

Design of Multi-Channel Non-Uniform Filter Bank



Engineering
KEYWORDS : Cosine modulation, non-uniform filter bank, near perfect reconstruction

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ABSTRACT
In this paper an improved and simplified approach is presented for designing multi-channel cosine modulated non-uniform filter bank. The method employs Blackman window family to design prototype filter. The amplitude distortion function, which is used as an objective function, is minimized by optimizing the cutoff frequency of the prototype filter. The results show significant reduction in amplitude distortion with improvement in stop band attenuation characteristics compared to earlier reported work.

1. Introduction

The theory of perfect reconstruction (PR) and near PR (NPR) M-channel uniform critically sampled filter bank is well established [1-3]. However, uniform filter banks do not provide appropriate time-frequency decomposition in some applications like, to approximate the time-frequency resolution of human ears, where the filter bank with non uniform frequency spacing is preferred. Therefore, efficient structures and design procedures for such filter banks are highly desirable. A number of design methods have been proposed for non-uniform filter bank (NUFB), over the years [4-20]. However, among various available methods, only a few poses linear phase (LP) property which is crucial in some applications such as image coding to avoid artifacts in the reconstructed image. Even though some methods exist for LP NUFBs [14-15], they are of high design cost and implementation complexity. Moreover, their performance may be not satisfactory.

In this paper, a design technique for NUFB is presented which is derived from a uniform CMFB by merging the relevant filters in the associated CMFB. The filters of NUFB so formed do not necessarily have linear phase even for a linear phased prototype filter but an alias-free \bar{M} -channel non-uniform CMFB can be derived from an M -channel uniform CMFB by combining consecutive analysis/synthesis filters that follow certain criteria.

2. Non-uniform CMFB

In this section we consider the alias-free design procedure for non-uniform CMFB which is derived from a uniform M -channel CMFB by merging the l_i adjacent analysis/synthesis filters, i.e., $H_k(z)$'s and $F_k(z)$'s, for $k = n_i$ through $k = n_i + l_i - 1$, where l_i and n_i are chosen such that n_i is an integral multiple of l_i for all $i = 0, 1, \dots, \bar{M} - 1$ which results in alias-free design of NUFB.

If $p(n)$ is impulse response coefficients of the prototype filter than the analysis/synthesis filters of M -channel CMFB take the closed form expressions:

$$h_k(n) = 2p(n) \cos \left[\frac{\pi}{M} (k + 0.5) \left(n - \frac{N}{2} \right) + (-1)^k \frac{\pi}{4} \right] \quad (1)$$

$$f_k(n) = 2p(n) \cos \left[\frac{\pi}{M} (k + 0.5) \left(n - \frac{N}{2} \right) - (-1)^k \frac{\pi}{4} \right]$$

for $0 \leq k \leq M - 1, 0 \leq n \leq N - 1$

and we define $\bar{H}_i(z)$ or $\bar{F}_i(z)$, analysis/synthesis filters of NUFB

$$\bar{H}_i(z) = \sum_{k=n_i}^{n_i+l_i-1} H_k(z) \quad 0 \leq i \leq \bar{M} - 1 \quad (2)$$

$$\bar{F}_i(z) = \frac{1}{l_i} \sum_{k=n_i}^{n_i+l_i-1} F_k(z) \quad 0 \leq i \leq \bar{M} - 1 \quad (3)$$

where $H_k(z)$ and $F_k(z)$ are frequency responses of the filters of analysis/synthesis section of the uniform CMFB. $\bar{H}_i(z)$ and $\bar{F}_i(z)$ form a new set of analysis and synthesis filters in the \bar{M} -channel non-uniform CMFB. Here, we assign the \bar{M} number of integer

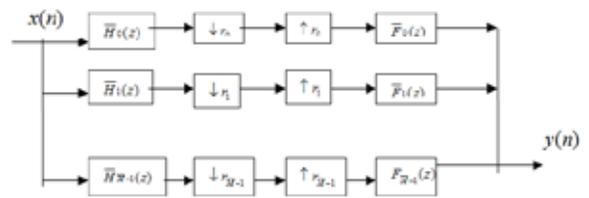


Fig.1 \bar{M} -channel non-uniform filter bank
 set $l_i (i = 0, 1, 2, \dots, \bar{M} - 1)$ to each band on the positive frequency range from 0 to π . Let L be defined as :

$$L = \sum_{k=0}^{\bar{M}-1} l_k \quad (4)$$

Then, the bandwidth of the analysis filters becomes

$$BW_i = \frac{2\pi l_i}{L} \quad (i = 0, 1, 2, \dots, \bar{M} - 1). \quad (5)$$

Note that $n_0 = 0 < n_1 < n_2 < \dots < n_{\bar{M}} = \bar{M}$, and $l_0 + l_1 + l_2 + \dots + l_{\bar{M}-1} = \bar{M}$.

Fig. 1 shows the resulting overall structure of the \bar{M} -channel non-uniform CMFB. In this figure $r_i = \frac{L}{l_i}$ for $i = 0, 1, 2, \dots, \bar{M} - 1$, represents

the maximum factor of decimation and interpolation on the i -th channel. When l_i is the divisor of L , the decimation factor becomes an integer.

Under alias-free condition distortion function takes the form:

$$\phi = \sum_{i=0}^{\bar{M}-1} \frac{1}{r_i} \bar{F}_i(z) \bar{H}_i(z) \quad (6)$$

3. Design and Optimization Procedure

If the prototype filter $P(e^{j\omega})$ has linear phase, the condition for approximate reconstruction can be stated in terms of $P(e^{j\omega})$ as follows [1,2]:

The chosen objective function is given below:

$$\phi_2 = \max \left| \left| P(e^{j\omega}) \right|^2 + \left| P(e^{j(\omega - \pi/M)}) \right|^2 - 1 \right|, \text{ for } 0 \leq \omega \leq \pi/M \quad (7)$$

Design approach for the optimized prototype filter involves linear optimization as described in Fig. 2 with the final objective function that is weighted sum of distortion function ϕ_1 and objective function ϕ_2 given by (8)

$$\phi = \alpha \phi_1 + (1 - \alpha) \phi_2, \text{ for } 0 \leq \alpha \leq 1 \quad (8)$$

The objective function given by (8) is used to obtain the alias-free NUFB.

An iterative linear optimization algorithm developed in [21,22] is modified to minimize the objective function ϕ . Initially, input parameters, i.e., sampling rate, number of channels (M and \bar{M}), pass band (ω_p) and stop band (ω_s) frequencies and filter length (N) of prototype filter are specified. Cutoff frequency (ω_c) and transition band ($\Delta\omega$) is then determined. Initialize, different optimization pointers like step size ($step$), search direction (dir), flag and initial ($error$) as well as expected minimum possible values (tol) of the objective function (8). Inside the optimization loop, design the prototype low pass filter and determine the corresponding band pass filters for analysis and synthesis sections using cosine modulation. Obtain the desired NUFB using merging of relevant band pass filters. The cutoff frequency of the prototype filter is gradually changed as per the search direction and calculates the corresponding value of the objective function (8). Algorithm halts when it attains the minimum value of the objective function.

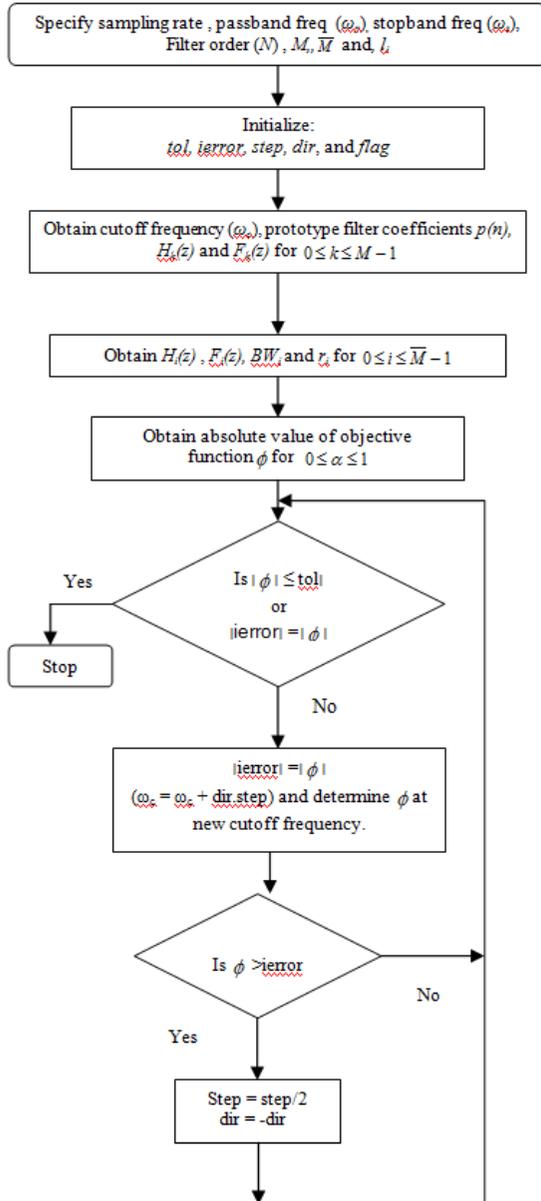


Fig. 2 Flow chart for design and optimization algorithm

4. Design examples

In this section, two design examples are presented to dem-

onstrate effectiveness of the design over the reported results [11,12,16,20,23]. The optimization algorithm is initialized with $tol = 2.0E-04$, $dir = 1$, $step = 0.05$, $error = 500$.

4.1. In this example, a 3-channel NUFB has been designed at the same filter length as given by Li *et al.* [11], Xie *et al.* [16] and Soni *et al.* [23]. The positive frequency range is clearly divided into three non-uniform channels. The ratios are set as $l_0 = 1, l_1 = 1, l_2 = 2$ and $n_0 = 0, n_1 = 1, n_2 = 2, n_3 = 4$. The bandwidth and the decimation factors become

$$BW_0 = \frac{\pi}{2}, BW_1 = \frac{\pi}{2}, BW_2 = \pi, \text{ and } r_0 = 4, r_1 = 4, r_2 = 2$$

Blackman window based low pass filter with filter length $N=63$ is used as prototype filter. The magnitude plot of the designed 3-channel filter bank with decimation factors (4,4,2) is shown in Fig.3. Fig.4 shows the peak amplitude distortion using Blackman-Nuttall 4 term function. The obtained parameters are reported in tabular form as given in Table 1.

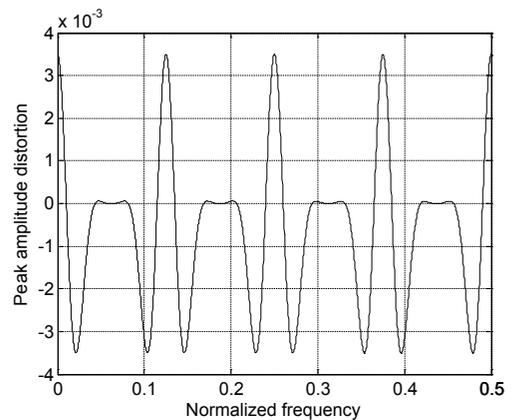
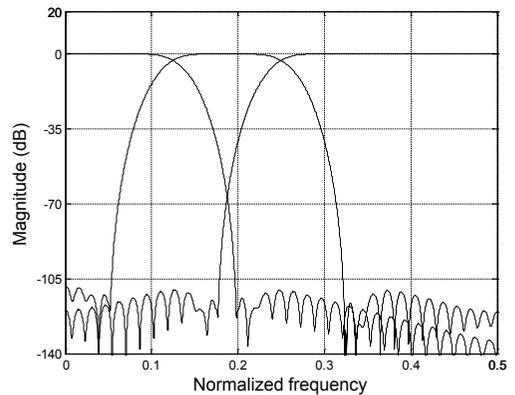


Fig.3 Magnitude plot of 3-channel NUFB using Blackman-Nuttall 4 term window. Fig.4 Peak amplitude distortion of 3-channel NUFB using Blackman-Nuttall 4 term window.

Table 1. Performance comparison with earlier reported works for 3-channel non-uniform filter bank (4,4,2)

Work	Channels	Technique used	Filter length	As (dB)	Peak amplitude distortion
Li <i>et al</i> [11] (1997)	Three channel (4,4,2)	Cosine modulation (PM Method)	64	-60	7.80E-03
Xie <i>et al</i> [16] (2005)	Three channel (4,4,2)	Recombination (PM Method)	63	-110	7.80E-03
Soni <i>et al</i> [23] (2010)	Three channel (4,4,2)	Tree Structure Blackman-Harris 3 term Blackman-Nuttall 4 term	63	-75	8.60E-04
			63	-70	1.15E-03
			63	-110	3.85E-03

Proposed	Three channel (4,4,2)	Cosine Modulation	63	-75	9.69E-04
		Blackman	63	-83	1.57E-03
		Blackman-Harris 3 term	63	-110	3.80E-03
		Blackman-Nuttall 4 term			

4.2. In this example a comparative performance has been made with the reported work of Zijing *et al.* [20]. A 5-channel NUFB is designed with the same specifications as taken in [20], with $l_0 = 2, l_1 = 2, l_2 = 1, l_3 = 1, l_4 = 2$ and $n_0 = 0, n_1 = 2, n_2 = 4, n_3 = 5, n_4 = 6, n_5 = 8$.

The bandwidth and the decimation factors become

$$BW_0 = \frac{\pi}{2}, BW_1 = \frac{\pi}{2}, BW_2 = \frac{\pi}{4}, BW_3 = \frac{\pi}{4}, BW_4 = \pi$$

and $r_0 = 4, r_1 = 4, r_2 = 8, r_3 = 8, r_4 = 4$. For this design Blackman-Harris 4 term windowed LPF with filter length $N = 163$ has been taken to design filter bank. The magnitude response of the filter bank and its amplitude distortion is shown in Fig.5 and Fig.6, respectively. The obtained values are tabulated in Table 2.

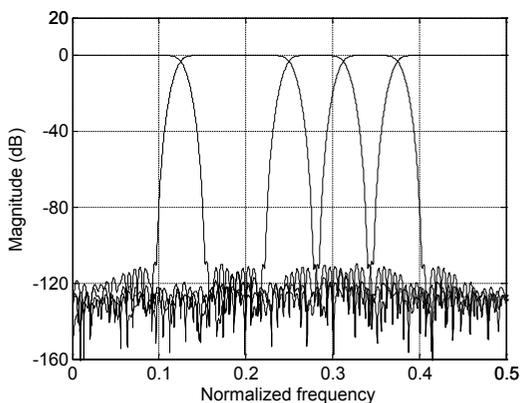


Fig.5 Magnitude plot of 5-channel NUFB using Blackman-Harris 4 term window.

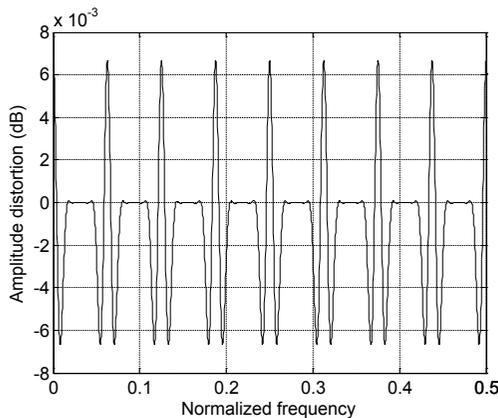


Fig.6 Amplitude distortion plot of 5-channel NUFB using Blackman-Harris 4 term window.

Table 2. Performance comparison with earlier reported works for 4-channel NUFB (8,8,4,2)

Work	Channels	Technique used	Filter length	As (dB)	Amplitude distortion (dB)
Lee & Lee [12] (1995)	Five channel (4,4,8,8,4)	Cosine Modulation (FIR)	41	-46.3	0.027
Zijing <i>et al</i> [20] (2007)	Five channel (4,4,8,8,4)	Cosine Modulation (IFIR)	163	-110	0.0068
Proposed	Five channel (4,4,8,8,4)	Cosine Modulation (FIR) Blackman-Harris 4 term	163	-110	0.0066

5. Discussion

From the values given in Table 1 and Table 2, it has been observed that, in the proposed work, peak amplitude distortions are 9.69E-04, 1.57E-03 and 3.80E-03 for Blackman, Blackman-Harris 3 term, and Blackman-Nuttall 4 term window functions, respectively, which are at par and better than the previously published results. Similarly in case of 5-channel NUFB, the peak amplitude distortion provided by the proposed method is smaller than the Zijing *et al* [20] and other results.

6. Conclusion

A simple and efficient design procedure of NUFB is presented. In traditional design approaches, it is difficult to design the NUFB at high stop band attenuation above 100 dB. The proposed work has eliminated this constraint by exploiting the design process of cosine modulation and obtained NUFB with a feasible partition property. The performance comparison of proposed one with previously reported work shows that the resulting amplitude distortion is smaller at higher stop band attenuation. Therefore this method is suitable for large number of channels where high attenuation filters with unequal pass bands have to be designed with alias free small amplitude distortion. Such filter banks are specifically used in geological or financial time-series analysis.

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