Research Paper

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Unsteady State Mathematical Model for Chlorine Concentration Decay in Drinking Water Pipeline Network

Mathematics

KEYWORDS :

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ABSTRACT

The unsteady-state mass conservation equation for the concentration of chlorine in drinking water distribution system considering diffusion and advection in the axial direction as well as chlorine decay within the bulk flow to the pipe wall is solved. The results obtained in the model analytically and numerically both are compared for various values of diffusivity, bulk flow velocity and axial distance. The model can be used to locate the point where more chlorination is required in the system to maintain the safe limit of drinking quality of the water.

INTRODUTION

The study of safe drinking water with in a municipal water distribution system is of great significations as it plays an important role in assure the good drinking water to the consumer. Munavali and Mohan (2005) presented a simulation-optimization model for water quality parameter estimation in the distribution system under dynamic state. Osman,and Metin (1999) solved two dimensional convection dispersive equation numerically for various boundary and initial conditions, considering the decay of chlorine in the bulk flow, but they did not consider the transfer of chlorine from bulk flow to the pipe wall. Jaipal, Bhadula and Kala (2012) presented two dimensional steady state mathematical model that accounts for transport in the axial direction of diffusion and that incorporates chlorine decay within the bulk flow and transport of the chlorine from bulk flow to the pipe wall to predict the chlorine concentration in a drinking water distribution system . Eran *et al* (2011) studied the chlorination and ultraviolet (UV) irradiation of rotating biological contractor in treating the light-grey water. They examined the ability of chlorine and UV to inactive indicator bacteria and specific Pathogens. Cherchi and Gu (2011) investigated the impact of the cell growth stage on chlorine disinfection efficiency and the impact of the growth stage on chlorination resistance by comparing the inactivation efficiencies of two indicator bacterial strains obtained from various growth Phases. Hoefel *et al* (2005) in micro trial resistant to chlorination has observed both of these in lab studies and in full scale chlorine disinfection Practice for water and Waste-water treatment. Wojcicka *et al* (2007) in previous studies have found that indigenous bacteria are related from different environment. Huang *et al* (2011) studied that the influence of chlorination on end toxin activities of secondary sewage effluent and Pure Cultured Gram-negative bacteria was instigated. Biswass et al. (1993) considered a steady-state model for chlorine concentration decay in pipes. They determine the cup-mixing average chlorine concentration at any location decay in pipes. The transport of chlorine from the bulk flow to the pipe wall (due to concentration decay at wall) was not considered in their model. Clark et al. (1994) showed how chlorine residuals can vary throughout the day at different locations in the distributive systems. Clark et al. (1995) used first order kinetics and rate of chlorine decay in their model. They showed that the fluid velocity and pipe radius affect the propagation of chlorine residual levels, disinfection efficiency and the formation of disinfection by-products. Reddy et al. (1996) discussed the weighted least-square method for some parameter estimation in water distribution network, like model pressure heads, pipe flow, head loss in pipes and consumptions in flows. David and Bryan (1996) developed an adjective transport model by neglecting the contribution of radials as well as axial diffusion terms.

MATHEMATICAL MODELLING

The unsteady-state mass conservation equation for the concentration of the chlorine in drinking water distribution system considering transport in the axial direction of diffusion and advection as well as chlorine decay within the bulk flow to the pipe wall for axially symmetric flow can be written as.

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2} - K_b C - \frac{K_f (C - C_w)}{r_h} \quad (1)$$

Where K_b and K_f are the chlorine decay rate constant for bulk flow (s^{-1}) and mass transfer coefficient (m/s) respectively. C_w is the chlorine concentration at the pipe wall (kg/m^3) and r_h is the hydraulic radius of the pipe.

Assuming that the reaction of chlorine at the pipe wall is of first order with respect to the wall concentration C_w and that it proceeds at the same rate as chlorine is transported to the wall gives the following mass balance equation for the chlorine at the wall.

$$K_f (C - C_w) = K_w C_w \quad (2)$$

Substituting the value of C_w from equation (2) into equation (1). We get

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2} - K_b C - \frac{K_f K_w}{r_h (K_f + K_w)} C \quad (3)$$

The initial and boundary conditions are

(i) C(x, t) = 0 x ≥ 0, t = 0 (4.i)
(ii) C(x, t) = C_o x = 0, t › 0 (4.ii)
(iii) C(x, t) = 0 x→ ∞, t ≥ 0 (4.iii)

Where C_o is initial concentration of chlorine.
Introducing the following transformations

$$C(x, t) = P(x,t)\exp\left[\left(\frac{U}{2D}x - \left(\frac{U^2}{4D} + K\right)t\right)\right] \quad (5)$$

Where $K = K_b + \frac{K_w K_f}{r_h (K_f + K_w)}$

Equation (3) reduced into

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x^2} \quad (6)$$

The initial and boundary conditions (4.i) to 4(.iii) become

(i) P(x, t) = 0 x ≥ 0, t = 0 (7.i)
(ii) P(x, t) = $c_0 \exp\left[\left(\frac{U^2}{4D} + K\right)t\right]$
x = 0, t › 0 (7.ii)
(iii) P(x, t) = 0 x ® ∞, t ≥ 0 (7.iii)

Solving equation (6) together with initial and boundary conditions (7.i) to (7.iii) by Laplace transformations method and then substituting the value of P(x, t) in equations (5), we get

$$C(x,t) = \frac{C_0}{2}\left[\exp\left(\frac{U - \sqrt{U^2 + 4KD}}{2D}x\right)\operatorname{erfc}\left(\frac{x - \sqrt{U^2 + 4KD}\,t}{2\sqrt{Dt}}\right) + \exp\left(\frac{U + \sqrt{U^2 + 4KD}}{2D}x\right)\operatorname{erfc}\left(\frac{x + \sqrt{U^2 + 4KD}\,t}{2\sqrt{Dt}}\right)\right] \quad (8)$$

RESULTS AND DISCUSSION

The numerical values of chlorine concentration given by equation (8) are obtained for different values of parameters such as fluid velocity, diffusivity and chlorine consumption rate and are shown in fig.1 and fig.2

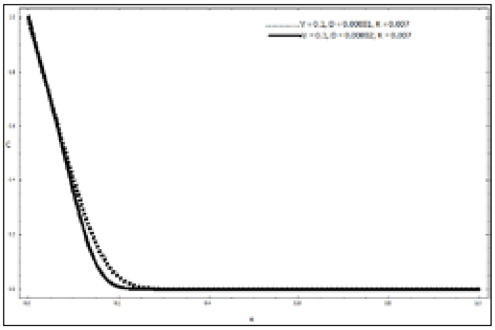

Fig.1 Variation of chlorine concentration with axial distance for different values of diffusivity at t = 100.

From fig.1 it is clear that when fluid velocity U=0.1, consumption rate k = 0.007 and diffusivity D=0.00001 chlorine concentration decreases rapidly from x=0 to x=0.2(approximately) then it decreases slowly from x=0.2 to x=0.25 and after that it becomes constant. But if value of fluid velocity U and chlorine consumption rate are same and diffusivity D=0.00002 then chlorine concentration decreases very fast from x=0 to x=0.18(approximately) then it decreases slowly from x=0.18 to x=0.19 and it becomes constant after x=0.2. This is true due to the fact that when diffusivity increases then more mixing of chlorine takes place in water and therefore chlorine concentration becomes constant at some earlier time and so at some earlier axial distance.

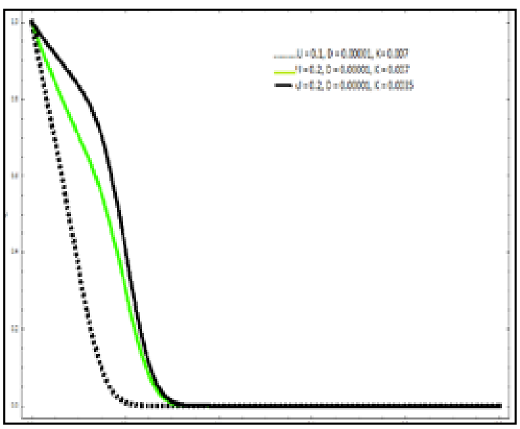

Fig.2 Variation of chlorine concentration with axial distance for different values of bulk flow velocity and K.

From fig.2 we can observe that when fluid (bulk) velocity increases while diffusivity and consumption rate are constant then decay of chlorine concentration is slower with respect to axial distance this is due to the fact that when bulk flow velocity increase then water can travel more distance in less time and mixing of chlorine in water is less. Again from fig.2 it is clear that when chlorine consumption rate decreases (keeping bulk flow velocity and diffusivity constant) then chlorine concentration increases at the same location from x = 0 to x = 0.3 and after that concentration becomes constant .This shows that when chlorine consumption rate to the pipe wall is higher then clearly there will be less chlorine in the water.

Equation (3) together with initial and boundary conditions (4.i) to (4.iii) is also solved numerically and result are shown graphically from fig.3 to fig.5.

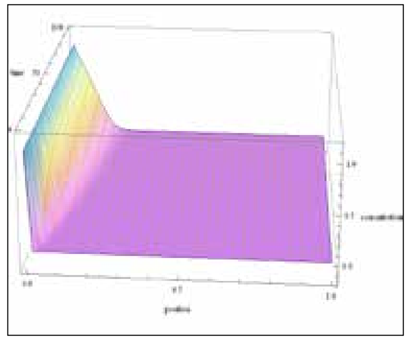

Fig.3 Variation of chlorine concentration with axial distance and time for K=0.007, U=0.001 and D=0.00001.

It is clear that from fig.3 that at t = 100 the variation of chlorine concentration with axial distance is almost same as obtained analytically given by equation (8) shown in fig.1 i.e. chlorine concentration decay is very fast from x = 0 to x = 0.21 and then it decreases slowly from x = 0.21 to x = 0.25 and after that it becomes constant. The same pattern follows for other values of t.

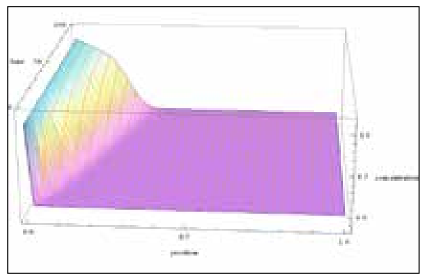

Fig.4 Variation of chlorine concentration with axial distance and time for K=0.0035, U=0.002 and D=0.00001.

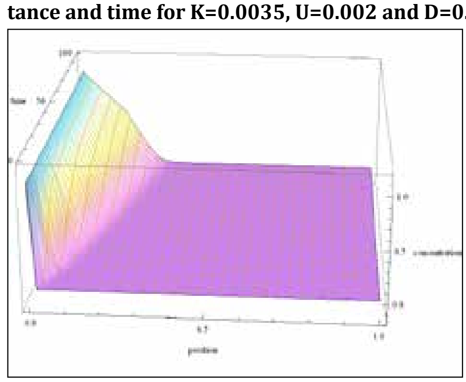

Fig.5 Variation of chlorine concentration with axial distance and time for K=0.007, U= 0.002 and D = 0.00001

On comparing fig. 4 and fig. 5 we see that when the values of bulk flow velocity and diffusivity are taken constant but the value of chlorine consumption rate increases from 0.0035 to 0.007 the level of chlorine concentration in drinking water decreases and the same nature is obtained in fig. 2, from our analytical result.

CONCLUSION

The unsteady-state mathematical model presented in the paper predicts well chlorine concentration in drinking water pipeline system .The result obtained from analytical results and numerical method are very much similar .The model can be use effectively to locate the position where more chlorination is required to maintain the safe limit.

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