

Theory and Geometry Comparison Between Inclined Free and Over-Sill Hydraulic Jumps



Engineering

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ABSTRACT

In this experimental study a comparison is presented between theory (onedimensional flow equations) and geometry (free surface water profiles) concerning the free and over - sill inclined (angle φ , $0 \leq \varphi \leq 140$) hydraulic jumps. A number of experimental results concerning the water profiles for the jump over - sills are dimensionlessly elaborated, showing that all these profiles are systematic. The comparison to the free jump profiles is based on a previous paper by the first author and shows that both corresponding pairs of profiles are lowering when angle φ is increasing, and that for larger inclination angles the respective profiles become almost identical.

INTRODUCTION

The steady inclined hydraulic jump has a considerable interest when designing open channels or small dam outflows. Although the jump has various forms, two particular jumps are examined and compared here, both from the point of view of theory (one dimensional equations) and geometry - especially in relation to their water free surface profiles.

Figs. 1a and 1b show the pertinent basic characteristics of the above hydraulic jumps, both the free jump and the weak jump - over a sill. The free jump has a length L and two conjugate depths d_1, d_2 (uniform flow cross sections), in a channel with an inclination angle φ (and slope $J_0 = \sin\varphi$), while the discharge (per unit width) is q and the typical water depth is d at a distance x . The main parameter is the Froude number $Fr_1 = q/g^{1/2} \cdot d_1^{3/2}$, which is larger than 1 (supercritical flow).

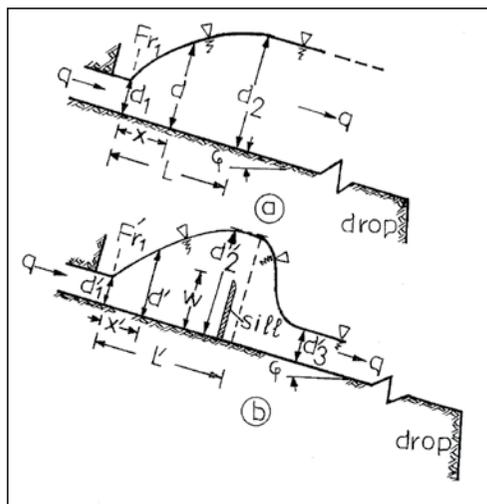


Figure 1: basic flow characteristics.

The jump over - sill (w) is more complicate. It has a minimum upstream water depth d_1' (uniform flow cross section), a maximum depth d_2' (no uniform flow cross section and not always over w - but including w), and it ends up to a uniform flow cross section with water depth d_3' , while $Fr_1' = q/g^{1/2} \cdot (d_1')^{3/2} (>1)$. At any distance x' the upstream water depth is d' , while the entire length of the ascending jump is L' . The profiles of the jump over - sill usually have

two shapes, one (upstream) shown in Fig. 1b (similar to the profile of the free jump) and another downstream (not shown here) which is a steady wave - like profile. In this study only the profile of Fig. 1b is examined and compared to the profile of Fig. 1a, since they have similar shapes.

Demetriou, 2005, [1], has experimentally examined the water surface profile of the free inclined jump, while the experimental profile results of the inclined jump over - sill are elaborated here, all in dimensionless terms.

THE ONEDIMENSIONAL EQUATIONS

The continuity equations along the flow (x, x') are, for the free jump (Fig. 1a) $q = V_1 \cdot d_1 = V_2 \cdot d_2$, and for the jump over - sill, $q = V_1' \cdot d_1' = V_3' \cdot d_3'$.

Since the jump over - sill is more complicated, the onedimensional momentum equation for this jump is first examined between d_1' and d_3' ,

$$P_x + W_t \cdot J_0 - G_x = \rho \cdot q^2 \cdot [(1/d_3') - (1/d_1')]$$

where, G_x = total force from the boundaries, while per unit channel width,

$$P_x = 0.5 \cdot \gamma \cdot [(d_1')^2 - (d_3')^2] \cdot \cos\varphi = \text{total pressure force along } x',$$

$$W_t = \text{total water weight under the flow profile} = K_w \cdot P_x,$$

$$G_x = N_x + F_x = K_G \cdot P_x,$$

$$N_x = \text{force exerted by the sill along } x',$$

$$F_x = \text{tractive force from the boundaries,}$$

$$(K_w, K_G) = \text{dimensionless weight and } G_x \text{ force coefficients,}$$

$$\gamma = \text{specific water weight.}$$

If the Froude number Fr_1' and

$$\lambda = d_3' / d_1', K_w \cdot P_x, K_G \cdot P_x,$$

are introduced, then the equation

$$\lambda^2 \cdot (T \cdot \cos\varphi) + \lambda \cdot (T \cdot \cos\varphi) - 2 \cdot (Fr_1')^2 = 0, \quad (1)$$

where $T = 1 + K_w \cdot J_0 - K_G$,

is received.

Eq. (1) may suitably be solved

$$\lambda = 0.5 \cdot \left[\left(1 + 8 \cdot \frac{(Fr_1')^2}{T \cdot \cos\phi} \right)^{1/2} - 1 \right] \quad (2)$$

For the jump without sill ($w=0$ -free jump), $N_x=0$, $G_x=F_x$, $d_1'=d_1$, $d_3'=d_2'=d_2$, $Fr_1'=Fr_1$, $L'=L$, $x'=x$, W_t =water weight between d_1 and d_2 , $\lambda=d_2/d_1$, eq. (2) is adjusted to

$$\lambda = 0.5 \cdot \left[\left\{ 1 + 8 \cdot \frac{F_1'^2}{(1 + K_w \cdot J_o - K_G) \cdot \cos\phi} \right\}^{1/2} - 1 \right] \quad (3)$$

while for horizontal channels ($K_w \cdot J_o = 0$, $F_x=0$, $K_G=0$, $\cos\phi=1$),

$$\lambda = d_2/d_1 = 0.5 \cdot \left[\left(1 + 8 \cdot F_1'^2 \right)^{1/2} - 1 \right]$$

which is the well known conjugate depths' equation for the classical free hydraulic jump.

Based on the previous comparison it can be seen that the above equations are similar, i.e. one comes from the other.

In eq. (2) d_1' (Fr_1'), d_3' and λ , K_w , should be experimentally measured in order that K_G (or G_x) is indirectly determined, since G_x is rather difficult to be directly measured.

THE EXPERIMENTS

The experimental results for the inclined free jump profiles were presented by Demetriou, 2005, [1], for $2 \leq F_1' \leq 19$, $0^\circ \leq \phi \leq 16^\circ$.

For the hydraulic jump over - sill a number of measurements were performed. A large number of d_1' , d' , d_2' , L' and q

were used, with the sill (heights $w=1, 3, 5$ cm) in various places from d_1' with $0.14 \leq d_1'/w \leq 1$. Fr_1' reached values more than 9,

all flows were fully turbulent, realized within a perspex tilting small channel with $\phi=0^\circ-3^\circ-6^\circ-9^\circ-12^\circ-14^\circ$. In all cases the flows were organized to be steady and freely developing, while no considerable air pockets were observed in the downstream faces of the sills. All hydraulic jump depths were completely recorded and all profiles were reconstructed later on paper.

All the above over - sill jumps were organized to be rather weak (small Fr_1') since a thin sill cannot withstand strong jumps.

JUMP PROFILES

In order to dimensionalize the profile results the terms

$$\bar{d}' = (d' - d_1') / (d_2' - d_1') \text{ and } \bar{x}' = x' / L'$$

were calculated in the field $0 \leq \bar{x}' \leq 1$, for all the inclined over - sill jumps. In this way L' and Fr_1' were also included in the analysis since L' is a function of Fr_1' , while w is included in d_2' .

Figs. 2 and 3 show the laboratory results in the form of clouds of experimental points through which a number of systematic profile curves are traced, for $\phi=0^\circ-6^\circ-12^\circ$ and $\phi=3^\circ-9^\circ-14^\circ$ correspondingly.

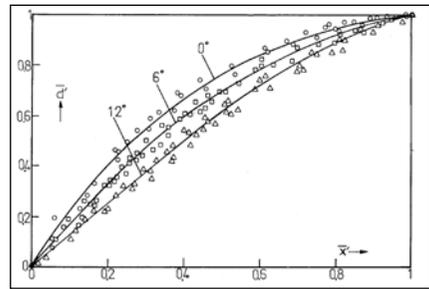


Figure 2: profiles for $\phi=0^\circ-6^\circ-12^\circ$.

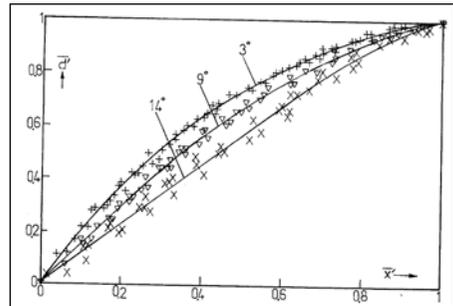


Figure 3: profiles for $\phi=3^\circ-9^\circ-14^\circ$.

Based on the above jump diagrams the general profile equation is suitably received,

$$\bar{d}' \cong (7.48 - 4.76 \cdot e^{J_o}) \cdot (\bar{x}') - (6.48 - 4.76 \cdot e^{J_o}) \cdot (\bar{x}')^{1.5} \quad (4)$$

which gives $\bar{d}' = 0$ ($d' = d_1'$) for $\bar{x}' = 0$ and $\bar{d}' = 1$ ($d' = d_2'$) for $\bar{x}' = 1$.

For the inclined free hydraulic jump, Demetriou, 2005, [1], has given an equation, in terms of $\bar{d} = (d - d_1) / (d_2 - d_1)$ and $\bar{x} = x / L$,

$$\bar{d} \cong (3.3 - 8.1 \cdot J_o) \cdot (\bar{x}) - (2.3 - 8.1 \cdot J_o) \cdot (\bar{x})^{1.5} \quad (5)$$

for $0 \leq J_o \leq 0.27$ and $1 \leq Fr_1' \leq 16$.

Figs. 4, 5, 6, 7, 8, 9, show a comparison between corresponding profiles (free and over - sill jumps) for $\phi=0^\circ-3^\circ-6^\circ-9^\circ-12^\circ-14^\circ$.

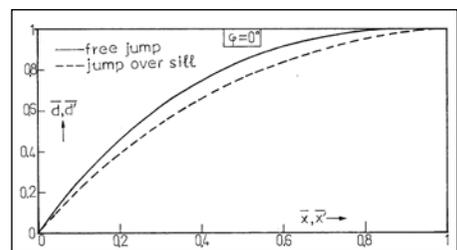


Figure 4: profile comparison for $\phi=0^\circ$.

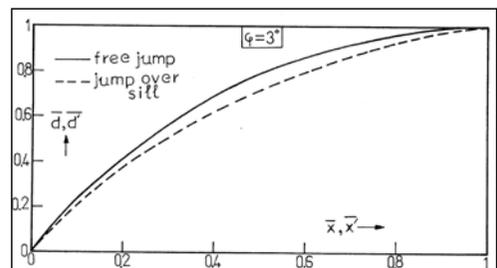


Figure 5: profile comparison for $\phi=3^\circ$.

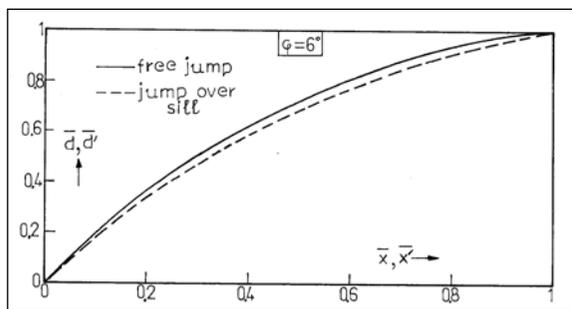


Figure 6: profile comparison for $\varphi=6^\circ$.

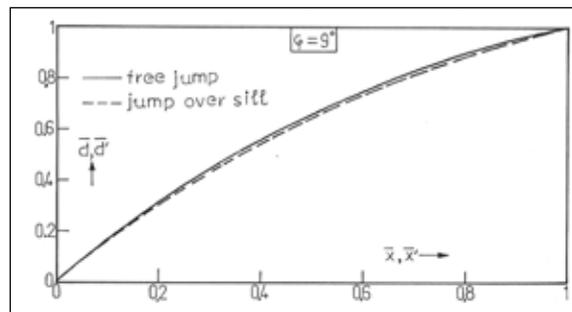


Figure 7: profile comparison for $\varphi=9^\circ$.

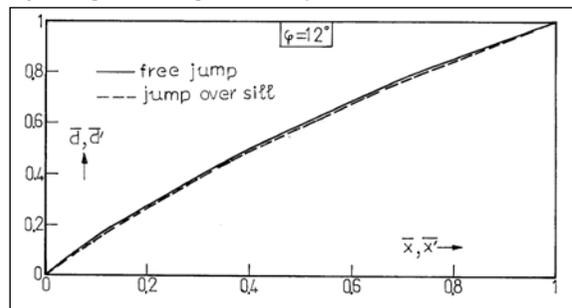


Figure 8: profile comparison for $\varphi=12^\circ$.

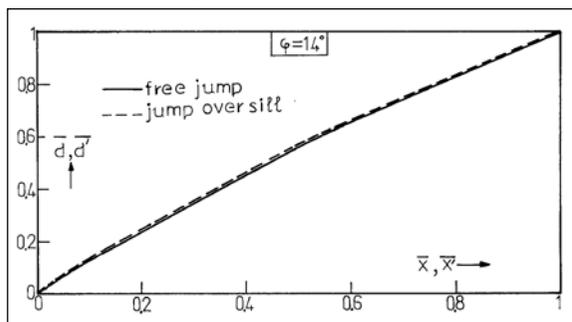


Figure 9: profile comparison for $\varphi=14^\circ$.

All figures show that the corresponding pairs of profiles are lowering when angle φ is increasing.

Fig. 4 shows that for $\varphi=0^\circ$ there is a discrepancy between corresponding profiles, for example for $\bar{x} = \bar{x}' = 0.5$ the free jump profile is above the jump over - sill profile at a percentage amount of about 11%.

In the next figures this discrepancy is decreasing (when φ is increasing) and for $\varphi=12^\circ$ and 14° the two corresponding profiles are almost coinciding.

The similarity of flow profiles does not mean that the jump over - sill is almost identical to the free jump, since L' and L , d'_2/d_1

and d_2/d_1 , are correspondingly quite different among them (because of w). These differences have further consequences in P_x , W_x , G_x , i.e. the two flow phenomena – although described by similar equations (2 and 3) – give quantitatively distinctive results, which are mainly due to the presence (or absence) of the sill. The similarity of flow profiles is mainly due to the dimensionless method which is followed here, and of course d (d')

may solved to d (d') to give the corresponding actual water depth at any place x/L (x'/L').

CONCLUSIONS

In this experimental study a comparison is presented between inclined (angle φ , $0^\circ \leq \varphi \leq 14^\circ$) free jump and jump over - sill free surface profiles. The corresponding onedimensional momentum equations are also compared as they are similar in relation to their structures, i.e. one comes from the other. A common method of dimensionless treatment concerning the free surface profiles is elaborated, followed by the presentation of the experimental flow profiles for all jumps over - sill. The previous profiles are also systematically compared to the corresponding profiles of the free jumps. The main conclusions are : (i) Corresponding onedimensional equations are similar, i.e. one comes from the other. (ii) The experimental measurements show a very systematic flow profiles' structure for the jump over - sills. (iii) When compared to corresponding free jump profiles, both profiles are lowering when angle φ is increasing. (iv) When angle φ is increasing the free jump and jump over - sill profiles are systematically coming closer among them, while for larger inclination angles they nearly coincide.

REFERENCE

[1]Demetriou, J.D., (2005), "Unique Length and Profile Equations for Hydraulic Jumps in Sloping Channnels". 17th Canadian Hydro Conference, Edmonton, Alberta, Canada, p.p. 891-898.