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Research Paper

Profit Analysis of a Two Similar Unit Cold Standby Redundant System with Different Failure Modes with Expert Repairman

Mathematics

KEYWORDS : Cold standby, partial failure, regenerative point, redundant system etc.

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ABSTRACT

In this paper we studied two similar unit cold standby redundant systems. Each unit has different failure modes. Initially first unit is in operative mode and second unit is in cold standby mode. First unit failed via partial failure mode and second unit cannot fail via partial failure mode. An ordinary repairman is considered to repair the failed (completely or partial) units of the system on FCFS (First come first serve) basis, if ordinary repairman is not capable to repair the unit then expert repairman repair that unit. The failure time distributions of the components are taken as exponential whereas the repair time distributions are general. Using regenerative point technique, important measures of the system effectiveness are obtained. The results are also analyzed through graphs in a particular case when repair time distributions are also exponential.

Introduction

In reliability analysis of redundant repairable systems, most of the research workers have concentrated their attention on system parameters like mean time to system failure, availability, etc. and have emphasized the importance of profit. In most of the chapters one of the basis assumptions is that the repair facility is perfect. However, it may not be true in all situations. Most of the researchers have studied two unit standby redundant systems Arora, J.R. (1976) studied on reliability of a two-unit standby redundant system with constant repair time. Gupta and Kumar (1981) studied on profit evaluation in two-unit maintenance redundant system. Kumar and Render (1993) carried out the reliability analysis of a complex redundant system.

In the present paper we study two similar unit standby redundant systems. Each unit has different failure rates. Initially first unit is in operative mode and second unit is in cold-standby mode. First unit fails via partial failure mode and second unit cannot fails via partial failure of mode. An ordinary repairman is considered to repair the failed (completely or partial) units of the system on FCFS (First come first serve) basis, if ordinary repairman is not capable to repair the unit then expert repairman repair that unit. If first unit fails, then standby unit start working immediately. Such system must have many practical examples in industry especially in the field of electronics.

The system fails only when both units of the system fails, repairs are done on the basis of first come, first served. The system is analyzed to determine various reliability measures by using semi-Markov process and regenerative point technique. Assuming the failure time distribution as exponential and other time distribution as general, the following measures of the system effectiveness is obtained:-

- Mean time to system failure (MTSF)
- Steady-state availability of the system.
- Expected busy period per unit time by the repairman.
- Expected number of visits by the repairman
- Expected profit incurred to the system.

Graphical study is made and cut-off points for various rates/costs to study the economic aspect have been obtained.

Model Descriptions and Assumption

(1) The system consists of two identical units. Initially one is operative and another is cold standby. The standby unit can't fail.
(2) Each unit of the system has two modes i.e. operative & failure
(3) First unit fails via partial failure mode and second unit cann't fails via partial failure mode.
(4) Whenever the operative unit partially fails, it goes under the ordinary repairman, who comes immediately whenever required; if ordinary repairman is not capable to repair the unit then expert repairman repair that unit.
(5) Each unit has exponential distribution of time of failure, while the distribution of repair time by the repairman is arbitrary.
(6) Unit recovers it's functioning perfectly after repair.
(7) System breakdown occurs when no unit is available for operation.
(8) After any repair a unit works like a new one.
(9) When both units fail, the system becomes inoperable.
(10) All the random variables are independent.

Notation and States of the System

E	:	Set of regenerative state $\{S_i, i = 0,2\}$
$\bar{E}$	:	Set of non-regenerative state $\{S_j; j = 1, 3\}$
a, b, g unit.	:	Constant failure rates of components of a
$g_1(t)$, $G_1(t)$	:	p.d.f. and c.d.f. of repair time of repairman for 1st component.
$g_2(t)$, $G_2(t)$	:	p.d.f. and c.d.f. of repair time of repairman for 2nd component.
P_{ij}	:	Transition probability from regenerative state S_i to S_j.
$P_{i,j}^{(k)}$	:	Probability that the system transit from regenerative state S_i to S_j. Passing through the non-regenerative state K.
m_i	:	Mean sojourn time in state S_i.

Symbols for the State of the System

O	:	Operative
Cs	:	Cold standby
f_{er}	:	Partially failed unit under repair of expert repairman.
F_{er}	:	Failed unit under repair of expert repairman.
F_{eR}	:	Failed unit under repair of expert repairman continued from earlier state.
F_{wr}	:	Failed unit waiting for repair

Transition Probabilities and Sojourn Times
State Transition Diagram

(a) The steady state transition probabilities are given as:

Good state Down state

(a) The steady state transition probabilities are given as:

$$P_{0,1} = 1; \quad P_{1,0} = \frac{\gamma}{\beta+\gamma}; \quad P_{1,2} = \frac{\beta}{\beta+\gamma}; \quad P_{2,0} = \frac{\eta}{\lambda+\eta}; \quad P_{2,3} = \frac{\lambda}{\lambda+\eta}; \quad P_{2,2}^{3} = \frac{\lambda}{\lambda+\eta}$$

By these probabilities, we can verify:

$$P_{0,1} = 1; \; p_{1,0} + P_{1,2} = 1; \; P_{2,0} + P_{2,3} = 1; \; P_{2,0} + P_{2,2}^{3} = 1.$$

Mean Time to System Failure (MTSF): $\phi_2 = \frac{N_1}{D_1}$ where, $N_1 = \mu_0 + \mu_1 + P_{1,2} . \mu_2$ and $D_1 = P_{1,2} . P_{2,3}$

Availability Analysis: The steady state availability is $A_0 = \lim_{s \to 0}(s A_0 *(s)) = \frac{N_2}{D_2}$

where, $N_2 = P_{2,0} . M_0 + P_{2,0} . M_1 + P_{1,2} . M_2$ and D_2 $P_{2,0} . \mu_0 + P_{2,0} . \mu_1 + P_{1,2} . \mu_3$

Busy Period Analysis of the Repairman:
In steady state; $B_0 = \lim_{s \to 0}(s B_0 *(s)) = \frac{N_3}{D_2}$

where, $N_3 = (p_{0,1} - P_{0,1} . P^3_{2,2}) W_1 + P_{0,1} . P_{1,2} . W_2$.

Expected Number of Visits by the Repairman: In steady state, $V_0 = \lim_{s \to 0}(s V_0 *(s)) = \frac{N_4}{D_2}$
where, $N_4 = P_{2,0} v_0 + P_{2,0}$ and D_2 is already specified.

Cost Benefit Analysis:
The expected total profit incurred to the system in steady state is given by $P_2 = C_0A_0 - C_1B_0 - C_2V_0$

where C_0 = revenue per unit uptime of the system;
C_1 = cost per unit time for which repairman is busy

C_2 = the cost per visit by the repairman

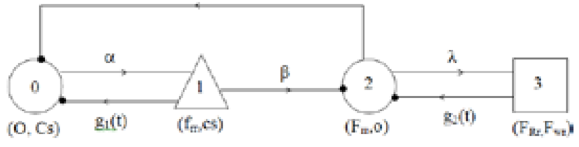

Fig. 1

Particular Cases
For graphical representation, the following particular case is considered $g_1(t) = \gamma e^{-\gamma t}$ and $g_2(t) = \eta e^{-\eta t}$

where γ and η are the repair rat eof the first and second component respectively. Thus, we have

$P_{0,1} = 1$; $P_{1,0} = \frac{\gamma}{\beta+\gamma}$; $P_{1,2} = \frac{\beta}{\beta+\gamma}$; $P_{2,0} = \frac{\eta}{\lambda+\eta}$; $P_{2,3} = \frac{\lambda}{\lambda+\eta}$; $P^3_{2,2} = \frac{\lambda}{\lambda+\eta}$; $\mu_0 = 1/\alpha$; $\mu_1 = \frac{1}{\beta+\gamma}$; $\mu_2 = \frac{1}{\lambda+\eta}$

Using the above equations, we can have the expression for MTSF, availability etc, for this particular cases.

On the basis of the numerical values taken as: $\alpha = 0.004$; $\beta = 0.003$; $\lambda = 0.03$; $\gamma = 0.04$; $\eta = 0.02$

The values of various measures of the system effectiveness are obtained as:

Mean time to system failure (MTSF) = 429.251; Availability (A_0) = 0.951625

Busy period analysis of repairman (β_0) = 0.279752

Expected number of visits by the repairman (V_0) = 0.009133.

For the graphical interpretation, the mentioned particular case is considered.
Fig. 2 depicts that availability (A_0) increase with increase in the value of repair rate (γ) for different value of failure rate (β). Following can also be observed from the graph:

Fig. 3 shows the behaviour of MTSF with respect to failure rate (β). From the figure, it is clear that MTSF get decrease as increase in the value of failure rate (β).

Fig. 4 depicts the profit increase with increase in the values of revenue (C_1) for different values of cost (C_0) and it is lower for higher value of cost (C_0).

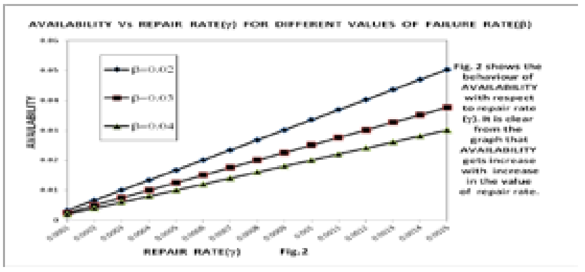

Fig.2

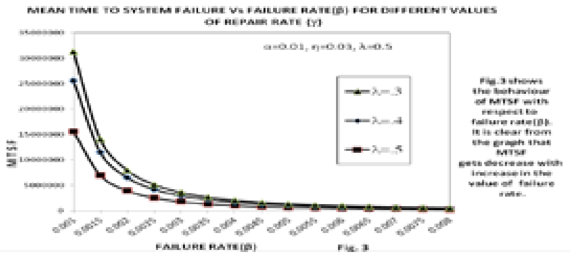

Fig. 3

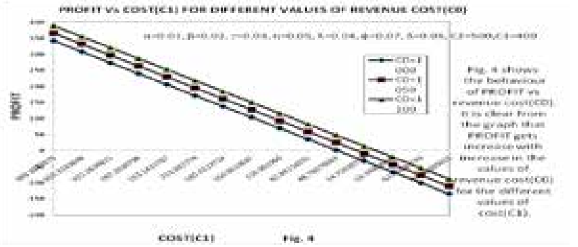

Fig. 4

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