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Mathematical Properties of Homogeneous and Isotropic Cosmological Models

Mathematics

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Pratima Ojha Department of mathematics sager institute of science and technology, Bhopal M.P., India

R.K.Dubey Govt.P G Science College Rewa (MP)

ABSTRACT

In the present paper we have discussed the mathematical properties of non static and non empty cosmological model with variable cosmological constant ( ) & variable Gravitatinal constant ( ). It may be concluded that the cosmological constant ( ) may be replaced by a non linear Scalar field. It has been also focused that there is close relation between model with Scalar field & model with perfect fluids.

NTRODUCTION

The cosmological constant denoted by the Greek letter Λ was first introduced by Einstein to obtain a static, homogeneous cosmological model in 1917. Einstein included this term in his equations for General Relativity because he was not sure that his equations allow for a static universe. Gravity affects the universe to contract. To avoid this possibility, he introduced term called the cosmological constant that would act as a repulsive form of gravity to balance the attractive nature of gravity. Einstein rejected hisintroduction of the cosmological constant after the expansion was discovered by Hubble.

At the time of Einstein , the cosmological constant Λ was having no physical meaning but recent cosmological observation indicates that the expansion of universe is accelerating and this has led to a great deal of theoretical activities. Non static cosmological expansion raises a variety of interesting mathematical questions different with kinds of non-empty universe. Here in the present paper we have tried to investigate the mathematical properties of cosmological constant.

2. HOMOGENEOUS & ISOTROPIC NATURE OF UNIVERSE.

In Einstein's theory [1], gravity is described by the spacetime metric $g_{\mu\nu}$, where the indices run over the time and three space coordinates and where the matric varies in spacetime .The infinitesimal, invariant , line element dт is given by

$$d\tau^2 = g_{\mu\nu}(x)dx^\mu dx^\nu \tag{1}$$

There are ten gravity field over the four spacetime coodinates.However, the symmetries of the theory stemming from the Equivalence principal redues that to two independent degree of freedom. Einstein used the mathematical theory of differential geometry to find the relevant tensors quadratic in spacetime derivatives of the matricfield, the Ricci tensor $R_{\mu\nu}$ and curvature scalar R, to drive the dynamical equations for the matric tensor. In the modified form with a cosmological constant Λ, the equations are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2}$$

Where G is Newton's constant which determines the strength of the gravity force, and $\mu\nu$ is the energy momentum tensor. Here as in the follwing, we have set the velocity of light to unity (c=1)

Einstein's equation (2) represent tencoupled differential equation with the Friedmann-Lemaitre-Roberston-Walkar assumption about the cosmological principal the matric simplifies to

$$d\tau^2 = dt^2 - a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right\} \tag{3}$$

Where a(t) is a scalar factor and k is constant that depends on the curvature of the space time. The constant k has been normalized to the values –1, 0 or 1 describing an open, flat or closed universe. The variables r, θ, and φ are so called co-moving coordinates, in which a typical galaxy has fixed values.

The physical cosmological distance for galaxies repeated by r at a given time t is a(t)r, which grow with time as the scale factor a(t) in an expanding Universe. We using two independent Friedmann equations

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} - \frac{\Lambda}{3} \tag{4}$$

And

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{5}$$

Where a dot means a time derivatives and H is the expansion rate of our Universe called the Hubble parameter,or the Hubble constant with its present value H_0 It is seen to depend on both the energy density of the universe as well as its curvature and a possible cosmological onstant. With k and set Λ to zero, one defines the critical density as

$$\rho_c = \frac{3H^2}{8\pi G}$$

In 1934, Lemaitre had already pointed out that the cosmological constant could be considered as a vacuum energy and hence a contribution to the energy density of the form We will assume that the universe is composed of a set of components I each having a fraction of the criti Ω_i density

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

3. COSMOLOGICAL MODEL WITH $\Lambda \neq 0$

Under the assumption of an homogeneous isotropic universe. Einstein's field equation reduces them elves to Einstein- friedmann Lemaitre equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \rho_\Lambda) - \frac{k}{a^2} \tag{6}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}(\rho - 2\rho_\Lambda + 3p) \tag{7}$$

We consider first the static solution ($\ddot{a} = \dot{a} = 0$).The equation are then written as (for $\rho = 0$)

$$8\pi G\rho = 2\rho_\Lambda \tag{8}$$

$$\frac{8\pi G\rho}{3a} + \frac{\rho_\Lambda a^2}{3} = k \tag{9}$$

From the first equation it follows that $\rho_\Lambda > 0$ and therefore the second equation has a solution for k=

$$a^2 = \frac{2}{8\pi G\rho} \tag{10}$$

From equation represents the equilibrium condition for the universe.

The attractive force due to ρ has a exactly compensate the repulsive effect of a positive cosmological constant in order to produce a static universe.

4. DIFFERENT TYPES OF MODELS

K → / Λ ↓	K=-1	K=0	K=+1
> 0	Monotone models of first kind	Monotone models of first kind	(i)Asymp. Models of I and II kind (ii) Monotone models of I and II kind (iii) Oscillating models of I kind
Λ=0	Oscillating models of I kind	Monotone models of first kind	Monotone models of first kind
Λ < 0	Oscillating models of first kind	Oscillating models of first kind	Oscillating models of first kind

CONCLUDING REMARKS

This paper provides the mathematical analysis of cosmological model with non-static expansion. The expansion with a review of results in the case of positive cosmological constant has be discussed in brief and it is highlighted that the cosmological constant. Λ may be replaced by a non linear scalar field, it has been alo focused that there are close relation between models with scalar field & models with perfect fluids whose equation of state is more or less exotic, and also in this paper we have discuss a set of concept on the cosmological constant we have also discussed cosmological model with non static.

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