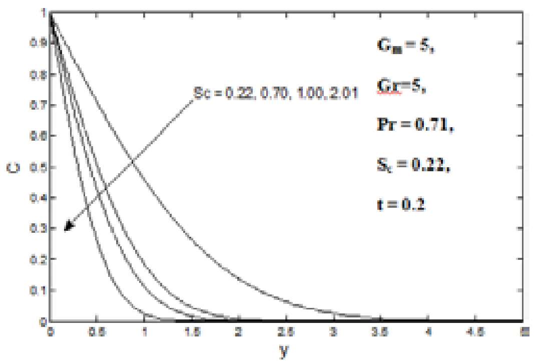

Figure.3. Effect of Sc on Concentration

Figure 1 depicts the effects of Schmidt number Sc on velocity while keeping the other parameters as constant. From this figure it is noticed that the velocity distribution attains a distinctive maximum value in the vicinity of the plate surface and then decreases properly to approach the free stream value zero. I t was found that an increase in the value of Sc velocity decreases.

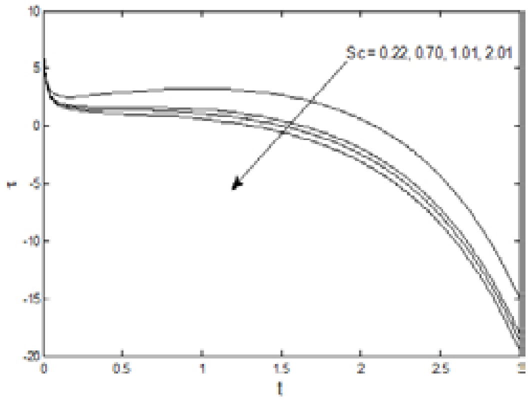

Figure.4. Skin- friction profiles in variation of Sc

In figures 2, temperature profiles are presented for different values of Pr, from this figures it is noticed that temperature distribution decreases with an increase in the values of Pr. Concentration profiles are shown through figure 3, with the variations in Schmidt number Sc. This figure evident that concentration decreases with the increasing values of Sc throughout the boundary layer. Another important aspect of the flow, coefficient of skin friction is presented in figure 4, for various values of Sc. Skin friction near the plate decreases with an increase in the cases of Sc

Conclusions:

In this paper we have studied heat and mass transfer effects on unsteady free convection boundary layer flow past an impulsively started vertical surface with Newtonian heating, in the closed form by using the Laplace transform technique. From this study we conclude that velocity, Concentration and Skin friction decrease with the increasing values of Sc where as temperature distribution decreases in the increase in Pr.

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