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Research Paper

Band Head Spin Assignment of Super Deformed Rotational Bands Using Vmi Model

Physics

KEYWORDS : Band head spin, VMI equation, Eγ energy ratio, Softness parameter

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ABSTRACT

A novel method for assignment of band head spin (I0) of super deformed rotational bands is proposed and in fact it turns out to be very efficient method. Only γ-ray energies are known experimentally for SD bands. The band head spin (Io) is determined in terms of gamma energies ratio (R) and softness parameter (σ) by using VMI equation. The softness parameter is taken from the literature. The results obtained are well in agreement when the ratio of transition energy over spin (RTEOS) versus spin is plotted. The application of VMI equation on 194Hg and 195Pb are compared with experimental data and also with energy power formula. This method brings some comprehensive interpretation about spin assignment for SD rotational bands.

INTRODUCTION

The super deformed (SD) nuclei is one of the most challenging and interesting area of the nuclear structure physics. Over the past, more than two decades, many super deformed nuclei have been observed in different region A=60, 80, 130, 150 and 190. The super deformed mass region A=60-90 are very interesting region as a limited number of particles in these nuclei and also the exhibit highest rotational frequencies. More than 320 SD bands have been established in the past few decades [1]. Many super deformed rotational bands have been observed but there are still many open problem like spins, parity and excitation energy relative to the ground state. The gamma ray energies are unfortunately the only spectroscopic information universally available. Because of the non observation of the discrete linking transitions between the SD states and the low lying states at normal deformation [2]. Due to this lacking in experimental information about band head spin and energy, the only way to obtain the value of the spin is doing theoretically. Many theoretical methods like energy expansion I (I+1), ω^2expansion, *ab* formula, and the super symmetric algebraic one including many-body interactions (SAM) have been proposed and they have achieved good results also. We use a simple VMI model equation to determine band head spin I_0 directly from experimental Eγ ratio which needs less computational time. The γ energies ratio R derived in term of VMI equation depends upon softness parameter (σ) and unknown band head spin I_0. Therefore band head spin is determined for SD bands. The scope of this work is to estimate the unknown band head spin of super deformed rotational bands. Obtained value of band head spin I_0 is verified on plotting ratio of transition energy over spin (RTEOS) versus spin. Both gave same band head (I_0) value, verifying the calculated value. In this paper, we will describe the formulation of VMI energy equation in section 2. In section 3, we will compare the result obtained by VMI model, energy power expression and experimental RTEOS. Finally, we will give a brief conclusion and discussion in section 4

METHODS

VMI model was first proposed by Mariscotti *et.al.,* 1969 to predict energies of even-even nuclei. Extended VMI model was reported by Goel.A. *et.al.* 1992, for estimating K values in odd-odd SD bands. Also they had produced Mallmann's curves for odd-odd nuclei [3,4].

As per VMI model equation, band head energy of rotational bands is given as,

$$E_{II_0} = E_{I_0} + \frac{1}{2\mathcal{J}}[I(I+1) - I_0(I_0-1)] + \frac{1}{2}C(\mathcal{J}_I - \mathcal{J}_{I_0})^2 \qquad \text{………… (1)}$$

Therefore,

$$E_I = \frac{I(I+1)-I_0(I_0+1)}{4\mathcal{J}_0}\left[\frac{3r_I-1}{r^2{}_I}\right] \qquad \text{……….. (2)}$$

$$r_I = \left\{\frac{1}{3}+\frac{2}{3}\cosh\left[\frac{1}{3}\cosh^{-1}\left\{\frac{27}{2}\left(\frac{\sigma(I(I+1)-I_0(I_0+1))}{(2I_0+1)}+\frac{2}{27}\right)\right\}\right]\right\} \qquad \text{… (3)}$$

Therefore

$$E(I) = \frac{I(I+1)-I_0(I_0+1)}{4\mathcal{J}_0}\left[\frac{\left\{2\cosh\left[\frac{1}{3}\cosh^{-1}\left\{27\times\frac{1}{2}\left(\frac{\sigma(I(I+1)-I_0(I_0+1))}{(2I_0+1)}+\frac{2}{27}\right)\right\}\right]\right\}}{\left[\frac{1}{3}+\frac{2}{3}\cosh\left[\frac{1}{3}\cosh^{-1}\left\{27\times\frac{1}{2}\left(\frac{\sigma(I(I+1)-I_0(I_0+1))}{(2I_0+1)}+\frac{2}{27}\right)\right\}\right]\right]^2}\right] \qquad \text{-------- (4)}$$

and then;

$$E(I-2) = \frac{(I-2)(I-1)-I_0(I_0+1)}{4\mathcal{J}_0}\left[\frac{\left\{2\cosh\left[\frac{1}{3}\cosh^{-1}\left\{27\times\frac{1}{2}\left(\frac{\sigma((I-2)(I-1)-I_0(I_0+1))}{(2I_0+1)}+\frac{2}{27}\right)\right\}\right]\right\}}{\left[\frac{1}{3}+\frac{2}{3}\cosh\left[\frac{1}{3}\cosh^{-1}\left\{27\times\frac{1}{2}\left(\frac{\sigma((I-2)(I-1)-I_0(I_0+1))}{(2I_0+1)}+\frac{2}{27}\right)\right\}\right]\right]^2}\right] \qquad \text{…. (5)}$$

Mallman's energy ratios in a SD band can be written as,

Where +2n is the energy in the nth state in the band relative to the band head energy.

Therefore energy ratio can be expressed in terms of gamma energies for SD bands as [5],

R ratio dependence upon band head spin and softness parameter (σ). Softness parameter (σ) is given as a parameter from two parameters energy expression formula [6,7].

The obtained value of from VMI equation are verified by plotting the ratio of transition energy over spin (RTEOS) versus spin.

For a rigid rotor, the energy of the band can be described in terms of the spin I as[2],

E(I) = A I (I+1) ……………… (6)

Where A is a constant which is related to the moment of inertia as A=1/2 .

The E2 transition γ-ray energy is

Eγ (I) = E (I) – E (I-2) …………... (7)

Eγ (I→I-2) = 4A (I-1/2) ……….. (8)

Then,

$$\frac{E\gamma\,(I\rightarrow I-2)}{(I-1/2)} = 4A \qquad \text{…………… (9)}$$

It is obvious that such a ratio of transition energy over spin (RTEOS) is a constant for a rigid rotor if the exit I0 spin is assigned correctly [2]. However, if I0 deviates from the exact value by ±1, RTEOS which maintains the energy expression changes to

$$\frac{E\gamma\,(I\rightarrow I-2)}{(I'-1/2)} = 4A\ (1\ \ 1/I'\text{-}1/2)\ \text{for}\ (I'_0=I_0\pm1) \qquad \text{………………..(10)}$$

When plotted versus spin, RTEOS is a horizontal line for the exact I0 and will shift to a hyperbola when I0±1 is assigned to I0. In Fig.1 RTEOS are plotted versus spin for rigid rotor.

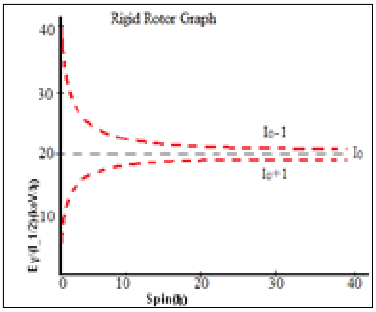


Fig.1: RTEOS plot for rigid rotor.

Oon plotting the ratio of transition energy over spin (RTEOS) which is

$$\frac{E\gamma(I\rightarrow I-2)}{(I-1/2)}$$ versus spin.

The horizontal line for the exact I_0 is obtained and will shift to hyperbola when =1.

Further we have compared our results with energy expression. Bohr and Mottelson reported that rotational energy E_{rot} (I,K) of a nuclei with angular momentum (I,K=0) in an even-even nuclei can be represented by a power series of I(I+1).

$$E_{rot}(I,K) = A[I(I+1)] + B[I(I+1)]^2 + C[I(I+1)]^3 + ... (11)$$

Where A, B, and C are parameters related to the moment of inertia. As B and C are very small as compared to A so neglecting C. The equation depends on two parameters. The value of A and B is obtained by least square fitting method.

We can write in term of Gamma energy as,

$$E\gamma(I\rightarrow I-2)=A\{[I(I+1)-(I-2)(I-1)]\}+B\{[I(I+1)]^2-[(I-2)(I-1)]^2\}+....$$

Then RTEOS reads

$$\frac{E\gamma(I\rightarrow I-2)}{(I-1/2)} = 4\{A + 2B(I^2-I+1)\}$$

We have compared this RTEOS plot with VMI RTEOS both are in agreement also with experimental RTEOS plot. This gave comprehensive interpretation of the spin assignment for SD bands. We have given in the paper the calculation and comparison with experimental data for ^{194}Hg (b1) (even-even) and ^{195}Pb (b2) (odd-A) nucleus.

RESULTS AND DISCUSSIONS

This study we plotted experimental RTEOS versus spin and compared with the calculations assigning band head spin by VMI equation for A=190 region mainly to ^{194}Hg (1), ^{195}Pb (2) as shown in Fig. 2 (a, b, c) and Fig. 3 (a, b, c). The band head spin for 194Hg is 8 and for 195Pb. Obtained results are listed in Table 1.

Table 1: Comparing different gamma energy values of ^{194}Hg

Spin (I)	Eγ/(I-1/2) (Experimental)	Eγ/(I-1/2) (VMI)	Eγ/(I-1/2) (Energy power Expression)
10	22.3	22.42	22.22
12	22.0	22.42	22.12
14	21.9	22.42	22.00
16	21.7	22.42	21.86
18	21.5	22.42	21.70
20	21.3	22.42	21.53
22	21.1	22.42	21.33
24	20.9	22.41	21.11
26	20.7	22.41	20.88
28	20.4	22.41	20.62
30	20.2	22.41	20.35
32	20	22.41	20.06
34	19.7	22.41	19.74
36	19.5	22.40	19.41
38	19.3	22.40	19.06
40	19	22.40	18.69
42	18.8	22.40	18.30
44	18.6	22.40	17.90
46	18.5	22.39	17.47
48	18.3	22.39	17.02
50	18.2	22.39	16.56

Table 2: Comparing gamma energy values for ^{195}Pb

Spin (I)	Eγ (Experimental)	Eγ (VMI)	Eγ (Energy power Expression)
8.5	20.38	20.40	20.43
10.5	20.30	20.40	20.44
12.5	20.33	20.40	20.46
14.5	20.36	20.40	20.48
16.5	20.38	20.40	20.50
18.5	20.39	20.40	20.53
20.5	20.40	20.39	20.56
22.5	20.41	20.39	20.59
24.5	20.42	20.39	20.63
26.5	20.42	20.39	20.67
28.5	20.36	20.38	20.71
30.5	20.37	20.38	20.76
32.5	20.34	20.38	20.81

Figure 2 (a, b, c): Graphs comparing values from different methods for ^{194}Hg (band 1)

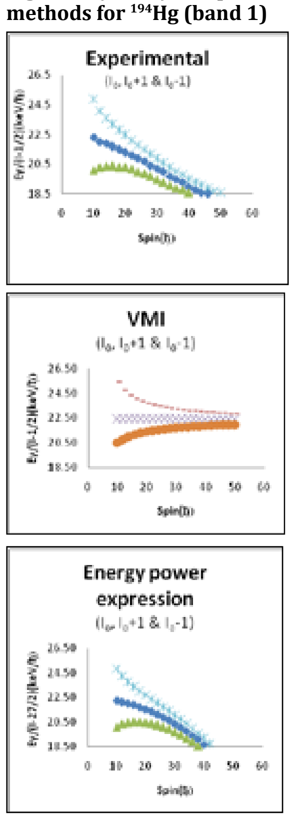
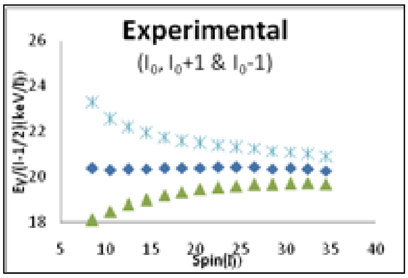
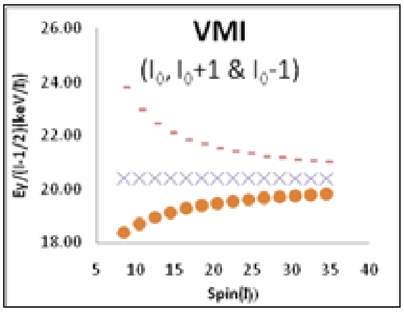
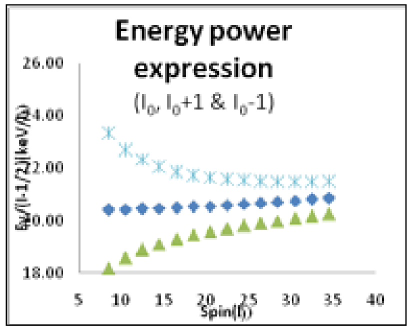


Figure 3 (a, b, c): Graphs comparing values from different methods ^{195}Pb (band 2)







CONCLUSION:

This study was an effort to assign band head spin (I_o) in terms of gamma energies ratio (R) and softness parameter (σ), using VMI equation. The softness parameter was assigned as found from literature. The results were verified through comparisons obtained through ratios of transition energy over spin (RTEOS) compared with spins. The results were used to assign values for ^{194}Hg and ^{195}Pb, through VMI equation and compared with experimental data and also with energy power formula. This method brings comprehensive interpretations about spin assignment for SD rotational bands. The research is going on to present some good results of this method.

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