Research Paper

Engineering

KEYWORDS : Cutting Forces, End milling, Response Surface Methodology, Multi response optimization, Desirability function.

Multi Response Optimization of Cutting Forces in End Milling Using Response Surface Methodology and Desirability Function

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ABSTRACT

End milling is the most important milling operation due to its capability of producing complex geometric shapes with reasonable accuracy and good surface finish. This paper focuses on multiple response optimization of cutting forces in end milling operation on AlSiC metal matrix composites to get minimum forces in tangential(Fx), radial(Fy) and axial(Fz) directions using response surface methodology. Cutting forces provide the basis for surface accuracy prediction and improvement, tool wear rate, the energy consumption within the machine tool, depending on power consumption and operating time. In this work, second-order quadratic models were developed for Fx, Fy and Fz considering the spindle speed, feed rate, depth of cut and immersion angle as the cutting parameters, using central composite rotatable design. The adequacies of the models were checked using ANOVA. The developed models were used for multiple-response optimization by desirability function approach in conjunction with RSM to determine the optimum machining parameters.

1. Introduction

In the highly competitive manufacturing industries now a day, the manufacturer's ultimate goals are to produce high quality product with less cost and time constraints. Due to the advances in machine tool, CNC, CAD/CAM, cutting tool and high speed machining technologies the volume and importance of milling have increased in key industries such as aerospace, aeronautical, biomedical, die and mold, automotive and component manufacturing[1-3]. Only the implementation of automation in end milling process is not the last achievement. It is also necessary to improve the machining process and machining performances continuously for effective machining and also for the fulfillment of requirements of the industries. Despite the recent advances in machining technology, productivity in milling is usually reduced due to the process limitations such as high cutting forces and less stability [4-5].

If milling conditions are not selected properly, the process may result in violations of machine limitations and part quality, or reduced productivity. The usual practice in machining operations is to use experience-based selection of cutting parameters which may not yield optimum conditions. The cutting forces affect the quality and the precision of the final component; therefore precise prediction of milling forces becomes an important factor to improve machining performance. Moreover, reliable quantitative prediction of cutting forces is essential for further prediction of the necessary power and torque, machine tool vibrations, work piece surface quality, geometrical accuracy and process stability. Cutting forces provides a basis for surface accuracy prediction and improvement, tool wear rate, the energy consumption within the machine tool, depending on power consumption and operating time [3]. So multi response optimization of end milling is carried out for forces as responses to optimize the machining parameters.

2. Literature review

C. Dhavamani(2011) et al.[9] gives review on latest optimization techniques such as fuzzy logics, ant colony technique, Taguchi method, scatter search technique, Response surface methodology etc. Design of Experiment methods like Taguchi technique and response surface methodology are being applied successfully in industrial applications for optimal selection of process variables in the area of machining. Experimentation and making inferences are the twin features of general scientific methodology. Statistics as a scientific discipline is mainly designed to achieve these objectives. Planning of experiments is particularly very useful in deriving clear and accurate conclusions from the experimental observations, on the basis of which inferences can be made in the best possible manner [6].

3.1 Response Surface Methodology

RSM is a collection of mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response. RSM is useful for developing, modeling and optimize the response variables [21]. It is a technique for determining and representing the cause and effect relationship between the responses and input control variables influencing the responses as a two- or three-dimensional hyper surface. The accuracy and effectiveness of an experimental design depends on careful planning and execution.

Fig.1: Procedure for response surface methodology

Cochran & Cox (1962) quoted Box and Wilson as having proposed response surface methodology for the optimization of experiments. There are mainly 2 types Design of Experiments for RSM. 1) Central Composite Design. 2) Box Behnken Design. Box & Hunter (1957) have proposed that the scheme based on central composite rotatable design fits the second-order response surfaces very accurately [6].

3.2 Central Composite Rotatable Design

Central composite designs (CCDs), also known as Box-Wilson designs, are appropriate for calibrating full quadratic models. Total design points in CCD are 2^k factorial (cube) points + 2*k axial (star) points + n_o centre points; where k is the number of factors in the experiments. Factorial points in CCD contribute to the estimation of the interaction terms. The factorial points do not contribute to the estimation of quadratic terms. Axial points are added to estimate the curvature. The axial points contribute in a large way to the estimation of quadratic terms. Centre points provide an internal estimate of pure error; contribute towards the estimate of quadratic terms and has an influence on the distribution of variance in the region of interest [22]. In the central composite rotatable design, standard error remains the same at all the points which are equidistant from the centre of the region.

3.3 Experimental procedure

1. Identifying the important process control variables.
2. Finding the upper and lower limits of the control variables, viz., spindle speed, feed rate, depth of cut and immersion angle.
3. Development of design matrix using central composite design and conducting the experiments as per the design matrix.
4. Recording the responses of Forces in tangential, radial and axial directions.
5. Development of second-order quadratic model.
6. Determining the coefficients of the second-order polynomials.
7. Checking the adequacy of the models developed.
8. Testing the significance of the regression coefficients.
9. Presenting the main effects and the significant interaction effects of the process parameters on the responses in two (contour) and three dimensional (surface) graphical form.
10. Determination of optimized machining process parameters for the multiple responses.

4.1 Mathematical modeling

Through the use of the design of experiments and applying regression analysis, the modeling of the desired response to several independent input variables can be gained. If all variables are assumed to be measurable, the response surface can be expressed as follows:

$$Y_u = f(X_1, X_2, X_3, \ldots X_k) + \varepsilon \qquad (1)$$

Where, Y_u is the corresponding response function (or response surface), $X_1, X_2, X_3 \ldots X_k$ are coded values of the machining process parameters, and ε is the fitting error of the u[th] observations. In this study a second-order polynomial regression model, which is called quadratic model, is proposed. The quadratic model of Y_u can be written as follows:

$$Y_u = b_o + \sum_{i=1}^{k} b_i X_i + \sum_{i<j}^{k} b_{ij} X_i X_j + \sum_{i=1}^{k} b_{ii} X_i^2 \pm \varepsilon$$

The coefficient bo is the free term, the coefficients bi are the linear terms, the coefficients bij are the interaction terms and bii are quadratic terms.

Using the results presented in Table.2, the derived quadratic model for Fx, Fy and Fz in terms of coded factors are given as:

$$F_x = 22.55 - 0.225A + 2.739B - 0.040C + 0.321D + 0.424AB + 0.99AC + 0.022AD + 0.238BC - 0.230BD + 0.441CD - 0.024A^2 - 0.985B^2 + 0.059C^2 - 0.050D^2 \qquad (3)$$

$$F_y = 45.93 - 0.707A + 6.587B - 0.099C + 0.757D + 1.206AB + 0.723AC - 0.328AD + 0.378BC - 0.852BD + 1.868CD - 0.104A^2 - 6.007B^2 + 0.118C^2 - 0.082D^2 \qquad (4)$$

$$F_z = 13.66 - 0.116A + 5.516B - 0.011C + 0.876D + 0.289AB + 0.533AC - 0.344AD + 0.569BC + 0.129BD + 0.613CD - 0.139A^2 + 0.442B^2 - 0.055C^2 - 0.187D^2 \qquad (5)$$

4.2 Model adequacy checking by ANOVA

The adequacies of the models are checked using the analysis of variance (ANOVA) technique. The analysis of the experimental data was done to statistically analyze the significance of the parameters depth of cut(D), feed(F), speed(V) and immersion angle(A) on the response variables tangential F_x, radial F_y and axial F_z forces respectively. The model has been developed for 95% confidence level.

As per this technique the F value of model Fx 9.07[tab.3] implies that the model is significant. There is only a 0.01% chance that a model F-Value this large could occur due to noise. Values of P less than 0.0500 indicate model terms are significant. In this case B,B^2 are significant model terms. Values greater than 0.1000 indicate the model terms are not

Table. 1: Factors and their Levels selected for central composite design:

Sl. No	Parameters	Units	Notations	Factor Levels				
				-2	-1	0	1	2
1	Depth of cut	mm	D	.4	.8	1.2	1.6	2
2	Feed rate	mm/sec	F	.3	.6	.9	1.2	1.5
3	Cutting speed	m/min	V	56	84	112	140	224
4	Immersion angle	degree	A	90	120	180	270	360

Table.2: Four factor Three response Central Composite Design Matrix:

Run	D(mm)	F(mm/sec)	V(m/min)	Φ(degree)	F_x(N)	F_y(N)	F_z(N)
1	0.80	0.60	84.00	120.00	19.868	34.903	9.0466
2	1.60	0.60	84.00	120.00	19.868	35.075	9.245
3	0.80	1.20	84.00	120.00	23.385	47.69	17.386
4	1.60	1.20	84.00	120.00	23.385	47.393	17.543
5	0.80	0.60	140.00	120.00	18.718	30.3916	4.703
6	1.60	0.60	140.00	120.00	15.373	20.086	4.555
7	0.80	1.20	140.00	120.00	24.053	46.873	19.201
8	1.60	1.20	140.00	120.00	24.271	48.075	19.806
9	0.80	0.60	84.00	270.00	21.051	39.246	10.455
10	1.60	0.60	84.00	270.00	17.603	25.82	5.6683
11	0.80	1.20	84.00	270.00	24.72	49.091	22.061
12	1.60	1.20	84.00	270.00	24.72	47.811	20.59
13	0.80	0.60	140.00	270.00	20.69	37.396	10.023
14	1.60	0.60	140.00	270.00	21.14	39.456	10.513
15	0.80	1.20	140.00	270.00	24.203	48.595	20.695
16	1.60	1.20	140.00	270.00	24.726	49.571	22.073
17	0.40	0.90	112.00	180.00	22.013	43.656	12.503
18	2.00	0.90	112.00	180.00	22.26	44.218	12.756
19	1.20	0.30	112.00	180.00	11.566	10.788	5.715
20	1.20	1.50	112.00	180.00	25.011	29.863	24.181
21	1.20	0.90	56.00	180.00	22.426	44.4783	12.851
22	1.20	0.90	168.00	180.00	22.508	45.176	13.038
23	1.20	0.90	112.00	90.00	22.506	45.408	13.168
24	1.20	0.90	112.00	360.00	21.645	42.468	12.405
25	1.20	0.90	112.00	180.00	22.208	42.638	12.475
26	1.20	0.90	112.00	180.00	22.611	43.975	12.818
27	1.20	0.90	112.00	180.00	22.711	45.9017	13.075
28	1.20	0.90	112.00	180.00	22.876	46.388	13.546
29	1.20	0.90	112.00	180.00	23.271	46.386	14.895
30	1.20	0.90	112.00	180.00	23.09	46.536	14.356

significant. If there are many insignificant model terms model reduction may improve the model. The Lack of Fit F-value of 7.65 implies the Lack of Fit is significant. There is only a 1.83% chance that a Lack of Fit F-value this large could occur due to noise. Insignificant lack of fit is good; so we want the model to fit. Adequacy Precision measures the signal to noise ratio. A ratio greater than 4 is desirable. The ratio of 13.127 indicates an adequate signal. This model can be used to navigate the design space.

4.3 Residual analysis for responses

The developed response surface models for Fx, Fy, & Fz have been checked using residual analysis.The residual plots for response parameters Fx, Fy & Fz are shown in fig.2,3,& 4 respectively. In normal plot of residuals the data are spread approximately in a straight line, which shows a good correlation between experimental and predicted values for responses.

4.4 Parametric influences and analysis

Based on RSM model the effects of parameters on forces can be seen from the graphs also. The effects of feed rate (F) and depth of cut (D) on F_x, while keeping the other parameters at the center level are shown in fig.5. The non linear nature of variation of F_x with respect to F is observed and F_x increases with increase in feed and Fx increase linearly as depth of cut increases but not much significant as feed rate.

The variation of Fx with respect to speed and immersion angle while keeping other two parameter at there center are

Table.3: ANOVA for response surface quadratic model for F_x

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	219.79	14	15.70	9.07	< 0.0001	significant
A-D	1.22	1	1.22	0.70	0.4145	
B-F	180.08	1	180.08	104.09	< 0.0001	significant
C-V	.039	1	0.039	0.022	0.8828	
D-A	2.48	1	2.48	1.43	0.2502	
AB	2.88	1	2.88	1.66	0.2168	
AC	0.16	1	0.16	0.092	0.7661	
AD	7.744E-003	1	7.744E-003	4.476E-003	0.9475	
BC	0.91	1	0.91	0.52	0.4799	
BD	0.85	1	0.85	0.49	0.4947	
CD	3.11	1	3.11	1.80	0.1998	
A^2	0.015	1	0.015	8.402E-003	0.9282	
B^2	26.61	1	26.61	15.38	0.0014	significant
C^2	0.097	1	0.097	0.056	0.8164	
D^2	0.070	1	0.070	0.040	0.8437	
Residual	25.95	15	1.73			
Lack of Fit	24.36	10	2.44	7.65	0.0183	significant
Pure Error	1.59	5	0.32			
Cor Total	245.74	29				

Table.4: ANOVA for response surface quadratic model for F_y

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	2207.86	14	157.70	10.58	< 0.0001	significant
A-D	12.01	1	12.01	0.81	0.3836	
B-F	1041.16	1	1041.16	69.87	< 0.0001	significant
C-V	0.24	1	0.24	0.016	0.9010	
D-A	13.74	1	13.74	0.92	0.3521	
AB	23.28	1	23.28	1.56	0.2305	
AC	8.36	1	8.36	0.56	0.4655	
AD	1.72	1	1.72	0.12	0.7390	
BC	2.28	1	2.28	0.15	0.7010	
BD	11.60	1	11.60	0.78	0.3915	
CD	55.82	1	55.82	3.75	0.0720	
A^2	0.30	1	0.30	0.020	0.8893	
B^2	989.81	1	989.81	66.43	< 0.0001	significant
C^2	0.38	1	0.38	0.026	0.8745	
D^2	0.19	1	0.19	0.013	0.9120	
Residual	223.51	15	14.90			
Lack of Fit	218.69	10	21.87	22.71	0.0015	significant
Pure Error	4.82	5	0.96			
Cor Total	2431.37	29				

Table.5: ANOVA for response surface quadratic model for F_z

shown in fig.6. The graph shows Fx increases slightly with increase in speed and immersion angle but not much affect as feed rate. Similar effect of parameters on response F_y is shown in fig.7& 8. The non- linear nature of variation of F_y with respect to feed is observed and F_y increases with increase in feed. After optimization it's shown that there is a slight decrease in forces as speed and immersion angle increse at higher levels.

The variation of F_z with respect to feed rate and depth of cut while keeping the other parameters at their center levels are shown in fig.9. Axial force increases with increase in depth of cut but not much effect as that of feed rate on F_z. Increase in speed and immersion angle slightly increase the axial force as shown in fig. 10.

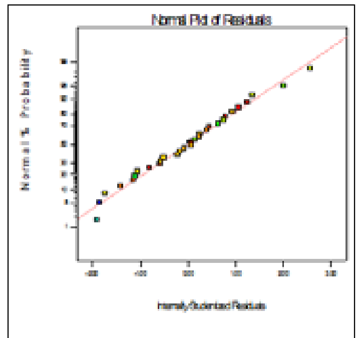

Fig.2: Normal Plot of Residuals for F_x

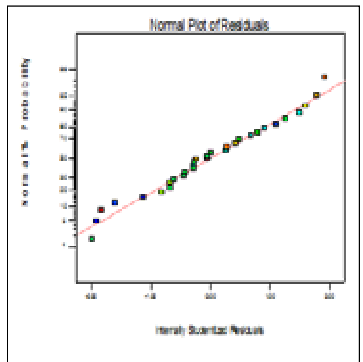

Fig.4: Normal Plot of Residuals for F_z

5.1 Multi response optimization by desirability function

A single-response optimization algorithm provides optimal solution for only one response. In multi response optimization, simultaneous optimization of more than one response is carried out and optimum solution is obtained. To overcome the problems of conflicting response of single response optimization multi response optimization is carried out

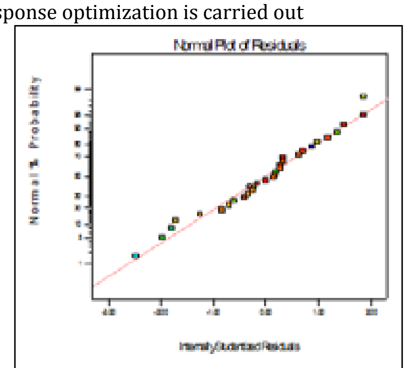

Fig.3: Normal Plot of Residuals for F_y

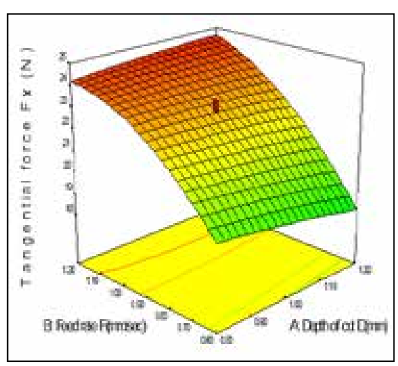

Fig.5: Tangential force V/s feed rate & depth of cut

Volume : 2 | Issue : 5 | May 2013 • ISSN No 2277 - 8179

by desirability function approach popularized by Derringer and

$$\begin{aligned} d_i &= 0 & i &< L_i \\ d_i &= [(i - L_i)/(T_i - L_i)]^{r_i} & L_i &\le i \le T_i \\ d_i &= 1 & i &> T_i \end{aligned} \qquad (10)$$

To minimize a response, the individual desirability is calculated

$$\begin{aligned} d_i &= 0 & i &> H_i \\ d_i &= [(H_i - i)/(H_i - T_i)]^{r_i} & T_i &\le i \le H_i \\ d_i &= 1 & i &< T_i \end{aligned} \qquad (11)$$

$$d_i = [(i - L_i)/(T_i - L_i)]^{r_i} \qquad L_i \le i \le T_i$$

desirability is calculated as:

$$\begin{aligned} d_i &= [(H_i - i)/(H_i - T_i)]^{r_i} & T_i &\le i \le H_i \\ d_i &= 0 & i &< L_i \\ d_i &= 0 & i &> H_i \end{aligned} \qquad (12)$$

Where,

- i predicted value of i[th] response
- y_i target value for i[th] response
- L_i lowest acceptable value for i[th] response
- H_i highest acceptable value for i[th] response
- d_i desirability for i[th] response
- r_i weight of desirability function of i[th] response

Table.6 shows the constraints of input parameters and that of responses and the goal and weights assigned to each parameter.

Table.7. shows the values of 21 levels combinations of process parameters that will give high value of composite desirability (ranged from 1 to 0.568), and the values of predicted responses obtained are also given.

The optimum parametric conditions are obtained by the composite desirability function as shown in the ramp diagram given in fig.11.

The optimum parameter combinations obtained for minimum forces are: 1.18mm depth of cut, 0.31mm/sec feed, 223.9m/min speed and 174.10 degree immersion angle. The minimum forces obtained are: Tangential force 11.52N, radial force 5.19N and axial force -1.88N.

Table 6: Constraints

Name	Goal	Upper Limit	Lower Limit	Upper Weight	Lower Weight	Importance
A:D	is in range	0.4	2	1	1	3
B:F	is in range	0.3	1.5	1	1	3
C:V	is in range	56	224	1	1	3
D:A	is in range	90	360	1	1	3
Fx	minimize	11.566	25.011	1	1	3
Fy	minimize	10.788	49.571	1	1	3
Fz	minimize	4.4	24.181	1	1	3

Table 7: Solutions

Number	D	F	V	A	Fx	Fy	Fz	Desirabilty	
1	1.18	0.31	223.59	174.10	11.5197	5.18725	-1.87602	1.000	Selected
2	0.92	0.30	171.08	126.90	11.3892	2.72669	-0.720505	1.000	
3	1.06	0.31	162.90	90.18	10.6787	0.890193	-0.782468	1.000	
4	0.51	0.31	204.35	108.44	10.8694	-1.76376	-6.10372	1.000	
5	0.41	0.31	187.57	101.03	11.4034	-0.162825	-4.98515	1.000	
6	1.97	0.30	58.44	357.84	11.5608	-2.34347	-0.951305	1.000	
7	1.06	0.33	172.67	122.01	11.4773	4.15092	-0.302296	1.000	
8	1.26	0.30	211.55	177.55	11.4965	5.44447	-0.783232	1.000	
9	0.57	0.31	220.57	98.20	10.109	-4.51619	-8.02949	1.000	
10	0.80	0.30	166.43	99.67	11.0757	0.851612	-1.56539	1.000	
11	0.99	0.33	213.44	103.99	10.1818	-1.071	-4.60074	1.000	
12	2.00	0.30	62.27	321.30	11.5757	-1.38523	-0.065023	1.000	
13	1.94	0.30	56.00	359.78	11.5913	-2.45478	-0.918739	0.999	
14	2.00	0.30	72.61	262.17	11.6037	0.672013	1.46461	0.999	
15	2.00	0.30	76.12	272.14	11.6151	0.816291	1.37967	0.999	
16	2.00	0.30	67.68	255.58	11.6177	0.449656	1.47349	0.999	
17	2.00	0.30	56.71	255.85	11.7047	-0.123706	1.23989	0.997	
18	2.00	0.31	64.31	346.57	11.7692	-0.719339	-0.472551	0.995	
19	0.40	0.30	105.89	92.40	14.3769	11.1985	3.7448	0.922	
20	0.40	0.30	70.72	90.00	16.1533	17.5317	7.43572	0.772	
21	0.40	0.91	224.00	90.00	20.0475	30.2881	2.41199	0.568	

6. CONCLUSION

A second order quadratic model was developed for each response; tangential, radial and axial forces by central composite design of RSM. Adequacy of the model was checked by ANOVA. Significant parameters for each model were identified. Feed rate is the most significant parameter for F_x, F_y and F_z. With the increase in feed rate there is a considerable increase in forces and with the increase in cutting speed, depth of cut and immersion angle there is a slight increase in forces F_x, F_y and F_z are shown in graphs. After optimization it was observed that forces decreases with increasing cutting velocity at higher speed and slight decrease in force with increase in immersion angle which will increase the productivity. Among the three force components tangential force component has the most important affect on machining process. Radial force is very low and axial force is negligible if optimum cutting conditions are selected. Optimum machining conditions were found out by desirability function at which machining parameters were observed as depth of cut: 1.18 mm, feed: 0.31mm/sec, speed: 223.9m/min and immersion angle: 174.10degre and optimum forces found as: 11.52N, 5.19N and -1.88N at tangential, radial and axial directions respectively. Using these results power consumption for minimum cost can be calculated.

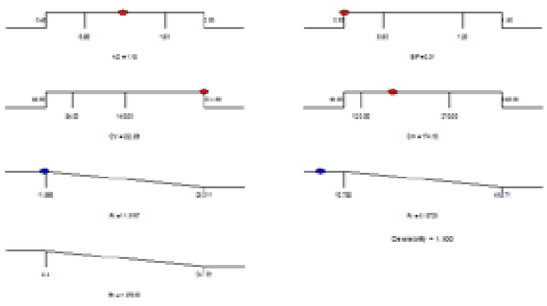

Fig 11: The ramp diagram for the maximum desirability

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