Volume : 2 | Issue : 6 | June 2013 • ISSN No 2277 - 8179

Research Paper

Ion Acoustic Solitary Waves in a Negative Ion Beam Plasma in Presence of Electron Inertia

Mathematics

KEYWORDS : Electron inertia, Solitary waves, Negative ion beam, Soliton.

R. Das	Department of Mathematics, Arya Vidyapeeth College, Guwahati –781016, Assam, India
K. Karmakar	Department of Mathematics, Don Bosco College Tura, Meghalaya –794002, India

ABSTRACT

Ion acoustic solitary waves have been investigated in a three component plasma consisting positive ions, negative beam ions and electrons. Existence of both compressive and rarefactive solitons is studied through perturbative technique. The amplitude of compressive soliton decreases rapidly with an increase of $Q'=\frac{m_b}{m_i}$ *(= beam to ion mass ratio). Further, only rarefactive solitons are found to exist in* $0.1 \le Q' < 1$, $0.3 \le \gamma \le 0.8$ *and* $0.1 \le \alpha \le 0.8$ *(where* $\gamma=\frac{V_b}{U}$ *= beam to phase velocity ratio and* $\alpha=\frac{n_{b0}}{n_{i0}}$ *= beam to ion equilibrium density ratio).*

INTRODUCTION

The propagation of non-linear ion acoustic waves in different types of plasma has been investigated extensively over the last several decades, starting from the work of Washimi and Taniuti [1]. The experimental investigations on the formation of solitons, carried out by Ikezi et al. [2] and Ikezi [3] are well appreciated by researchers. Theoretical studies on the process of heating and amplification of the linear ion acoustic waves by an ion beam were pursued by Gresillon and Doveil [4], Ostrovoski et al. [5] and the experimental part was performed by Okutso et al. [6]. Heating of Plasma to generate various characteristic aspects of it is a different branch of study. Gell and Roth [7] concentrated their studies on the effects of an ion beam on the characteristics of ion acoustic solitons. The investigations on the effects of ion beam on the propagation of ion acoustic solitons in an electron beam-plasma system with trapped and free electrons are the works of Abrol and Tagore [8, 9].

Among the several workers, Uberoy and Das [10] have shown that the execution of ion acoustic waves in plasma is dramatically influenced by the negative ions. They observed that in a plasma with cold or warm ions, a wave is broken into a number of solitons by the influence of the negative ions. The negative ions also play a major role in the formation of ion acoustic solitons and this was theoretically studied by Watanabe [11] and experimentally observed by Ludwig et al. [12]. Burger [13] has investigated negative ion beam-plasma system for the determination of H^- concentration. Tagare [14] has investigated modified KdV solitons with isothermal electrons while Tagare and Reddy [15] have studied the case with non isothermal electrons in a plasma with negative ions. Kalita and Kalita [16] have established the existence of modified KdV solitons only for negative to positive ion mass ratio $Q'>1$. Nakamura and Tsukayabashi [17] have made the observation on modified KdV solitons in a plasma with negative ions. The existence of a functional relation between the amplitude of the solitons and the density of the negative ions at finite ion temperature was observed by Nakamura [18]. Baboolal et al. [19] have investigated the role of negative ions in a plasma with double Maxwellian electrons, positive ions and negative ions. Their work also includes the establishment of the existence pattern for solitons and double layers by the three way interactions between the twin electrons and the negative ion component. An et al. [20] experimentally investigated low frequency lower-hybrid (LH) ion-ion mode in a plasma consisting of K^+ positive ions, SF_6^- negative ions and electrons. Kalita et al. [21] have investigated the existence of ion acoustic solitons in a magnetized ion beam plasma. Islam et al. [22] have discussed the ion acoustic solitary waves in magnetized plasma consisting of non-thermal and isothermal electrons.

The role of electron inertia in most of the investigations in plasmas is ignored for simplicity. But Kalita et al. [23] have considered the effect of electron inertia in the formation of solitary waves in a plasma, subject to its drifting effect $v_d^2 < 44.72 + \frac{M}{k_z}$ (M-mach number and k_z direction of propagation). Besides, Kalita and Das [24] have reported the existence of both compressive and rarefactive Korteweg de Vries (KdV) solitons of high amplitude in a warm plasma with electron inertia under the influence of electrons' drift motion for different values of Q' $(=m_j/m_i$, negative to positive ion mass ratio) greater or less than one. Kalita and Barman [25] have investigated the existence of ion acoustic solitons in a magnetized ion-beam plasma under the influence of electron inertia and constant ion-beam drift. Recently Das [26] has established the existence of both compressive and rarefactive solitons in a magnetized plasma model consisting of ions, electrons and positive ion beams using the Korteweg-de Vries (KdV) equation. In this investigation, both fast and slow modes are shown to exist due to the presence of ion temperature in the plasma.

BASIC EQUATIONS AND DERIVATION OF KdV EQUATION

We consider the propagation of ion-acoustic waves in a plasma with positive ions, beam ions and electrons. The fluid equations of motion, governing the collisionless plasma in one dimension are:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_i)=0 \quad (1)$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}+\frac{\partial \phi}{\partial x}=0 \quad (2)$$

$$\frac{\partial n_b}{\partial t}+\frac{\partial}{\partial x}(n_b v_b)=0 \quad (3)$$

$$\frac{\partial v_b}{\partial t}+v_b\frac{\partial v_b}{\partial x}=\frac{1}{Q'}\frac{\partial \phi}{\partial x} \quad (4)$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}(n_e v_e)=0 \quad (5)$$

$$\frac{\partial v_e}{\partial t}+v_e\frac{\partial v_e}{\partial x}=\frac{1}{Q}\left(\frac{\partial \phi}{\partial x}-\frac{1}{n_e}\frac{\partial n_e}{\partial x}\right) \quad (6)$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e+\frac{\alpha}{1-\alpha}n_b-\frac{1}{1-\alpha}n_i \quad (7)$$

where i, b and e stand for positive ion, negative ion beam and electron respectively, $Q=\frac{m_e}{m_i}$ (= electron to ion mass ratio), $Q'\left(=\frac{m_b}{m_i}\right)$ is the beam to ion mass ratio and $\alpha\left(=\frac{n_{b0}}{n_{i0}}\right)$ is the beam to ion equilibrium density ratio.

We have normalized densities n_i, n_e and n_b by the unperturbed densities n_{e0}, time t by the inverse of the characteristic ion plasma frequency i.e., $\omega_p^{-1}=\left(\frac{m_i}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, distance x by the electron Debye length $\lambda_D=\left(\frac{k_b T_e}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, velocities by the ion-acoustic speed $C_s=\left(\frac{k_b T_e}{m_i}\right)^{\frac{1}{2}}$, and the potential f by $\frac{k_b T_e}{e}$; k_b is the Boltzmann constant.

For deriving the KdV equation , we introduce the stretched variables

$$\xi=\varepsilon^{\frac{1}{2}}(x-U),\ \tau=\varepsilon^{\frac{3}{2}}t \quad (8)$$

Volume : 2 | Issue : 6 | June 2013 • ISSN No 2277 - 8179

where U is the phase velocity of the ion-acoustic wave in (x, t) space and ε is a small dimensionless expansion parameter.

We expand the flow variables asymptotically about the equilibrium state in terms of the parameter ε as follows:

$$n_i = 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \dots ,$$

$$n_b = 1 + \varepsilon n_{b1} + \varepsilon^2 n_{b2} + \dots ,$$

$$n_e = 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \dots , \; v_i = \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \dots , \quad (9)$$

$$v_b = V_b + \varepsilon v_{d1} + \varepsilon^2 v_{d2} + \dots ,$$

$$v_e = \varepsilon v_{e1} + \varepsilon^2 v_{e2} + \dots , \; \phi = \phi_1 + \varepsilon^2 \phi_2 + \dots\dots$$

With the use of the transformation (8) and the expansion (9) in the normalized set of equations (1) – (7), we get the expression for the phase velocity and the KdV equation as follows-

$$U^2 = \frac{\alpha + Q'(1-\gamma)^2}{(1-\alpha)Q'(1-\gamma)^2 + Q\{\alpha + Q'(1-\gamma)^2\}} \quad (10)$$

where $\gamma = \frac{V_b}{U} = \frac{beam \;\; velocity}{phase \;\; velocity}$

$$= 1 \pm \left[\frac{\alpha(1-QU^2)}{Q'\{(1-\alpha)U^2 + QU^2 - 1\}} \right]^{\frac{1}{2}}$$

and

$$\frac{\partial \phi_1}{\partial \tau} + p\phi_1 \frac{\partial \phi_1}{\partial \xi} + q \frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \quad (11)$$

where $p = \frac{A}{2B}$ and $q = \frac{1}{2B}$

with

$$+ \frac{3}{(1-\alpha)U^4}$$

and

$$+ \frac{1}{(1-\alpha)U^3}$$

It is observed that the nonlinear ion-acoustic solitons, in this model of plasma, exist when $U^2 \neq \frac{1}{Q}$.

SOLITARY WAVE SOLUTION

Using the transformation $\chi = \eta - V\tau$, the KdV equation (11) can be simplified to give the solitary wave solution as

$$\phi_1 = \frac{3V}{p} \sec h^2 \left(\frac{1}{2} \sqrt{\frac{V}{q}} \chi \right)$$

where V is the velocity with which the solitary waves travel to the right.

Thus, the wave amplitude of the soliton is given by $\phi_0 = \frac{3V}{p}$ and the corresponding width by $\Delta = 2\sqrt{\frac{q}{V}}$.

DISCUSSION

In this unmagnetised plasma under consideration, both compressive and rarefactive solitons of interesting characters are found to exist for fixed $Q = 0.00054$ depending on Q', V, γ and α. The amplitude of the compressive fast ion acoustic solitons exist only for smaller values of $\gamma(<1)$, $V(=0.05)$ and $\alpha(=0.01, 0.02, 0.03)$ [Fig. 1 a] and its amplitude increases as γ increases. On the other hand, rarefactive fast ion acoustic solitons exist [Fig. 1 a] only in the upper regime of $\gamma(<1)$ for the same set of smaller values of V and α. However, the character of the fast compressive solitons changes to fast rarefactive solitons after certain $\gamma^*(<1)$ characterizing an unaccountable regime. The corresponding widths [Fig. 1 b] of the fast (compressive and rarefactive) ion acoustic solitons decreases with the increase of $\gamma(<1)$. The figure 1 b further shows that this increase is monotonic in the lower regime but strict in the upper regime of γ. The amplitude [Fig. 2 a] of the fast ion acoustic compressive solitons for different values of α is found to decrease in the lower regime of $\gamma(>1)$ and almost remains constant in the upper regime of $\gamma(>1)$ for $V = 0.0$ and $Q' = 2$. The corresponding widths [Fig. 2 b] of the fast compressive ion acoustic solitons show the same pattern. Besides, their amplitudes and widths are found to increase with α for fixed γ. For higher value of $V = 0.0$, the amplitude [Fig. 3 a]of the fast ion acoustic rarefactive soliton increases (numerically) with Q'. Moreover, the magnitude of the amplitudes gets comparatively smaller when γ increases. The corresponding widths [Fig. 3 b] of the fast compressive ion acoustic solitons shows an interesting behaviour. For $\gamma = 0.7$ though the width initially tend to decrease upto certain value of Q', it increases again after crossing that value at Q'. For $\gamma = 0.8$, the width decreases initially upto certain value Q' and then remains almost constant for remaining values of Q'. On the contrary for $\gamma = 0.9$ the width decreases strictly for the initial values of Q' and then it further decreases monotonically for remaining values of Q'. For $Q'(\geq 1)$ with lighter ion beam mass, it is seen that compressive solitons of high amplitude [Fig. 4 a] is produced in the smaller regime of Q' whereas it remains almost constant in the upper regime of $Q'(\geq 1)$ of course with smaller amplitudes. The corresponding widths [Fig. 4 b] of the fast compressive ion acoustic solitons (for fixed $V = 0.0$ and $\alpha = 0.0$) for $Q'(\geq 1)$ is noticed to increase sharply for $\gamma = 0.6$ and uniformly for $\gamma = 0.4, 0.5$ in the lower regime of $Q'(\geq 1)$. Interestingly, the amplitude [Fig. 5 a] of the rarefactive solitons increases linearly with $Q'(\geq 1)$ for higher $\gamma = 0.9$ and uniformly for smaller $\gamma = 0.7, 0.8$ in the lower regime of Q'. On the other hand, the amplitude [Fig. 5 a] of the compressive solitons decreases uniformly in the upper regime of $Q'(>1)$. The corresponding widths [Fig. 5 b] of the (compressive and rarefactive) solitons increases uniformly with Q' for $\gamma = 0.7, 0.8$ and almost linearly for $\gamma = 0.9$ of course with slight decrease for some values of Q' in the beginning.

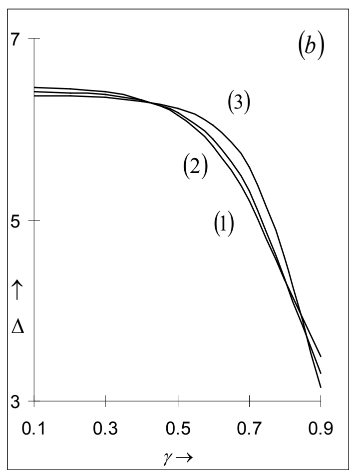

Figure1. Amplitudes (a) and widths (b) of compressive and rarefactive fast ion-acoustic solitons versus γ for fixed $V = 0.0$ and $Q' = 0.5$ for different values of $\alpha = 0.0$ (1), 0.0 (2), 0.0 (3).

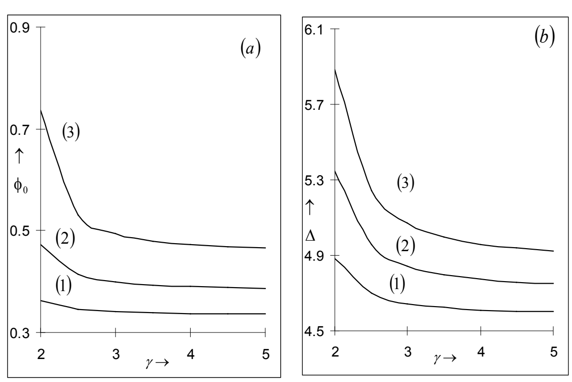

Figure 2. Amplitudes (a) and widths (b) of compressive fast ion-acoustic solitons versus γ for fixed $V = 0.0$ and $Q' = 2$ for different values of $\alpha = 0.1$ (1), 0.2 (2), 0.3 (3).

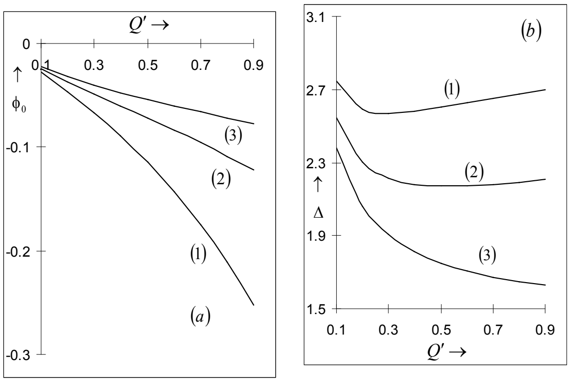

Figure 3. Amplitudes (a) and width (b) of compressive fast ion-acoustic solitons versus Q' for fixed $V = 0.2$ and $\alpha = 0.3$ for different values of $\gamma = 0.7(1)\ 0.8(2),\ 0.9(3)$.

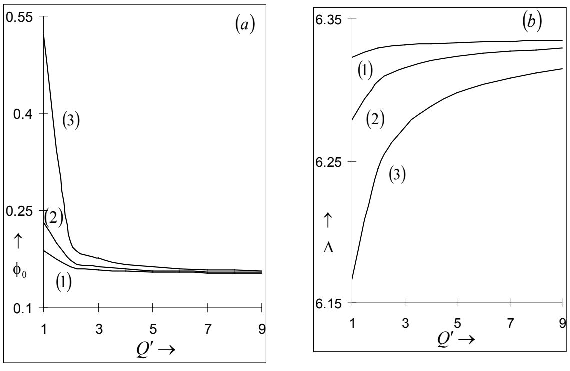

Figure 4. Amplitudes (a) and widths (b) of compressive fast ion-acoustic solitons versus Q' for fixed $V = 0.6$ and $\alpha = 0.0$ for different values of $\gamma = 0.4(1),\ 0.5(2),\ 0.6(3)$.

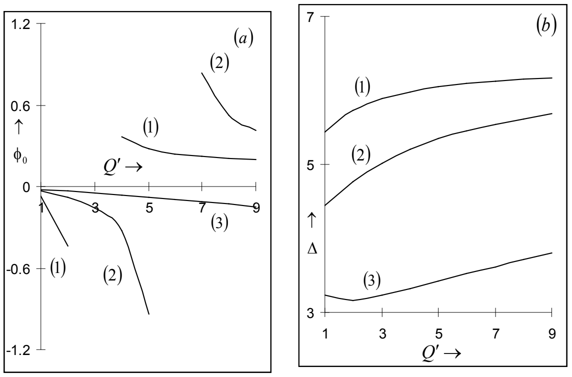

Figure 5. Amplitudes (a) and width (b) of compressive and rarefactive fast ion-acoustic solitons versus Q' for fixed $V = 0.6$ and $\alpha = 0.3$ for different values of $\gamma = 0.7(1),\ 0.8(2),\ 0.9(3)$.

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