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Improved Next to Next Minimum Penalty Method for Transportation Problems

Mathematics

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K.Jaikumar — Department of Mathematics, Dharmapuram Adhinam Arts College, Dharmapuram, Mayiladuthurai, Tamil nadu, India-609 001

R. Subash — Department of Mathematics, A.V.C College of Engineering, Mannampandal, Mayiladuthurai, Tamilnadu, India 609305.

S. Sunantha — Department of Mathematics, Vivekananda Arts and Science College for Women, Sirkali, Nagapattinam, Tamilnadu, India-60911

ABSTRACT

The transportation model is a special case of linear programming models, widely used in the areas of inventory control, employment scheduling. Aggregate planning, and personnel assignment, among others. Due to its special structure, the stepping-stone method is commonly adopted in order to improve the computational efficiency instead of the regular simplex method. This paper proposes a new approach, new algorithm named, Improved Next to Next Minimum Penalty Method is proposed for solving Transportation problem. This algorithm is more efficient than other existing algorithms. The procedure for the solution is illustrated with a numerical example.

Introduction:
The transportation problem is one of the oldest applications of linear programming problems. The basic transportation problem was originally developed by Hitchcock [3]. Efficient methods of solution derived from the simplex algorithm were developed in 1947, primarily by Dantzig [4] and then by Charnes and Cooper [1]. The transportation problem can be modeled as a standard linear programming problem, which can be solved by the simplex method. We can get an initial basic feasible solution for the transportation problem by using the North-West corner rule, Row Minima, Column Minima, Matrix minima or the Vogel's Approximation Method. To get an optimal solution for the transportation problem, we use the MODI method (Modified Distribution Method). Charnes and Cooper [1] developed the Stepping Stone Method, which provides an alternative way of determining the optimal solution.

The LINDO (Linear Interactive and Discrete Optimization) package handles the transportation problem in explicit equation from and thus solves the problem as a standard linear programming problem. Consider m origins (or sources) Oi (i=1,…,m) and n destinations Dj (j=1,…,n). At each origin Oi , let ai be the amount of a homogeneous product that we want to transport to n destinations Dj , in order to satisfy the demand for bj units of the product there. A penalty cij is associated with transport in a unit of the product from source i to destination j. The penalty could represent transportation cost, delivery time, quantity of goods delivered, under-used capacity, etc. A variable xij represent the unknown quantity to be transported from origin Oi to destination Dj. The transportation problem can be represented as a single objective transportation problem or as a multi-objective transportation problem.

Two more methods for solving transportation problems were introduced by Nagoorgani [2], namely Next to Next Minimum Penalty Method and Modified Vogel's Approximation Method was introduced by Edward [5].

In this paper, we propose an algorithm namely Improved Next to next Minimum Penalty Method is proposed for solving transportation problems, which is more efficient than other existing algorithm. The procedure for the solution is illustrated with a numerical example.

The Transportation Model:
The purpose of transportation modeling is to find the least cost means of shipping supplies from several origins to several destinations. Origins can be factories, warehouses, or any other locations from which goods are shipped. Destinations are any locations that receive goods. The transportation problem is usually presented as a matrix as shown in Figure 1. The unit transportation cost is generally indicated on the northeast corner in each cell.

	D_1	D_2	………	D_n	capacity
O_1	C_{11} X_{11}	C_{12} X_{12}	…….	C_{1n} X_{1n}	A_1
O_2	C_{21} X_{21}	C_{22} X_{22}	……	C_{2n} X_{2n}	A_2
…….	………	………	………..	………	…………
O_m	C_{m1} X_{m1}	C_{m2} X_{m2}	………	C_{mn} X_{mn}	A_m
Demand	B_1	B_2		………	B_n

This problem can be expressed as a linear programming model as follows:

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} c_{ij}$$

Subject to;

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \ldots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \ldots n.$$

Here, all a_i and b_j are assumed to be positive, and a_i are normally called capacities (supplies) and b_j are called demands, as shown in Figure 1. The cost c_{ij} are all nonnegative. If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ it is a balanced transportation problem.

Improved Next to Next Minimum Penalty Method:
Step 1: Balance the given transportation problem if either (total supply > total demand) or (total supply < total demand)

Step 2: Obtain the TOC matrix. The TOC matrix is obtained by adding the "row opportunity cost matrix" (for each row, the smallest cost of that row is subtracted from each element of the same row) and the "column opportunity cost matrix" (for each column of the original transportation cost matrix the smallest cost of that column is subtracted from each element of the same column).

Step 3: The smallest entry from the first row is chosen and it is subtracted from the third smallest entry. This value is written against the row on the right. This value is calculated as the penalty for the first row. Similarly, the penalty for each row is computed. Likewise column penalties are calculated and they are written on the bottom of the TOC matrix below their corresponding columns.

Step 4: The highest penalty is selected and the row or column for which this corresponds is verified. Min (a_i , b_j) allocation is made to the cell having the lowest cost from the selected row or column.

Step 5: The satisfied row or column is eliminated fresh penalties for the remaining sub matrix are calculated as in Step 3 and allocations made as mentioned in Step 4. This is continued until one row or column remains to be satisfied.

Step 6: Compute total transportation cost for the feasible allocations using the original balanced Transportation cost matrix.

Numerical Example:
To solve the following transportation problem of minimal cost with the initial basic feasible solution obtained by Improved Next to Next Minimum Penalty Method whose cost and requirement table is given below.

Origin/Destination	D1	D2	D3	D4	Capacity
O1	1	2	3	4	6
O2	4	3	2	0	8
O3	0	2	2	1	10
Demand	4	6	8	6	24

The basic solution is obtained with objective is 22, with x_{12}=6, x_{23}=2, x_{24}=6, x_{31}=4 and x_{33}=6.

Conclusion:
From the investigations it is clear that Modified Next to Next Minimum penalty Method is better than any other methods for solving the Transportation problem.

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