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Research Paper

Two-Dimenstional Stress Variations at the Interface in Orthotropic Elastic Half-Spaces

Mathematics

KEYWORDS : non-uniform slip, orthotropic , stress-ratio.

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ABSTRACT

: In this paper, the model of two welded orthotropic elastic half-spaces has been considered. The stresses at the interface has been studied due to different non-uniform slips (parabolic, linear, elliptic and cubic) along a very long vertical strike-slip fault lying in the upper elastic half-space.. The interface between two elastic half-spaces is assumed to be horizontal and parallel to one plane of elastic symmetry of the orthotropic elastic medium. It has been also observed that some stress components are not required to be continuous across the interface between two elastic half-spaces. However, stress ratio of stress components at the interface due to each non-uniform slip profile is square of ratio of the orthotropic elastic constants. Numerically, it is shown that the different slip-profiles has significant effect on the stresses at the interface.

1. INTRODUCTION

Slip and rupture length are the most readily observed parameters used to describe an earthquake source. In case of long faults, one is justified in using a two-dimensional approximation, which has simplified the algebra to a great extent. The static deformation of a semi-infinite elastic isotropic medium due to a very long strike-slip and dip-slip fault has been studied by many researchers (e.g. Kasahara [1], [2]; Rybicki [3],[4]; Savage [5], and Mavko [6]). However, most of these studies assumed to be uniform slip on fault.

A problem with uniform slip models is that they predict stress singularities around the edges of the fault. Furthermore uniform slip is not sufficient to explain complicated surface deformation. For this reason, uniform slip models cannot be used in the near field. There are a number of interesting phenomenon that occur near the edge of the fault zone; e.g., vertical movements associated with strike-slip faulting. In order to study these phenomena, it is necessary to consider models of earthquake faulting with non-uniform slip on a fault. In the variable slip model both the amount of slip in a given place and the length of rupture may vary from earthquake to earthquake.

Madan et al [7] obtained static deformation field due to non-uniform slip (parabolic, linear, cubic and elliptic) on a long vertical strike-slip fault in an orthotropic elastic half-space. Madan et al [8] also obtained static deformation of two coupled orthotropic elastic half-spaces due to non-uniform slip along a very long vertical strike-slip fault.

In this paper, the model of two orthotropic elastic half-spaces in perfect contact has been considered and the effect of the stress ratio at the interface of the model due to different non-uniform slips (parabolic, linear, elliptic and cubic) along a very long vertical strike-slip fault lying in the upper elastic half-space has been obtained. The interface between two elastic half-spaces is assumed to be horizontal and parallel to one plane of elastic symmetry of the orthotropic elastic medium. The coupling between two elastic half-spaces is assumed to be 'welded'. It has been also observed that some stress components are not required to be continuous across the interface between two elastic half-spaces. However, stress ratio of stress component at the interface due to each non-uniform slip profile is square of ratio of the orthotropic elastic constants. Numerically, the variation of shearing stresses at the interface due to different slip profiles have been depicted.

2. FORMULATION AND SOLUTION OF THE PROBLEM

For an orthotropic elastic medium, with coordinate planes coinciding with the planes of symmetry and one plane of symmetry being horizontal, the stress-strain relation in matrix form is [9]

$$\begin{bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{22} \\ e_{33} \\ 2e_{23} \\ 2e_{13} \\ 2e_{12} \end{bmatrix} \quad (1)$$

where the two-suffix quantities c_{ij} are elastic constants of the medium.

A transversely isotropic elastic medium, with the z-axis coinciding with the axis of symmetry is a particular case of an orthotropic medium for which

$$c_{11} = c_{22}\ ,\ c_{13} = c_{23}\ ,\ c_{55} = c_{44}\ ,\ c_{66} = \frac{1}{2}\left(c_{11} - c_{12}\right), \qquad (2)$$

and the number of independent elastic constants reduce from nine to five. When the medium is isotropic

$$\begin{aligned} c_{11} &= c_{22} = c_{33} = \lambda + 2\mu\ , \\ c_{12} &= c_{13} = c_{23} = \lambda\ , \\ c_{44} &= c_{55} = c_{66} = \mu\ , \end{aligned} \qquad (3)$$

where λ and μ are Lame's constants.

Let (u, v, w) be the components of the displacement vector. Let the elastic medium under consideration be under the conditions of antiplane strain deformation in the yz-plane due to a very long vertical strike-slip dislocation parallel to the x-axis. In this case, the displacement vector is parallel to the direction of the fault strike and depends upon y and z coordinates only. Thus, under the state of antiplane strain deformation, v = w = 0 and u= u (y, z). The equilibrium equation in terms of non-zero displacement component u is [10]

$$\frac{\partial^2 u}{\partial y^2} + \frac{1}{\alpha^2}\frac{\partial^2 u}{\partial z^2} = 0 \qquad (4)$$

for zero body forces. The non-zero stresses are

$$\tau_{12} = c\alpha^2 \frac{\partial u}{\partial y},\ \tau_{13} = c\frac{\partial u}{\partial z} \qquad (5)$$

where

$$c = c_{55}\quad ,\quad \alpha = \left(c_{66} / c_{55}\right)^{1/2}. \qquad (6)$$

The values of elastic constants α and c depend upon the values of c_{66} and c_{55}. We assume that the values of α and c are positive and real. In case of an isotropic elastic medium, $c = \mu$ and $\alpha = 1$. When the coupling between two orthotropic elastic half-spaces at the interface $z = 0$ is 'welded', displacements u and stresses τ_{13} across the horizontal plane $z = 0$ are continuous [11]. That is,

$$\begin{aligned} u\left(z = 0^+\right) &= u\left(z = 0^-\right), \\ \tau_{13}\left(z = 0^+\right) &= \tau_{13}\left(z = 0^-\right). \end{aligned} \qquad (7)$$

Consider an orthotropic elastic infinite medium comprising two elastic half-spaces. The upper half-space z > 0 (termed as medium I) and lower half-space z < 0 (termed as medium II) with z-axis vertically upwards. The origin of a cartesian coordinate system Oxyz is placed on the interface z = 0 (Fig. 1). Further assume that both elastic media are homogenous and orthotropic. The upper orthotropic semi-infinite elastic medium is in 'welded' contact with another lower orthotropic semi-infinite elastic medium.

Madan et al [8] obtained the closed-form analytical expressions of the displacement and stress for uniform slip and various non-uniform slip-profiles. The stress component τ_B is continuous across the interface $(z=0)$ due to the welded contact boundary conditions.

At the interface $z=0$, the stress component τ_2 is not continuous. For different slip profiles, at any point of the interface $z=0$, we have

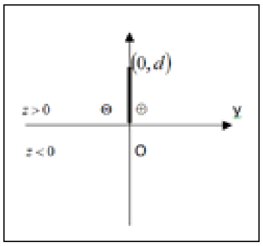

Figure 1. Section x = 0 of the model consisting two orthotropic elastic half-spaces with a long vertical strike-slip fault in the upper half-spcae of region $z>0$. ⊕ and ⊖ indicate the displacements in the positive x direction and negative x-direction, respectively.

2.1. Uniform Slip

The dimensionless stresses at any point of the interface $z=0$ are obtained as below:

$$\sigma_2^{(1)} = \frac{\alpha_1^3}{\pi(1+m)(Y^2+\alpha_1^2)} \quad (8)$$

for the upper half space, and

$$\sigma_2^{(2)} = \frac{\alpha_1\alpha_2^2}{\pi(1+m)(Y^2+\alpha_1^2)} \quad (9)$$

for the lower half-space, where

$$m = \frac{c_2\,\alpha_2}{c_1\,\alpha_1},$$

$$\sigma_2^{(1)} = \frac{\tau_2^{(1)} d}{b_0\, c_1}, \quad \sigma_2^{(2)} = \frac{\tau_2^{(2)} d}{b_0\, c_2} \quad (10)$$

and $b(h)=b_0$ is the uniform slip.

2.2 Parabolic slip

Let the slip on the fault vary according to law

$$b(h) = b_0\left(1-\frac{h^2}{d^2}\right), \qquad 0 \le h \le d. \quad (11)$$

The stresses at any point z=0 of the interface are

$$\sigma_2^{(1)} = \frac{-2\alpha_1}{\pi(1+m)}\left[-1+\frac{Y}{\alpha_1}\tan^{-1}\frac{\alpha_1}{Y}\right] \quad (12)$$

for the upper half-space, and

$$\sigma_2^{(2)} = \frac{-2\alpha_2^2}{\pi\,\alpha_1(1+m)}\left[-1+\frac{Y}{\alpha_1}\tan^{-1}\frac{\alpha_1}{Y}\right], \quad (13)$$

for the lower half-space.

3.3 Linear slip

Let the slip on the fault vary according to law

$$b(h) = b_0\left(1-\frac{h}{d}\right), \qquad 0 \le h \le d \quad (14)$$

The dimensionless stresses are obtained as under

$$\sigma_2^{(1)} = \frac{\alpha_1}{2\pi(1+m)}\left[\log\left(1+\frac{\alpha_1^2}{Y^2}\right)\right], \quad (15)$$

for the upper half-space, and

$$\sigma_2^{(2)} = \frac{\alpha_2^2}{2(1+m)\pi\,\alpha_1}\left[\log\left(1+\frac{\alpha_1^2}{Y^2}\right)\right] \quad (16)$$

for the lower half-space.

2.4 Elliptic slip

Let the slip on the fault vary according to law

$$b(h) = b_0\left(1-\frac{h^2}{d^2}\right)^{1/2}, \qquad 0 \le h \le d. \quad (17)$$

The stresses are

$$\sigma_2^{(1)} = \frac{\alpha_1}{2(1+m)}\left[1 \mp \frac{Y}{\sqrt{\alpha_1^2+Y^2}}\right] \quad (18)$$

for the upper half-space, and

$$\sigma_2^{(2)} = \frac{\alpha_2^2}{2(1+m)\alpha_1}\left[1 \mp \frac{Y}{\sqrt{\alpha_1^2+Y^2}}\right] \quad (19)$$

for the lower half-space.

The upper sign '-' is for Y>0 and the lower sign '+'is for Y<0.

2.5 Cubic slip

For the cubic slip profile

$$b(h) = b_0\left(1-\frac{h^2}{d^2}\right)^{3/2}, \quad 0 \le h \le d. \quad (20)$$

The stresses are obtained as

$$\sigma_2^{(1)} = \frac{3}{2(1+m)\alpha_1}\left[\frac{1}{2}\alpha_1^2 + Y^2 \mp Y(\alpha_1^2+Y^2)^{1/2}\right] \quad (21)$$

for the upper half-space, and

$$\sigma_2^{(2)} = \frac{3\alpha_2^2}{2(1+m)\alpha_1^3}\left[\frac{1}{2}\alpha_1^2 + Y^2 \mp Y(\alpha_1^2+Y^2)^{1/2}\right] \quad (22)$$

for the lower half-space.

Further, at any point of the interface, the stress ratio of stress component due to uniform slip and each non-uniform slip (parabolic, linear, elliptic and cubic) is obtained as under

$$\frac{\sigma_2^{(1)}}{\sigma_2^{(2)}} = \frac{\alpha_1^2}{\alpha_2^2}, \quad (23)$$

which is square of the ratio of the orthotropic elastic constants.

Therefore,

$$\frac{\tau_2^{(1)}}{\tau_2^{(2)}} = \frac{c_1\alpha_1^2}{c_2\alpha_2^2}. \quad (24)$$

On taking $\alpha_1 = \alpha_2 = 1, \quad c_1 = \mu_1 \ \ and\ c_2 = \mu_2$ we can obtain the results for entirely isotropic elastic medium.

3. NUMERICAL RESULTS AND CONCLUSIONS

In this study, the model of two welded orthotropic elastic half-spaces has been considered and the variation of the stresses at the interface of the model due to different non-uniform slips (parabolic, linear, elliptic and cubic) along a very long vertical strike-slip fault lying in the upper elastic half-space has been depicted. It has been also observed that some stress components are not required to be continuous across the interface between two elastic half-spaces. However, stress ratio of stress component at the interface due to each non-uniform slip profile is square of ratio of the orthotropic elastic constants.

For numerical computations, we use the values of orthotropic elastic constants for olivine materials for the medium I for which α_1 = 0.9894, $c_1 = 8.0 \times 10^{11}\ dynes/cm^2$. For medium II, we use the values of orthotropic elastic constants of Baryte type materials for which $\alpha_2 = 0.9824$, $c_1 = 2.8 \times 10^{11}\ dynes/cm^2$. In Fig.2, The variations of stress- es due to different slip-profiles are shown and the notations SU,SP,SL,SE and SE shown in the figure are:

SU-stresses due to uniform slip. SP-Stresses due to parabolic slip, SL-stresses due to linear slip, SE-stresses due to elliptic slip, and SC-stresses due to cubic slip. It shows that the slip profile influence significantly at the interface.

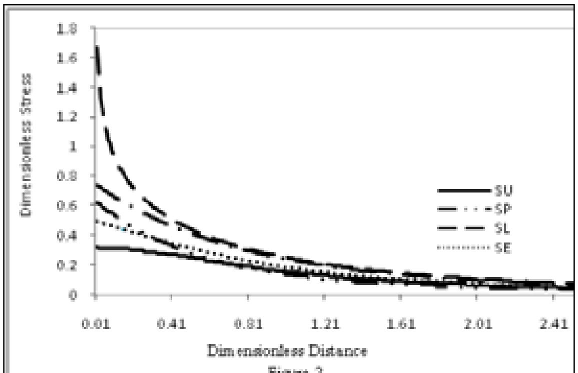

Figure 2

REFERENCE

[1] K. Kasahara, Static and dynamic characteristics of earthquake faults, Bull. Earthq. Res. Ins., University of Tokyo (1960) 74. | [2] K. Kasahara, A strike-slip fault buried in a layered medium, Bull. Earthq. Res. Inst., Vol. 42(1964) 609. | [3] K. Rybicki, Static deformation of a laterally inhomogenous half-space by a two-dimensional strike- slip fault, J. Phys. Earth, 26(1978) 351. | [4] K. Rybicki, Dislocations and their geophysical applications, Continuum theories in Solid Earth Phys.,Elsevier Amsterdam, The Netherlands (1986)18-186. | [5] J. C. Savage, Dislocation in seismology, In: Dislocations in solids 3-Moving dislocations (edited by F.R.N Nabarro) North-Holland Ameterdam, The Neitherlands (1980)251. | [6] G. M. Mavko, Mechanics of motion on major faults, Annual Review Earth Planet. Scs., Vol. 9(1981)81. | [7] D. K. Madan, K. Singh, R. Aggarwal, A. Gupta, Displacements and stresses in anisotropic medium due to non-uniform slip along a very long strike-slip fault, Indian Soc. Earth. Tech., Vol. 42(2005) 1. | [8] D. K.Madan, S. Chugh, K. Singh, Static deformation of two coupled orthotropic elastic half-spaces due to non-uniform slip along a very long vertical strike-slip fault, Int. J. Appl. Maths. Engg. Sci., Vol. 3(2009) 113. | [9] T. J. Chung, Applied Continuum Mechanics, Cambridge University Press, New York, 1996. | [10] N. R. Garg, D. K. Madan, R. K. Sharma, Two- dimensional deformation of an orthotropic elastic medium due to seismic sources, Phys. Earth. Planet. Int., Vol. 94(1996) 43 . | [11] S. J. Singh, Static deformation of a multilayered half-space by internal sources, J. Geophys. Res., Vol. 75(1970) 3257. |