Research Paper

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Third Ordered Transformations of Ramanujan Type Series for Pi

Mathematics

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ABSTRACT

Srinivasa Ramanujan had established several hypergeometric series for the number 1/π out of which 17 of them were sent to Hardy (1913) through his letter in his first letter. This article is to give a general transformations for his series of order 3(order defined by me) and using that establish many new hypergeometric series converging to 1/π.

Notations and definitions –
The general form of Ramanujan series for is

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} \frac{x^n * (\frac{1}{2})_n (s)_n (1-s)_n \, (a+bn)}{(n!)^3}$$

Here x, s ,a , $\frac{\Gamma(s+n)}{\Gamma(s)}$ a constants defined by Ramanujan.
Here (s)n = $\frac{\Gamma(s+n)}{\Gamma(s)}$ Where and is the Euler Gamma function.

Ramanujan generally kept s= $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ which I call of order 3, 4 and 6 respectively.

Aim – The main aim of this paper is to establish several new formulas of through transformation of existing Ramanujan type formulas.

Main Content – Assume that we have Ramanujan type formula

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} \frac{x^n * a_n \, (a+bn)}{(n!)^3}$$

And here = $a_n = (\frac{1}{2})_n (s)_n (1-s)_n$

We make a substitution of another but a specific type of numbers A and B with another function and a constant M. Such that the previous series is transformed into –

$$\sum_{n=0}^{\infty} A_n (A+Bn) \varpi^n$$

Such that

$$A = \left(\frac{b*M}{2} + \frac{a(1-M)}{1}\right)(1-M)^{\frac{-1}{2}}$$

$$B = b(1-M)^{\frac{-3}{2}}$$

$$\varpi = \left(\frac{x}{1-M}\right)^n$$

For this transformation to work we need an appropriate value of M and the function .

Main Series –
Let M= 108 and

$$A_n = 108^n \sum_{k=0}^{n} \binom{\left(\frac{-2}{3}\right)}{k} \binom{\left(\frac{-1}{6}\right)}{k} \binom{\left(\frac{-1}{3}\right)}{n-k} \binom{\left(\frac{-5}{6}\right)}{n-k}$$

$$s_n = \frac{1}{3}$$

Thus this transform can be considered as an third order series.
Now a series of Is transformed into Where is as defined earlier .

Here the following values converge when transformed accordingly.

No	x	a	b	ϖ	A	B
1	$\frac{-1}{192}$	$\frac{\sqrt{3}}{4}$	$\frac{5\sqrt{5}}{4}$	$\frac{1}{300}$	$\frac{\sqrt{3}}{50}$	$\frac{16\sqrt{3}}{25}$
2	$\frac{-1}{1728}$	$\frac{7\sqrt{3}}{36}$	$\frac{17\sqrt{3}}{12}$	$\frac{1}{1836}$	$\frac{11\sqrt{51}}{306}$	$\frac{48\sqrt{51}}{153}$
3	$\frac{-1}{8640}$	$\frac{\sqrt{15}}{12}$	$\frac{3\sqrt{15}}{4}$	$\frac{1}{8748}$	$\frac{85\sqrt{3}}{486}$	$\frac{400\sqrt{3}}{243}$
4	$\frac{-1}{108*2^{10}}$	$\frac{53\sqrt{3}}{288}$	$\frac{205\sqrt{3}}{96}$	$\frac{1}{110700}$	$\frac{527\sqrt{123}}{18450}$	$\frac{3072\sqrt{123}}{9225}$
5	$\frac{-1}{108*3024}$	$\frac{13\sqrt{7}}{108}$	$\frac{55\sqrt{7}}{36}$	$\frac{1}{326700}$	$\frac{9989\sqrt{3}}{54450}$	$\frac{127008\sqrt{3}}{54450}$
6	$\frac{-1}{108*500^2}$	$\frac{827\sqrt{3}}{4500}$	$\frac{4717\sqrt{3}}{1500}$	$\frac{1}{27000108}$	$\frac{97659\sqrt{267}}{4500018}$	$\frac{1500000\sqrt{267}}{4500018}$
7	$\frac{1}{1458}$	$\frac{8}{27}$	$\frac{20}{9}$	$\frac{-1}{1350}$	$\frac{52\sqrt{3}}{225}$	$\frac{36\sqrt{3}}{25}$

Conclusion –
We conclude that Ramanujan type formulas can be still be extended by transformation of themselves and still not losing their convergence rates.

REFERENCE

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