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Research Paper

A Single Period Inventory Model with Discrete and Poisson Demand

Statistics

KEYWORDS : EOQ model, Discrete demand, Poisson demand

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ABSTRACT

In this paper we investigate inventory models where the demand is discrete and holding cost incurred only for the period during which the inventory items are in the stock. First we consider model with deterministic uniform demand then we also consider a model with Poisson demand. We obtain optimal order quantities for both the models. We give the comparison of these two models with each other and also with classical EOQ model. We also indicate a scenario in which the models proposed in this paper provide better results.

1. Introduction:

The traditional EOQ model assumes that demand is uniform, continuous and occurring at deterministic rate. Because of the simplicity of this model, it is often used as an approximation in practice even when demand is stochastic or also when demand is discrete. In this paper we consider models which are specifically designed for discrete demand. We investigate models for deterministic demand as well as for stochastic demand.

Several such models have been developed in the literature of inventory theory.

Archibald(1981),Archibald & Silver(1978), Hill & Johansen(2004) consider batch sized demand, and treat this problem in a very general manner without making explicit assumptions about the probability distribution of the demand.

Katy et al.(2012) assume two types of demand viz demand arriving according to a compound Poisson process and the case of mixture of deterministic demand and compound Poisson demand. Presman & Sehi(2006), Sobel & Zhang(2001) also assume mixture demand models. Johan & Inneke(2005) consider models where purchase quantities are Poisson distributed while investigating the decision problem for a retailer.

We consider a continuous review inventory system and assume zero lead time. The work presented in this paper differs from the previous literature in the sense that holding cost incur only for the period during which the inventory items are in the stock. In section II, we discuss the model with deterministic uniform demand. In section III, we assume that the demand is generated according to a Poisson process with every customer having a unit demand. Due to our assumptions, the actual holding cost is random and depends on the time points at which actual demands occur. Also the replenishment is assumed to be instantaneous and hence the new order is placed only when (and as soon as) inventory level reaches zero. This also results into the random cycle length. In section 4 we compare these two models and also compare them with classical EOQ model.

2. Model with deterministic discrete uniform demand.

In this section, we investigate inventory model where the demand is discrete and holding cost incurs only for the period during which the inventory items are in the stock. In classical EOQ model the demand is assumed to be continuous and occurring at uniform rate over the period. This results is a linearly decreasing inventory level (as shown in figure 2(a)), until the reorder is placed. When the demand is assumed to be a discrete, the inventory level decreases as shown in figure 2(b)[2].

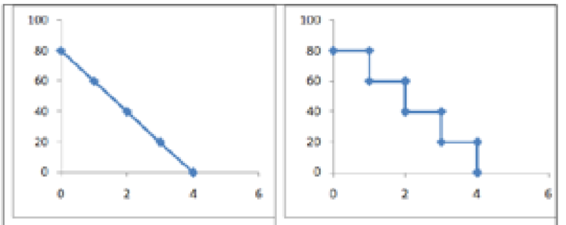

Fig 2(a) Fig 2(b)

The following are the basic assumptions of the proposed model.

1. Demand is discrete occurring at constant rate.
2. Supply is instantaneous. / Lead time is zero.
3. Reorder is placed as soon as inventory level reaches zero.
4. Holding cost is incurred only for the period during which the inventory item is in the stock.
5. The replenishment rate is constant.

Following notations are used in this section.

t : Cycle length

C_0 : Ordering cost per order.

C_h : Holding cost per unit per unit time.

C : Cost of inventory per unit

n : Lot size. (Also the Initial inventory level)

D : Demand rate per unit time.

Next we compute the total cost of inventory per unit time which will be minimized with respect to lot size in order to obtain the optimum lot size. .

Total cost of inventory for one cycle is computed as

Ordering cost $= C_0$ ... (2.1)

Cost of inventory $= nC$ ... (2.2)

Total holding cost $= DC_h + 2DC_h + \cdots + (t-2)DC_h + (t-1)DC_h + tDC_h)$

$= DC_h \frac{t(t+1)}{2}$ ... (2.3)

Thus the total cost per unit time is

$TC(n) = \frac{C_0}{t} + DC + DC_h \frac{(t+1)}{2}$ ... (2.4)

The optimal lot size is obtained by solving the following inequalities.

$(TC(n) \le TC(n-1)$ as well as

$TC(n) \le TC(n+1)$

Equation (2.4) implies that

$t(t-1) \le \frac{2C_0}{DC_h} \le t(t+1)$

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Therefore, for obtaining optimal value t*, we solve

$$t(t-1) = \frac{2C_0}{DC_h}$$

And take t* as the largest integer smaller than or equal to the solution.

$$\text{There for } t^* = \left\lfloor \frac{1+\sqrt{1+\frac{8C_0}{DC_h}}}{2} \right\rfloor \qquad \dots (2.5)$$

Hence $n = Dt^*$

$$= D\left\lfloor \frac{1+\sqrt{1+\frac{8C_0}{DC_h}}}{2} \right\rfloor \qquad \dots (2.6)$$

Is the optimal order quantity.

3. The model with Poisson demand

In most real world inventory systems, the demand is not found to be uniform, and the inventory level decrease as shown in figure 3(b)[3] instead of as shown in figure 3(a). This essentially requires that demand be modeled as a random process. In this section, we assume that customer arrival process is a Poisson process with each customer having demand of one unit. This leads to a random cycle length. This is contrary to the stochastic inventory models such as those described in [Hilliser & Liberman], where the cycle length is assumed to be fixed and total demand for a given cycle is taken as a random variable.

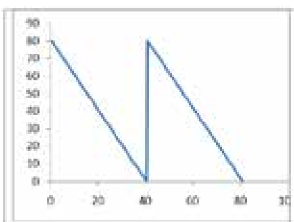

Figure 3(a)

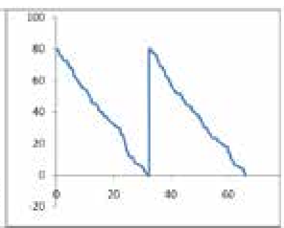

Figure 3(b)

The following are the basic assumptions of the model proposed in this section.

1. Customer arrival process is Poisson with each customer having demand of one unit.
2. Supply is instantaneous.
3. Reorder is placed as soon as inventory level reaches zero.
4. Holding cost is incurred only for the period during which the inventory item is in the stock.

Following notations are used in this section.

T: Cycle length

C_0: Ordering cost per order.

Ch: Holding cost per unit per unit time.

C: Cost of inventory per unit

n : Lot size.

$\frac{1}{\theta}$: Customer arrival rate.

Initially at time $t = 0$, the inventory level is raised to n units. Suppose 1[st] customer arrives at time t_1, and demand for 1 unit. So at time t_1, inventory level reduces to $(n-1)$. Similarly, 2[nd] customer arrives at time $t_1 + t_2$, and demand for 1 unit, at time $t_1 + t_2$, inventory level reduces to $(n-2)$ units, and so on.

At time $t_1 + t_2 + t_3 + \cdots + t_n = T$, inventory level becomes zero. At this time reorder is made and the second cycle starts. Clearly, the cycle length T is a random variable.

As the customer arrival process is a Poisson process, the inter-arrival times $t_1, t_2, t_3, \dots t_n$ are identically and independently distributed (iid) exponential random variables.

Since, the arrival rate of the arrival process is $\frac{1}{\theta}$,

$t_i \sim Exp(\theta)$, i=1, 2, …, n.

Then the expected value of Cycle length T = Σt_i is given by

$E(T) = E(t_1 + t_2 + t_3 + \cdots + t_n) = n\theta$

In order to obtain the optimal order quantity, we minimize the expected total inventory cost per unit time. The total inventory cost for one cycle is given by

Total cost = Ordering cost + Inventory cost + Holding cost

Ordering cost $= C_0$ … (3.1)

Cost of inventory $= nC$ … (3.2)

Total holding cost $= C_h(n\,t_1 + (n-1)\,t_2 + (n-2)\,t_3 + \dots\dots + t_n)$ … (3.3)

Equations 3.1 – 3.3 imply that the total cost per unit time, when initial inventory level is n, is

$$TC(n) = \frac{C_0 + nC}{T} + \frac{nC_h t_1 + (n-1)C_h t_2 + (n-2)C_h t_3 + \dots\dots + C_h t_n}{T}$$

Hence expected total cost per unit time is

$$E\big(TC(n)\big) = (C_0 + nC)E\left(\tfrac{1}{T}\right) + nC_h\, E\left(\tfrac{t_1}{T}\right) + (n-1)C_h\, E\left(\tfrac{t_2}{T}\right) + \cdots + C_h\, E\left(\tfrac{t_n}{T}\right).$$

Note that $t_i \sim Exp(\theta)$, and $T = \sum_{i=0}^{n} t_i \sim Gamma(n,\theta)$

Now, $E\left(\frac{t_i}{T}\right)$ is computed as

$$E\left(\frac{t_i}{T}\right) = E(E\left(\frac{T_i}{T} \mid T = t\right)$$

It can be shown that the conditional distribution of $t_i|T = t$ is given by p.d.f.

$$f_{(ti|T=t)}(y) = \frac{n-1}{t}\left(1 - \frac{y}{t}\right)^{n-2} \;;\; 0 < y < t$$

Thus, we have

$E(t_i|T = t) = \frac{t}{n}$.

This, further, implies that

$$E\left(\frac{t_i}{T}\right) = E\left(\frac{1}{T}\right)E(t_i|T = t) = E\left(\frac{1}{T}\frac{T}{n}\right) = E\left(\frac{1}{n}\right) = \frac{1}{n} \qquad \dots (3.4)$$

Also, for the Gamma random variable T, we have

$$E\left(\frac{1}{T}\right) = \frac{1}{\Gamma n}\int_0^{\infty} \frac{\frac{1}{t}e^{-t/\theta}}{\theta^n} t^{n-1}dt = \frac{1}{\theta(n-1)} \qquad \dots (3.5)$$

From (3.4) and (3.5),

$$E(TC(n)) = (C_0 + nC)\,E\left(\tfrac{1}{T}\right) + C_h\, E\left(\tfrac{t_i}{T}\right)(n(n-1)(n-2)\cdots 1)$$

$$= (C_0 + nC)\,\frac{1}{\theta(n-1)} + C_h\,\frac{1}{n}\left(\frac{n(n+1)}{2}\right)$$

$$E\big(TC(n)\big) = \frac{(C_0 + nC)}{\theta(n-1)} + C_h\,\frac{(n+1)}{2} \qquad \dots (3.6)$$

An optimal value of n is the one that minimizes E(TC(n)). This is the smallest value of n that satisfies

E(TC(n)) ≤ E(TC(n − 1)) as well as

E(TC(n)) ≤ E(TC(n − 1))

Equation (3.6) implies that the optimal value of n is obtained by solving the inequality

$$(n-1)(n-2) \le \frac{2(C_0+C)}{\theta C_h} \le n(n-1)$$

Therefore, for obtaining optimal value n, we solve

$$n(n-1) = \frac{2(C_0+C)}{\theta C_h}$$

and take n^* as the smallest integer greater than or equal to the solution.

There for $$n^* = \left\lceil \frac{1+\sqrt{1+\frac{8(C_o+C)}{\theta C_h}}}{2} \right\rceil \qquad \ldots (3.7)$$

4 An Example

Suppose a company stocks an item that is consumed at the rate of θ = 0.05 per unit time. The holding cost per unit per unit time is Rs.10, ordering cost is Rs.100 per order. Suppose shortages are not allowed and the purchasing cost per unit is Rs.1500. Determine expected total cost and optimal quantity.

This example solved with the help of a computer.

$C_o = 100$

$C_h = 10$

$C = 1{,}500$

$a = \frac{1}{\theta} = 1/0.05 = 20$

Using the classical EOQ formula we get $n^* = 20$ resulting in E(TC) = 31789.4483

Using the model in sec II, we get $n^* = 32.36 \simeq 32$ resulting in E(TC) = 31197.2581

Using the model in sec III, we get $n^* = 80.501 \simeq 81$ resulting in E(TC) = 30810

From the result obtain for above example it is clear that the use of approximate EOQ model results in the lot size for which the expected total cost is higher than that for both the models proposed in this paper. Also the model proposed in sec III gives best result.

5. Conclusion:

The example discussed in this paper, clearly indicates that the demand process is Poisson the use of EOQ model as well as the model in sec II gives unsatisfactory result. Hence we suggest that instead of using approximate models the exact model proposed in sec III should be used to obtain optimal results.

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