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Research Paper

Fuzzy Graph in Geology

Mathematics

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ABSTRACT

Fuzzy sets involved in many real applications. Uncertainty is an important matter of decision whether one would like to model uncertainty in an explicit way. In this paper we review traditional and recent approaches for handling. Uncertainty in geological investigations. Many types of uncertainties are classified and we outline traditional mathematical methods frequently applied in geology. Fuzzy graph seems to be one of the most efficient for geological purposes.

Introduction

Uncertainty is involved in many real applications. Apart from the definition, it is an important matter of decision whether one would like to model uncertainty in an explicit way. If not then it is typical to use a deterministic model as an approximation of the uncertain phenomenon. In this paper we shall focus on those cases in which the decision is to model uncertainty explicitly and also we follow him in the sense that we shall focus on the human-related, subjective interpretation of uncertainty which depends on the quantity and quality of information which is available to a human being about a system or its behavior that the human being wants to describe, predict or prescribe" The choice of an appropriate uncertainty calculus should depend on the causes of uncertainty, quantity and quality of information available, type of information processing required by the respective uncertainty calculus. The paper is organized

The paper is organized as follows. In the next section we summarize some of the fundamental concepts, especially those ones related to uncertainty. Then we briefly recall traditional methods for handling uncertainty, together with their shortcomings.

Basic Concepts

The collection of well defined objects are called set. The objects in a set called elements or members of a set.

If an objects X belongs to the set A we write $x \in A$ otherwise we write $x \notin A$

Example

1. $A = \{x : x$ is an even positive integer less than 100}
2. $B = \{x : x$ is a close friend of Ramesh}

This is not a set as closeness is not well defined. But this can be defined by using Fuzzy set. In Fuzzy set every element is assigned a membership value between 0 and 1 (both inclusive).

$$A^* = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$$

$$B^* = \{(x, \mu_B(x)), x \in B, \mu_B(x) \in [0,1]\}$$

Fuzzy set

A Fuzzy set A^* is defined by $A^* = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$ In the pair $(x, \mu_A(x))$ the first element x belongs to the classical set A, the second element $\mu_A(x)$ belongs to the interval [0,1] calledmembership function. The Fuzzy set can also denoted by

$$A^* = \{\mu_A(x)/x : x \in A, \mu_A(x) \in [0,1]\}$$

where symbol '/' is not a division sign but indicates that the top number $\mu_A(x)$ is the membership value of the element x in the bottom.

Example

Consider the fuzzy set $A^* = \{(x_1,0.1),(x_2,0.5),(x_3,0.3),(x_4,0.8),(x_5,1.0),(x_6,0.2)\}$ The mebership function $\mu_A(x)$ of set A^* takes the following values on [0,1].

$$\mu_A(x_1)=0.1, \mu_A(x_2)=0.5, \mu_A(x_3)=0.3, \mu_A(x_4)=0.8, \mu_A(x_5)=1.0, \mu_A(x_6)=0.2.$$

The elements x_5 is a full member of the fuzzy set A^* while the elements x_1 is the member of A^* a little ($\mu_A(x_1)=0.1$ near 0), x_6, x_3 and are a little more members of A^*, the element x_4 is almost a full member of A^*.

Basic Operations

Consider the Fuzzy sets A & B in the universe U

- **Union**

The operation union of A and B denoted as $A \cup B$ is defined

$$\mu_{A\cup B}(x) = \max\{(\mu_A(x), \mu_B(x)), x \in U\}$$

If $a_1 < a_2, \max(a_1, a_2) = a_2$

- **Intersection**

The operation intersection of A & B denoted as $A \cap B$ is defined by $\mu_{A\cap B}(x) = \min\{(\mu_A(x), \mu_B(x)), x \in U\}$ If $a_1 < a_2, \min(a_1, a_2) = a_1$

Example

Consider the $U = \{x_1, x_2, x_3, x_4\}$ and the fuzzy sets A & B defined by the table

x	x_1	x_2	x_3	x_4
$\mu_A(x)$	0.2	0.7	1.0	0
$\mu_B(x)$	0.5	0.3	1.0	0.1
$\mu_{A\cup B}(x)$	0.5	0.7	1.0	0.1
$\mu_{A\cap B}(x)$	0.2	0.3	1.0	0

- **Complementation**

The Fuzzy sets A and $\bar{A}$ are complementary if $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$ (or)

$$\mu_{\bar{A}}(x) + \mu_A(x) = 1$$

Fuzzy Graph

The Fuzzy node fuzzy graph G is defined by $G = (V, E)$, $V = \{v_i(u_i)\}$, $E = \{e_{ij}\}, 0 \le e_{ij} \le 1$ where u_i is the fuzziness of the node v_i, e_{ij} is the fuzziness of the arc from the node v_i to the node v_j

Example:

The fuzzy node fuzzy graph $G = (V, E)$ is illustrated in below figure $V = \{v_1(0.2)\ v_2(0.3)\ v_3(0.2)\ v_4(0.4)\}$

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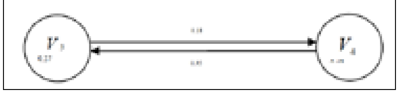
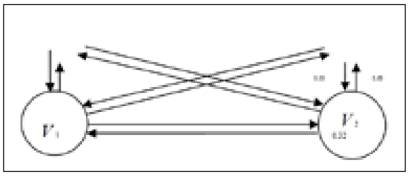
$$E = \begin{pmatrix} & v_1 & v_2 & v_3 & v_4 \\ v_1 & 1 & 0.48 & 0.32 & 0.12 \\ v_2 & 0.74 & 1 & 0.29 & 0.63 \\ v_3 & 0.20 & 0.81 & 1 & 0.18 \\ v_4 & 0 & 0.03 & 0.95 & 1 \end{pmatrix}$$





Geology is the study of the earth, the materials of which it is made, the structure of those materials, and the processes acting upon them. An important part of geology is the study of how earth's materials, structures, processes and organisms have changed over time. Uncertainty is a general term expressing lack of certainty and of precision in describing a geological object, a feature or a process.

Types of Uncertainties are given below

1. ***Imprecision or inaccuracy***, expressing the deviation of measurements from the true value.
2. ***Vagueness or ambiguity***, the uncertainty of non-measurable objects or properties.
3. ***Incompleteness***, the uncertainty due to incomplete information.
4. ***Conflicting evidence***, the uncertainty arising from contradicting evidences present in the studied system.
5. ***Presumption or belief***, when all available information is subjective. The well known „expert's opinion" belongs to this group.

The above classification is valid for geological investigations as well.

However, the complexity of most geological problems requires a more detailed and specific classification. The one elaborated by us distinguishes two main groups

A. Uncertainties due to natural variability (called also ***aleatory*** uncertainties).
B. Uncertainties due to human shortcomings and incomplete knowledge (called also ***epistemic*** uncertainties).

Natural variability is a property of nature, existing independently of us. It can be described and quantified by mathematical methods, but not diminished by further empirical studies (although it may be better characterized). The term aleatory uncertainty intends to emphasize its relation to the randomness in gambling and games of chance.

On the other hand, the second kind of uncertainty is the incertitude that comes from scientific ignorance, measurement uncertainty, inobservability, censoring, or other lack of knowledge

Review of Uncertainty Oriented Mathematical Methods

New mathematical methods have been developed since the sixties with the aim to efficiently handle the uncertainties; especially, to represent what is known about real valued but uncertain quantities. For situations in which the uncertainty is purely aleatory, probabilistic and statistical methods are usually preferred. When the gaps in our knowledge involve both aleatory and epistemic uncertainty, several competing approaches have been suggested. Let us stress that the frequently used term of „uncertainty analysis" has been applied so far only for probabilistic evaluations. Thus it could not offer the entire evaluation of all types of uncertainties. The most important new methods consist of interval analysis, fuzzy set theory, neuro-fuzzy systems, possibility theory, probability bounds theory, and different hybrid methods. The common feature of these methods is that instead of single valued crisp input data they apply different new types of data expressing the amount of uncertainty related to the given input data. The basic properties of these methods and their advantages as well as their limitations are shortly outlined below.

Interval analysis (Moore 1979) replaces crisp numbers by uncertainty intervals (Figure 1). The method guarantees that the true value will always remain within the interval, but this goal is achieved at cost of precision. During the calculations the intervals become wider and wider and the final results become too conservative.

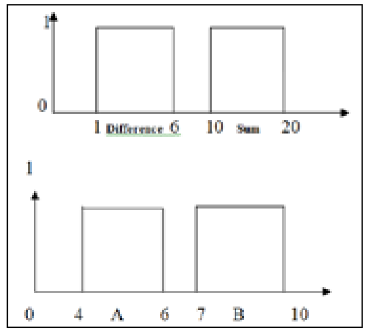


Figure 1: Two intervals and their sum (A+B) and difference (B-A)

The related fuzzy set theory expresses uncertainty very often by the use of fuzzy numbers. They represent estimates of uncertainty at different levels of possibility (or membership degree). Membership functions of fuzzy numbers are by definition unimodal and they have to reach at least in one point the possibility level one, that is, the full possibility. In geology mainly trapezoidal and triangular fuzzy numbers are applied. They can be both symmetrical and asymmetrical. The smallest and the largest possible values of the given variable represent the lower and the upper bounds of the support of the fuzzy number. All values of the variable must be within these boundaries. The values reaching the possibility level one are considered as the most possible estimates of the given variable, and this interval is called the core of the fuzzy number. Fuzzy numbers are generalizations of traditional real numbers, as the latter ones can be regarded as fuzzy numbers with a single point support.

All usual arithmetic calculations can be carried out with fuzzy numbers. The frequent transitions of the geological populations can be also represented by fuzzy numbers (Figure 2).

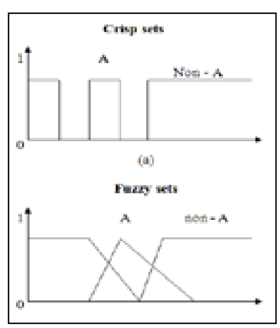


Figure 2 (a): Crisp set A and its complement non-A. Their intersection is empty, and their union is the set of all elements of the universe.
(b): Fuzzy set A and its complement non-A. They overlap.

Uncertainty of Risk Analysis in Geology

Risk is a common term in science, economy and industry. According to the definition of the Society of Risk Analysis, risk is the potential for realization of unwanted consequences of a decision or an action. Risk analysis is defined by the same society as „the process of quantification of the probabilities and expected consequences of risks". Risk analysis has been applied to several problems in geology, such as mineral exploration, mining projects, landslides, floods, volcanic and earthquake hazards. The safety assessments of toxic and radioactive waste repositories represent particularly important applications of risk analysis. All these calculations have been carried out so far by traditional deterministic and probabi-listic methods .At our knowledge, no uncertainty oriented methods have been applied for risks of geological problems so far.

The basic requirement of risk analysis is to exclude the possibility of underestimation of risk at the given conditions. With the traditional methods measures of central tendency (mean, median etc.) are produced. However, experience showed that not these measures, but the tail of the distributions are of paramount importance, as they represent risks of low proba-bility, but of severe consequences. Dependency bounds analysis, suggested by Ferson (1996), seems to assure sufficiently secure estimates of these tail-probabailities. The methods of interval analysis and fuzzy arithmetic have been first applied to risk analysis by Ferson and Kuhn (1992) for ecological problems. Our aim is to apply these methods for the calculation of geological risks as well.

Possible Applications in Geology

In geology the transitions are very frequent. When we apply a probabilistic evaluation, we are obliged to draw sharp boundaries and to cut transitional zones into different pieces. Obviously this is a distortion of the natural reality. Membership functions can be applied with success to solve this problem. The degree of preference refers to a set of more or less preferred objects and $\mu_A(x)$represents the degree of preference in favor of the object x. This case may occur in the exploration of mineral deposits, or of groundwater resources, when we have to choose among several potential regions. The degree of our preference for a given region can be represented by a membership function. The choice between suitable locations for commercial, toxic or radioactive waste disposal can also be represented by membership functions, instead of the traditional ranking of crisp numbers. Note that when using the degree of preference, the choice of the preferred object is ours.

Conclusions

In geology, uncertainty has long been considered a removable adverse circumstance that should gradually disappear with the overall development of the Earth-Sciences. However, one must recognize that a part of this uncertainty is an inherent feature of Nature. Therefore, understanding and appropriate handling of uncertainties should be part of all future geological investigations. Traditional mathematical methods – deterministic and probabilistic – applied so far in geological investigations are mathematically correct, but by far not optimal for the treatment of all kinds of uncertainties. New mathematical methods summarized in this paper are suitable to evaluate in a mathematically correct way semi-quantitative and qualitative („linguistic") input data and to determine the uncertainties and errors connected with them. A thorough study of the geological objects and processes is indispensable for any mathematical evaluation in geology with Fuzzy Graph.

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