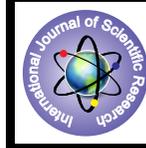


# Hypergraph Interference Models in Wireless Ad-Hoc Networks with Low Complexity Distributed Scheduling



## Engineering

**KEYWORDS :** Low Complexity Distributed Scheduling, hypergraph interference, Wireless Ad Hoc Network

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### ABSTRACT

*In this paper low complexity distributed scheduling approach is used in wireless Ad hoc networks to consider the conflicts caused by interference models. We explain the successful transmissions under any graph model can be improved by a hypergraph in the random network. The proposed one is different from the global signal-to- interference-plus-noise ratio (SINR) model, the hypergraph model protect a localized graph-theoretic structure and, therefore utilize the existing graph-based efficient scheduling algorithms to be extended to the cumulative interference container. In some networks, a hypergraph can double the uniform throughput compared to the disk graph. As an application of the propose models, we believe the performance of maximal scheduling, i.e., a low complexity distributed scheduling algorithm achieves accuracy of throughput, in wireless networks. We propose a Queuing Model to get average throughput that gives finest stability of the link pair. The scope of the project is to develop a hypergraph based model for the scheduling problem in wireless networks which is quite flexible in modelling sum interference constraints, which can include both the graph model and the physical SINR model as special cases thus achieving the low complexity scheduling in Wireless Ad Hoc Networks.*

### I. Introduction

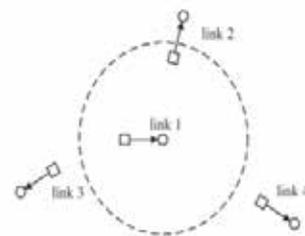
In wireless Ad Hoc network in particular vehicular adhoc network (VANETs) are plays vital role in the present transportation system [3]. The VANETs are formed spontaneously in between vehicles on the road. These VANETs delever both critical traffic accident or traffic jam messages [4] and useful commercial information [5] to the drivers using direct V2V communications.

To development any large scale wireless network like VANETs the design of MAC (Medium Access Control) mechanism is challenging one. This means the maximising the throughput of the network is key design goal in wireless network [2]. Tassiulas[4] Shah et al [4] proposed linear complexity randomized scheduling mechanism in that they achieve ed the maximum throughput region but it require centralized control. To designing a distributed scheduling strategy that achieves the throughput region in wireless adhoc network (WAN) is remained limited. Different WANs have considerably different interference limitations.

In this paper, we are try to address the low complexity distributed scheduling mechanism in VANETs. We consider the distributed algorithm for link scheduling in wireless networks. The main motive is the interfering links in VANET can't transmit at the same time, so that a scheduling strategy is needed to get better the latency, increase the network throughput and then get the higher energy efficiency. To understand the proposed scheduling algorithm, we need to provide a brief literature review on the previous scheduling mechanisms.

In past research based on the adopted interference model, in that we consider [5] the flow contention graph based scheduling (i.e., protocol interference model), and the physical signal-to-interference-plus-noise ratio (SINR) based (i.e., the physical interference model) scheduling.

The flow connection graph is used to express the interference between the links. In such a graph  $G=(V,E)$ , where  $V$  is vertices (node)set and  $E$  is edge(link) set, the two links  $i, j$  are accepted to share common resource only when  $(i, j)$  are not belongs to  $E$ . It is assumed that the interference is causes the failure of another link. This model is well known due to its simple structure for understanding and exists efficient graph algorithms in applications. For example consider the linked wireless network figure 1, here the flow connection graph can be constructed. Two links form an edge in the flow contention graph if one's transmitter is in the within region and associated with the other. In this case, in the Figure1of the flow contention



**Figure 1. A Sample wireless network with four links, where square nodes are the transmitters, and round nodes are the receivers. The dashed circle is the guard zone associated with link 1.**

only one edge  $\{1, 2\}$ . So that, a transmission schedule is applicable as long as links 1 and 2 are not chosen concurrently. Based on this approach of graph interference model, many interesting scheduling algorithms [6]–[9]. This is particularly convenient for the important class of low complexity distributed scheduling, such as maximal scheduling [6] and the longest queue first (LQF) scheduling [10], [11], where the throughput performance is often specified by the metrics of the underlying interference graph, such as the “interference degree” and the “local pooling factor.”

For example, in Figure 1, it is possible that link 1 fails when links  $\{1, 3, 4\}$  are scheduled due to the sum interference from both links 3 and 4. In such a case, the graph model can only guarantee that the transmission at link 1 is successful when only one of the other two links is transmitting due to its binary nature. On the other hand, if one conservatively builds the graph by increasing the size of the guard zone such that two additional edges  $\{1, 3\}$  and  $\{1, 4\}$  are included (note that both links 3 and 4 have the same distance to link 1 in this example), the network capacity is reduced, because when link 1 transmits, neither link nor link 3 is allowed to transmit, although there is no collision if only one of them transmits.

Our contribution is to characterize the maximum throughput attained by a distributed scheduling strategy under arbitrary network topologies and interference models. We consider the simple scheduling policy that is maximal scheduling, it ensures that if transmitter in Figure 2  $T$  has packed to transmit to a receiver  $R$  either  $(T,R)$  or transmitter and receiver pair that can't transmit simultaneously.

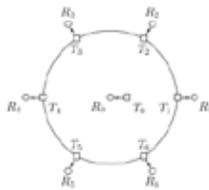


Figure 2. A “star” network with the transmitters of the 6 outside links lying on a hexagon centered at link 0’s receiver. The squares denote the transmitters, and the circles denote the receivers.

In this paper remaining sections are structured as follows. In Section II, we introduce the system model, terminology and in Section III, We characterize the maximal scheduling policy with hypergraph model for arbitrary wireless networks and in the Section IV describes the numerical analysis and finally in Section V we concludes this paper.

II. System Model

The network topology in a wireless network can be modelled as a directed graph  $G = (V,E)$ , where  $V$  and  $E$  denote the sets of nodes and links respectively. A link exists from one node ( $n_1$ ) to another node( $n_2$ ) if and only if  $n_2$  can receive  $n_1$ 's signals. The link set  $E$  depends on the transmission power levels of nodes and the propagation conditions in different directions. We imagine a TDMA system, each time slot, the transmission of a scheduled link  $i$  is successful if and only if the signal to interference plus noise ratio (SINR) at its receiver satisfies the following limitation:

$$\frac{S_i}{N_i + \sum_{k \in E} (I_{ki})} \geq \lambda_i \quad \text{----- (1)}$$

Where  $S_i$  is the received signal at link  $i$ ,  $I_{ki}$  is the received interference from  $i$  to transmit link  $i$ ,  $N_i$  is the noise power and  $\lambda$  is the SINR threshold .

The following terminology is used throughout the paper.

**Definition 1:** A node  $i$  is a neighbour of a node  $j$ , if there exists a link from  $i$  to  $j$ , i.e.,  $(i,j) \in E$ . The degree of a node  $u$  is the number of links in  $E$  originating from or ending at  $u$ . The degree of a link  $e = (u,v)$  is defined as the sum of the degrees of  $u$  and  $v$ . The maximum link degree in  $G$ ,  $\delta_G$ , is the maximum degree of any link in  $E$ .

The out-degree of a node  $u$  is the number of links in  $E$  originating from  $u$ . The in-degree of a node  $u$  is the number of links in  $E$  ending at  $u$ . The directed degree of a link  $e = (u,v)$  is defined as the sum of the out-degree of  $u$  and in-degree of  $v$ . The maximum directed link degree in  $G$ ,  $\Delta_G$ , is the maximum directed degree of any link in  $E$ .

At the MAC layer, each session traverses only one link. In this section, we only consider single-hop sessions (generalization of our results to multi-hop sessions is discussed in next Sections). We allow multiple sessions to traverse the same link. If a session  $i$  traverses link  $(u,v)$  then  $u$  and  $v$  are  $i$ 's transmitter and receiver respectively, and the session is completely specified by the 3-tuple,  $(i,u,v)$ . Without loss of generality, we assume that every node in  $V$  is either the transmitter or the receiver of at least one session.

**Definition 2:** The interference hypergraph  $I^N = (V^N, E^N)$  of a network  $N$  is an undirected graph in which the vertex set  $V^N$  corresponds to the set of sessions in  $N$  and there is an edge between two vertices  $i$  and  $j$  if  $j \in S_i$ .

We illuminate these definitions through example . Now we describe the arrival process. We assume that at most  $\beta_{max} > 1$  pack-

ets arrive for any session in any slot. Let  $A_i(n)$  be the number of packets that session  $i$  generate in interval  $[0,n], i=1,2,\dots,N$ . The arrival process  $\{A_i(\cdot), i=1, \dots, N\}$  satisfies a strong Law of large Number(SLLN), thus existing non negative real numbers  $\lambda_i, i=1, \dots, N$  such probability is equal to 1.

$$\lim_{n \rightarrow \infty} \frac{A_i(n)}{n} = \lambda_i = 1, \dots, N. \quad (2)$$

**Definition 3:** The arrival rate of session  $i$  is  $\lambda_i, i = 1,\dots,N$ . The arrival rate vector  $\vec{\lambda}$  is an  $N$ -dimensional vector whose components are the arrival rates

**Definition 4:** The network is said to be stable if with probability 1,

$$\lim_{n \rightarrow \infty} \frac{D_i(n)}{n} = \lambda_i = 1, \dots, N. \quad (3)$$

**Definition 4:** The maximum throughput region  $\Lambda$  is the set of feasible arrival rate vectors. Note,  $\Lambda$  depends on the network  $N$ .

**Example 1:** Consider the network shown in Figure. 3. Consider a scheduling policy  $\pi_1$ , that serves session  $(t \bmod 9)+1$  in slot  $t$ , where “mod” is a modulo operator. Under  $\pi_1$ , each session  $i \in \{1,\dots,9\}$  can transmit at the rate of at most  $1/9$ . Thus, the throughput region of  $\pi_1, \Lambda_{\pi_1}$ , is characterized as follows:

$$\Lambda^{\pi} = \{(\lambda_1, \dots, \lambda_9) : \lambda_i \leq 1/9 \text{ } i\} \quad (4)$$

In this case, since only scheduling constraint is that session 1 cannot be scheduled simultaneously with any of the session 2, 3, ... 9, the maximum throughput region  $\Lambda^{\pi}$  is given by

$$\Lambda^{\pi} = \{(\lambda_1, \dots, \lambda_9) : \lambda_i + \max_{2 \leq i < 9} \lambda_i \leq 1\}. \quad (5)$$

III. maximal scheduling policy with hypergraph Interference models

In this section, we consider the system model and also considered scheduling problem at the MAC layer in VANET. We assumes that the wireless ad hoc network  $N$ , interference hypergraph  $I^N$  is an extension of a graph in the sense that each hyperedge can connect more than two vertices ( $V^N$ ). The main difference between graph and interference hypergraph is that in a graph an edge can have no more than two vertices, but this restriction does not hold for a hypergraph. In our model, the set of vertices ( $V^N$ ), corresponds to possible transmission links. A node  $i$  is a neighbour of a node  $j$ , if there exists a link from  $i$  to  $j$ , i.e.,  $(i,j) \in E$ .

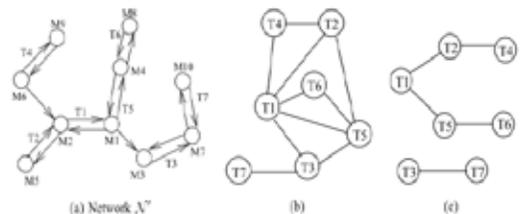


Figure 3. A shows a directed graph with  $V = \{M1,\dots,M10\}$ . The arrows between the nodes indicate the directed links. There are 7 sessions:  $T1,\dots,T7$ . Nodes  $M2,M5,M3,M6,M1,M8$  and  $M10$  are the transmitters of sessions  $T1,T2,T3,T4,T5,T6$  and  $T7$ , respectively. Node  $M2$  has 3 neighbors:  $M1, M5, M6$ . Nodes  $M1$  and  $M2$  have degree 5; hence the degrees of edges  $(M1,M2)$  and  $(M2,M1)$  are 10. Here,  $\delta_G = 10$ . Both the out-degree of  $M1$  and in-degree of  $M2$  are 3. Thus, the directed degree of  $(M1,M2)$  is 6. Here,  $\Delta_G = 6$ . Sessions  $T5$  and  $T6$  interfere with each other, as  $M4$  has a single transceiver.

The hyperedge  $E^N$  is a collection of links, all of which cannot communicate simultaneously. Our hypergraph representation is shown in Figure 4.2. In this graph,  $V = \{(1 \rightarrow 5), (7 \rightarrow 1), (10 \rightarrow 3), (8 \rightarrow 11), (4 \rightarrow 9), (2 \rightarrow 12), (6 \rightarrow 2)\}$  and  $E = \{(1 \rightarrow 5, 7 \rightarrow 1), (7 \rightarrow 1, 10 \rightarrow 3, 8 \rightarrow 11), (8 \rightarrow 11, 4 \rightarrow 9, 2 \rightarrow 12), (2 \rightarrow 12, 6 \rightarrow 2)\}$ . From this graph it can be concluded that links  $7 \rightarrow 1$ ,  $10 \rightarrow 3$  and  $8 \rightarrow 11$  cannot be established at the same time slot due to high interference. The links  $(7 \rightarrow 1, 10 \rightarrow 3)$ ,  $(7 \rightarrow 1, 8 \rightarrow 11)$  and  $(8 \rightarrow 11, 10 \rightarrow 3)$  can be established simultaneously since they do not form hyperedges.

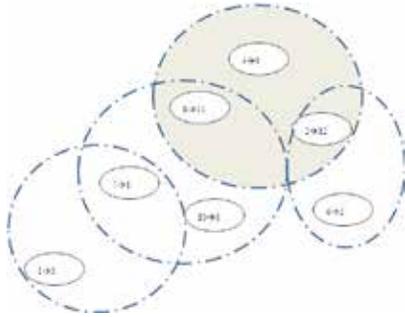


Figure 4. Hypergraph Model Example

In the above Figure 4, links  $(1 \rightarrow 5)$  and  $(10 \rightarrow 3)$  can form an independent set since they do not contain any hyperedges. And the use of the same time slot for these two transmissions does not violate the interference constraint. But independent set which contains links  $(1 \rightarrow 5)$  and  $(10 \rightarrow 3)$  is not maximal since we can add a new transmission link, e.g.,  $(8 \rightarrow 11)$ , to this independent set without breaking the interference constraint. New independent set  $\{(1 \rightarrow 5), (10 \rightarrow 3) \text{ and } (8 \rightarrow 11)\}$  cannot form maximal independent set since enlargement of this set is still possible. Adding link  $(2 \rightarrow 12)$  makes the set  $\{(1 \rightarrow 5), (10 \rightarrow 3) \text{ and } (8 \rightarrow 11)\}$  maximal independent set. All maximal independent set of given hypergraph in the figure 4 as follows

- Here MIS – Maximal Independent Set.
- MIS-1:  $(1 \rightarrow 5), (10 \rightarrow 3), (8 \rightarrow 11), (4 \rightarrow 9)$  and  $(6 \rightarrow 2)$
  - MIS-2:  $(1 \rightarrow 5), (10 \rightarrow 3), (8 \rightarrow 11)$  and  $(2 \rightarrow 12)$
  - MIS-3:  $(7 \rightarrow 1), (4 \rightarrow 9), (8 \rightarrow 11)$  and  $(6 \rightarrow 2)$
  - MIS-4:  $(7 \rightarrow 1), (10 \rightarrow 3), (4 \rightarrow 9)$  and  $(6 \rightarrow 2)$
  - MIS-5:  $(7 \rightarrow 1), (10 \rightarrow 3), (4 \rightarrow 9)$  and  $(2 \rightarrow 12)$
  - MIS-6:  $(1 \rightarrow 5), (10 \rightarrow 3), (4 \rightarrow 9)$  and  $(2 \rightarrow 12)$

An algorithm for determining all maximal independent sets of a hypergraph is described [12]. We use the same algorithm in order to list all maximal independent sets of the interference hypergraph.

We next show that for an arbitrary wireless network and interference model the throughput region of maximal scheduling,  $\Lambda^{MS}$ , can be tightly characterized in terms of  $K(N)$ .

**Theorem 1:** In any wireless network  $N$ , if  $\epsilon \in \Lambda$  in  $N$ ,

$$1/K(N) \in \Lambda^{MS} \text{ in } N.$$

For gravity of the theorem 1 we prove in appendix A.

**Specific Interference Model**

We characterize  $K(N)$  for some representative interference models. These characterizations together with Theorems 1 characterize the throughput regions of maximal scheduling for these models with the following Lemma 2

- 1) For the bidirectional equal power model,  $K(N) \leq 8$  for any network  $N$ , and there exists a network  $N$  such that  $K(N) = 8$ .
- 2) For the unidirectional equal power model, given any constant  $Z$ , there exists a network  $N$  such that  $K(N) > Z$ .
- 3) For the node-exclusive spectrum sharing model,  $K(N) \leq 2$  for any network  $N$ , and there exists a network  $N$  such that  $K(N) = 2$ .

We prove that  $K(N) \leq 8$  for the bidirectional equal power model

in disc graph [5]. We present the intuition behind the result here. From the interference constraints, for any  $i$ , at least one end point of each session in  $S_i$  must be within a distance  $d$  (transmission radius) from either  $i$ 's transmitter or  $i$ 's receiver. Also, the distance between  $i$ 's transmitter and receiver is at most  $d$ . Thus, at least one end point of each session in  $S_i$  must be in the union of two circles of radius  $d$  and centered around  $i$ 's transmitter and receiver respectively (Figure 5 (a)). We refer to the area in this union as  $i$ 's interference area. We prove using geometric arguments that at most 8 points can be present in this interference area such that the distance between any two points exceeds  $d$ . Clearly, if session's  $j$  and  $k$  need to simultaneously transmit packets, the distance between an end point of  $j$  and an end point of  $k$  must exceed  $d$ . The result follows. It is worth noting that several results on packing of unit disk graphs in the existing literature show that  $K(N)$  must be upper-bounded by a constant in the bi-directional equal power model. In particular, results in [8] show that  $K(N)$  cannot exceed 12. Furthermore, results in [11] imply that  $K(N)$  must be upper-bounded by 9. We obtain a better upper-bound on  $K(N)$  in this case, namely 8, which turns out to be tight.

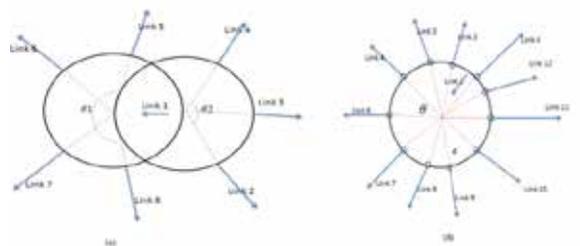
Here we prove the rest of the assumption .

*Proof:* Consider the bidirectional equal power model. We prove that  $K(N) \leq 8$  in appendix B. Figure. 5(a) shows a network  $N$  with bidirectional equal power model such that  $K(N) = 8$ . Consider the unidirectional equal power model and any constant  $Z$ . In the network  $N$  of Figure 4 (b), for  $\theta < 2\pi/(Z + 2)$ ,  $K(N) > Z$  under unidirectional equal power model.

Consider the node exclusive spectrum sharing model, and a session  $(i, u, v)$ . Any session  $j$  in  $S_i$  must traverse either  $u$  or  $v$ . Thus, if  $|S_i| \geq 3$ , then at most 2 of any 3 sessions in  $S_i$  must traverse the same node, and hence must interfere. Thus,  $K(N) \leq 2$ . Figure 4(c) shows an example of a network  $N$  under node exclusive spectrum sharing model with  $K(N) = 2$ . The hypothesis follows.

We now describe the significance of the above results. For the bidirectional equal power model, it follows from part (1) of assumption and Theorems 1, that (a) if  $\lambda \in \Lambda$ ,  $\lambda/8 \in \Lambda^{MS}$ , and (b) for any constant  $Z < 8$ , there exists a network  $N$  and an arrival rate vector  $\vec{\lambda}$ , such that  $\vec{\lambda} \in \Lambda$  in  $N$ , but  $(\vec{\lambda} - \epsilon)/Z \in \Lambda^{MS}$  in  $N$ . Thus,  $\Lambda^{MS}$  is  $1/8$ th of the maximum throughput region  $\Lambda$  in this case.

For the unidirectional equal power model, it follows from part (2) of assumption and Theorem 1, that for any positive constant  $Z$ , there exists a network  $N$ , an arrival rate vector  $\vec{\lambda}$ , such that  $\vec{\lambda} \in \Lambda$  in  $N$ , but  $\vec{\lambda}/Z \in \Lambda^{MS}$  in  $N$ . Thus, maximal scheduling can't attain any constant fraction (however small) of the maximum throughput region. Next note that for the node exclusive spectrum sharing model, maximal scheduling is the same as maximal matching. Lin *et al.* [9] has proved that maximal matching attains at least  $1/2$  the maximum throughput region in this model.



**Figure 5.** Fig. (a) shows a network with interference constraints given by the bidirectional equal power model and transmission range  $d$ . There are 9 sessions:  $1, \dots, 9$ . Session  $i$  has transmitter and receiver Link $_i$ (link exists between transmitter T and receiver R). The interference area of session 1 is the union of circles  $C_1$  and  $C_2$ . Here,  $\theta_1 \theta_2$  degree . Distance between (i) Transmitter and receiver for link is  $d$  for every  $i = 1, \dots, 8$ , (ii) Link $_9$  is  $\epsilon > 0$ , where  $\epsilon$  is a small positive number, (iii) Transmitter 1 and transmitter

$i$  is  $d$  for every  $i = 2, \dots, 9$ , (ii) Transmitter  $j$  and Transmitter  $k$  is greater than  $d$  for every  $j, k \in \{2, \dots, 9\}, j \neq k$  and (iv) T9 and T1 is  $\in$ . Thus, session 1 interferes with all the other 8 sessions, but none of the other sessions interfere with each other.

Fig. (b) shows a network with interference constraints given by the unidirectional equal power model and transmission range  $d$ . There are 12 sessions: 1, ..., 12. Session  $i$  has transmitter and receiver Link  $i$ . The distance between transmitter and receiver is  $d$  for every  $i$ . Thus, session 1 interferes will all the other 11 sessions, but none of the other sessions interfere with each other. We refer to sessions 2, ..., 12 as non-interfering sessions. Here,  $\theta$  is  $\pi/6$ . Note that  $2\pi/\theta - 1$  non-interfering sessions can be accommodated. Thus, for any given  $Z, Z + 1$  non-interfering sessions can be accommodated by choosing  $\theta = 2\pi/(Z + 2)$ .

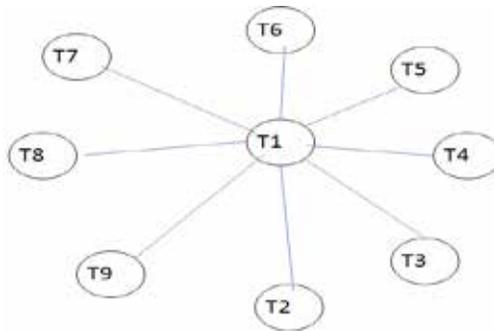
This result also follows from part (3) of assumption and Theorem 1. In addition, part (3) of assumption and Theorem 2 (defined in Appendix A) shows that this characterization is tight. Specifically, for the node-exclusive spectrum sharing model, for any positive constant  $Z$  such that  $Z < 2$ , there exists a network and an arrival rate vector  $\lambda \rightarrow$  such that  $\lambda \in N$ , but  $\lambda \notin Z \Lambda^{MS}$  in  $N$ . It is worth noting here that in the context of input-queued switches, Chuang et al. [13] have proved a result that is related to (although significantly different from) part (3) of Lemma 2. More precisely, the authors in [13] show that for an  $N \times N$  input-queued switch, a speed-up of  $2 - 1/N$  is necessary to emulate an output-queued switch with FIFO scheduling discipline. Thus, the performance guarantees for maximal scheduling will critically depend on the interference relations, and slight changes in interference conditions can significantly alter the guarantees.

**IV. Numerical Analysis**

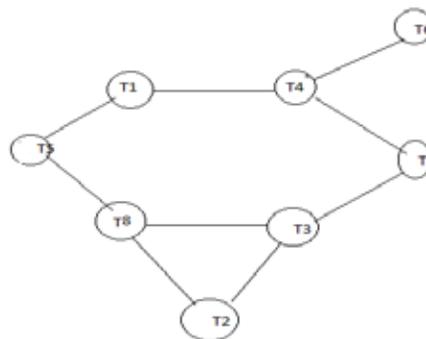
We proposed that the lower bound of throughput region of maximal scheduling presented in Theorem 1 by using the specific network topologies, specific scheduling policies within the range of scheduling policy. Here we demonstrate that the performance attained by the maximal scheduling with respect to the throughput optimality policy in presence of the random network and scheduling models.

We consider that the two network architectures, first network, has 9 single hop sessions T1,T2,T3,...,T9 and the interference graph of this network is shown in Figure-6(a). The second network with 8 single hop sessions, T1,T2,T3,...,T8, whose interference graph is shown in Figure -6(b). The packet arrival rate at all sessions is the same and equal to  $\mu$  if the arrival process is Bernoulli. So that the throughput region is characterized by a single parameter  $\mu^*$ . The network one and two can be composed as (1/2),(1/3) respectively. The maximum throughput attained by scheduling algorithm A is measured as  $\mu_A$ , the maximum value of arrival rate  $\mu$  per second leads to bounded delays under A. The Table I shows  $\mu/\mu^*$  of three different maximal scheduling algorithms which described as follows.

**Greedy Maximal Scheduling :** In Greedy MS, the sessions are picked for scheduling greedily according to a predetermined order, skipping over sessions that are not backlogged or interfere with a session that has already been chosen. For network one (n1), the sessions are chosen according to the sequence T2, T3, ..., followed by T1, i.e., the scheduling policy gives preference to links that correspond to the peripheral nodes in the interference graph, over the link that corresponds to the central node. Note that this is the same scheduling policy that achieved the lower-bound of (1/8) on the throughput guarantee attained in network topology one. For network two, the sessions are chosen according to the sequence T1, T2, T3,T4,...., Randomized Maximal Scheduling : In this case, sessions are chosen at random, ignoring sessions that are not backlogged or interfere with a session that has already been chosen. Distributed Maximal Scheduling: Here we use the randomized distributed maximal schedule construction algorithm described in [14]. This algorithm constructs a maximal schedule in  $O(\log |V|)$  communication rounds.



(A) Interference graph One (N1)



(b) Interference graph Two (N2)

**Figure 6. (a) and (b) are Interference graphs**

**TABLE 1. Various ratios of the performance of maximal scheduling with respect to optimum**

Algorithm A	$\mu_A/\mu^*$ for network One (N1)	$\mu_A/\mu^*$ for network Two (N2)
Greedy MS	0.38	0.75
Randomized MS	0.59	0.92
Distributed MS	0.54	0.9

The results demonstrate that the throughput ratio attained by the maximal scheduling algorithms with respect to the optimum is significantly better than  $1/K(N)$  under randomized traffic patterns and scheduling policies.

**V. Conclusions**

We have proposed a hypergraph interference model for scheduling problem in wireless adhoc network. The proposed hypergraph is throughput guarantees with distributed scheduling in wireless network which includes both graph model and physical SINR model as special case. We examine performance of scheduling algorithms with hypergraph model and proposed lower bound on the stability region. This class of maximal scheduling policy quite broad and our performance bounds apply to all policies in this class. However it remains to attain better performance bounds, while still being liable to low complexity distributed implementation.

**Appendix**

*Appendix A: Proof of Theorem 1*

We prove Theorem 1 using the following supporting lemmas.

Lemma 1:

Let  $\lambda \rightarrow \in \Lambda$ . Then,  $\sum_{j \in S U \{i\}} \lambda_j \leq K(N)$  for all  $i = 1, \dots, N^2$

Lemma 2: Let  $\vec{\lambda} \in \{ \vec{\lambda} : \text{if } \lambda_i > 0, \sum_{j \in S \cup \{i\}} \lambda_j \leq 1, i = 1, \dots, N \}$ .  
 $\vec{\lambda}$  Then  $\in \Lambda^{MS}$ .

Theorem 1 follows from Lemmas 1 and 2, proved below.

Appendix A.1: Proof of Lemma 1:

We assume that there exists a session  $i$  such that

$$\sum_{j \in S \cup \{i\}} \lambda_j \geq K(N) \text{ And show that } \vec{\lambda} \notin \Lambda. \quad (6)$$

Consider an arbitrary scheduling policy  $\pi$ . Under  $\pi$ ,  $\sum_{j \in S \cup \{i\}} D_j(n) \leq nK(N)$  for every  $n \geq 0$  as at most  $K(N)$  nodes among  $S \cup \{i\}$  can be scheduled concurrently.

Thus,  $\lim_{n \rightarrow \infty} \sum_{j \in S \cup \{i\}} \frac{D_j(n)}{n} \leq K(N)$ .

$$\sum_{j \in S \cup \{i\}} \lim_{n \rightarrow \infty} \frac{D_j(n)}{n} \leq K(N),$$

$$\lim_{n \rightarrow \infty} \frac{D_i(n)}{n} \leq \lambda_i \text{ for some } j \in S \cup \{i\}.$$

Thus, if  $\lim_{n \rightarrow \infty} \frac{D_i(n)}{n}$  exists, then its value is less than  $\lambda_j$ . Thus, the network is not stable under  $\pi$ .

Alternatively, if the limit does not exist, then also the network is not stable under  $\pi$

Appendix A.2: Proof of Lemma 2:

Recall that  $Q_i(n)$  denotes the queue length of session  $i$  in the beginning of the  $n^{\text{th}}$  slot. Then, for any

scheduling policy,

$$Q_i(n + 1) = Q_i(0) + A_i(n) - D_i(n) \quad \forall n \geq 1 \text{ and } i = 1, \dots, N. \quad (8)$$

We first define fluid limits.

**Definition 5: Fluid Limits**

We briefly introduce the framework of fluid limits, which is used to prove the rate stability [15].

We first rewrite the queueing equation in (8) as follows:

$$Q_i(n) = Q_i(0) + A_i(n) - D_i(n) \quad (9)$$

Consider any scheduling policy. From any sender  $i$ , at most one packet can be served in a slot. Also, the maximum number of packets arriving in a slot at  $i$  is bounded by  $\alpha_{\text{max}}$ . Thus, for every  $i, \omega, t \geq 0$  and  $\delta > 0$ .

$$A_i(t + \delta, \omega) - A_i(t, \omega) \leq \delta_{\alpha_{\text{max}}} \quad (10)$$

$$D_i(t + \delta, \omega) - D_i(t, \omega) \leq \delta, \quad (11)$$

Where (19) occurs because of the (functional) SLLN, and (20) is because any regular point  $t$  with  $\bar{Q}(t) = 0$  achieves the minimum value (since  $\bar{Q}(t) \geq 0$ ) and, therefore, has zero derivative. We further have the following lemma, which provides a sufficient condition about rate stability [15].

Lemma : The network is rate stable if any fluid limit with

$$\bar{Q}(0) = 0 \text{ has } \bar{Q}(t) = 0 \quad \forall t \geq 0.$$

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