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Research Paper

A Study of Algorithm for the Optimal Scheduling of the Job Sequencing Problem

Statistics

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ABSTRACT

This paper studies flow shop scheduling problem in which job processing time on each machine are given. The objective of the study is to get optimal sequence of jobs in order to minimize the total elapsed time. The given paper focuses on to the study of the algorithm for finding optimal scheduling of the job sequencing problem. The method is illustrated with the help of numerical example.

1] Introduction

Many applied & experimental situations which generally arise in manufacturing concern to get an optimal schedule of jobs in set of machines. Sequencing has been a topic of considerable research in the past few years. Sequencing is a technique to order the jobs in a particular sequence. There are different type of sequencing which are followed in industries such as first in first out basis, priority basis, job size basis and processing time basis.

By scheduling we assign a particular time for completing a particular job. The main objective of scheduling is to arrive at a position where we will get minimum processing time.

Flow shop sequencing problem is commonly defined as determining the sequence in which n jobs is to be processed on m machines in order to minimize the total elapsed time. In flow shop scheduling the objective is to obtained a sequence of jobs which when processed in a fixed order of machines will optimize some well defined criteria.

The classical flow shop scheduling problem is one of the most well known scheduling problems. The problem can be described as follows.

There are a set of jobs and set of machines. Each job consist of chain of operation, each of which needs to be processed during an uninterrupted time period of a given length on a given machine. Each machine can process at most one operation at a time. A schedule is an allocation of operation to time intervals of the machine. The problem is to find schedule of minimum length.

It is a typical combinatorial optimization problem where each job has to go through processing in each and every machine. The number of possible schedules of the flow shop scheduling problems involving n jobs and m machines is $(n!)^m$. The optimal solution for the problem is to find the sequence of jobs on each machines in order to complete all the jobs on all the machines in the minimum total time provided each job is processed on machines 1,2,3,.....,m in that order. In this paper our main focus on to implement branch and bounding method to the solution of flow shop scheduling problem.

2] Branch and bounding technique used in flow shop scheduling problem

The Branch and Bounding algorithm are time intensive tree based exploration methods for solving to optimality combinatorial optimization problem. In order to use the branch and bound technique one must be able to describe the problem as a tree in which each node represents a partial solution. The first node in the related tree structure is called as the root node. From this node there are n branches corresponding to the n possible sub problems. Branching is the process of partitioning a large problem into two or more sub problem and bounding is the process of calculating lower bound on the optimal solution of the given problem.

In this section application of branch and bounding technique is applied to obtain an optimal sequence for the flow shop scheduling problem. A branch and bounding scheme is given as below.

Let S_0 denotes root node at level 0. At level 1 the node S_0 is partition in to n different sub problems $\{S_1^1, S_2^1, S_3^1, \ldots, S_n^1\}$ by assigning each job to the last position in the sequence. Clearly these sub problems are smaller than S_0 and each S_1^1 is a partially solved version of S_0. Next each of the sub problems can be partitioned further. For instance S_2^1 can be partitioned into S_{21}^1, S_{23}^2, S_{24}^2 ....., S_{2n}^2. In S_{23}^2 job 2 & 3 can occupy the last two positions in the sequence. At level k then each sub problems contains k fixed positions in the sequence and can be partitioned in to (n-k) sub problems.

At any point of time we compare the lower bounds of all the terminal nodes and select the node with minimum lower bound for further branching. If there is a tie on the minimum lower bound then select the node at the lower level for further branching. If the node with the minimum lower bound lies at the $(n-1)^{th}$ level then the optimality is reached.

3] Notations and terminology used in Branch and Bounding method

Let σ denotes partial permutation occurring at the beginning of the sequence.

σ' denotes set of jobs that are not contained in the partial permutation.

t_1, t_2,t_3 denotes the completion time of the last job on machine 1, machine 2, machine 3 respectively among the jobs in σ.

Hence a lower bound on the make span with respect to machine 1 is

$$b_1 = t_1 + \sum_{i\varepsilon\sigma'} A_i + \min_{i\varepsilon\sigma'}\{B_i + C_i\} \ldots..(3.1)$$

The second lower bound on machine 2 is given by

$$b_2 = t_2 + \sum_{i\varepsilon\sigma'} B_i + \min_{i\varepsilon\sigma'}\{C_i\} \quad \ldots..(3.2)$$

The third lower bound on machine 3 is given by

$$b_3 = t_3 + \sum_{i\varepsilon\sigma'} C_i \quad \ldots..(3.3)$$

The final lower bound for σ is

$$LB(\sigma) = Max \begin{bmatrix} b_1 = t_1 + \sum_{i\varepsilon\sigma'} A_i + \min_{i\varepsilon\sigma'}\{B_i + C_i\} \\ b_2 = t_2 + \sum_{i\varepsilon\sigma'} B_i + \min_{i\varepsilon\sigma'}\{C_i\} \\ b_3 = t_3 + \sum_{i\varepsilon\sigma'} C_i \end{bmatrix} \ldots\ldots(3.4)$$

Where A_i,B_i & C_i are the processing times on machine I, machine II, machine III respectively.

4] Application of Branch and Bounding method to flow shop scheduling problem

Let us explain the application of branch and bounding method to flow shop scheduling problem with the help of numerical example. Consider 4 jobs has to be processed on 3 machines with their processing time is given below.

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Job	Machine I	Machine II	Machine III
1	10	15	23
2	8	10	7
3	12	7	10
4	15	20	6

We start with the root node

At the root node σ = {0} and σ' = {1,2,3,4} The lower bound B for the root node = 0. The set of nodes created under the root node, S_0 is $S_1^1, S_2^1, S_3^1, S_4^1$ Initially $(q_1,q_2,q_3) = (0,0,0)$

The lower bound calculations using the equations (3.1),(3.2),(3.3) are as given below.

First level

Node S_1^1 **σ = {1}** **σ'**={2,3,4}

$(t_1,t_2,t_3) = (10,25,48)$

$b_1 = 10 + 35 + 17 = 62$

$b_2 = 25 + 37 + 06 = 68$

$b_3 = 48 + 23 = 71$

LB(1) = max { 62, 68, 71} = 71

Node S_2^1 **σ = {2}** **σ'**={1,3,4}

$(t_1,t_2,t_3) = (8, 18, 25)$

$b_1 = 8 + 37 + 17 = 62$

$b_2 = 18 + 42 + 06 = 66$

$b_3 = 25 + 39 = 64$

LB(2) = max { 62, 66, 64} = 66

Node S_3^1 **σ = {3}** **σ'**={1,2,4}

$(t_1,t_2,t_3) = (12, 19, 29)$

$b_1 = 12 + 33 + 17 = 62$

$b_2 = 19 + 45 + 06 = 70$

$b_3 = 29 + 36 = 65$

LB(3) = max { 62, 70, 65} = 70

Node S_4^1 **σ = {4}** **σ'**={1, 2, 3}

$(t_1,t_2,t_3) = (15, 35, 41)$

$b_1 = 15 + 30 + 17 = 62$

$b_2 = 35 + 32 + 07 = 74$

$b_3 = 41 + 40 = 81$

LB(4) = max { 62, 74, 81} = 81

In the first level nodes with lower bounds are as given below

S_1^1	S_2^1	S_3^1	S_4^1
71	66	70	81

The node with minimum lower bound is S_2^1 so branch from this node on the second level.

Second level

Node S_{21}^2 σ = { 2,1} σ' = {3,4}

$(t_1,t_2,t_3) = (18, 33, 56)$

$b_1 = 18 + 27 + 17 = 62$

$b_2 = 33 + 27 + 06 = 66$

$b_3 = 56 + 16 = 72$

LB(21) = max { 62, 66, 72} = 72

Node S_{23}^2 σ = { 2,3} σ' = {1,4}

$(t_1,t_2,t_3) = (20, 27, 37)$

$b_1 = 20 + 25 + 26 = 71$

$b_2 = 27 + 35 + 06 = 68$

$b_3 = 37 + 29 = 66$

LB(23) = max {71, 68, 66} = 71

Node S_{24}^2 σ = { 2,4} σ' = {1, 3}

$(t_1,t_2,t_3) = (23, 43, 49)$

$b_1 = 23 + 22 + 17 = 62$

$b_2 = 43 + 22 + 10 = 75$

$b_3 = 49 + 33 = 82$

LB(24) = max { 62, 75, 82} = 82

S_{21}^2	S_{23}^2	S_{24}^2
72	71	82

The node with minimum lower bound on the second level is the node S_{23}^2. So branch from this node.

Third level

Node S_{231}^3 σ = { 2,3,1} σ' = {,4}

$(t_1,t_2,t_3) = (30, 45, 68)$

$b_1 = 30 + 15 + 26 = 71$

$b_2 = 45 + 20 + 06 = 71$

$b_3 = 68 + 6 = 74$

LB(231) = max {71, 71, 74} = 74

Node S_{234}^3 σ = { 2,3,4} σ' = {1}

$(t_1,t_2,t_3) = (35, 55, 61)$

$b_1 = 35 + 10 + 38 = 83$

$b_2 = 55 + 15 + 23 = 93$

$b_3 = 61 + 23 = 84$

LB(234) = max {83, 93, 84} = 93

S_{231}^3	S_{234}^3
74	93

The node with minimum lower bound on the third level is on the node S_{231}^3 lies at the $(n-1)^{th}$ level hence the optimality is reached to the given problem. Hence the optimal sequence for the given flow shop scheduling problem is 4,2,3,1.

The whole procedure is summarized in the following table.

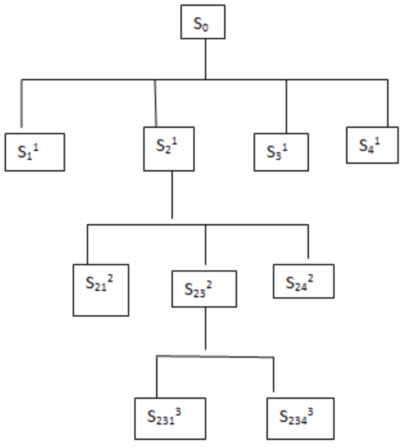
Table 1

Partialse-quence	(t_1,t_2,t_3)	Lower bounds (b_1,b_2,b_3)	Maximum lower bound LB(σ)
1	(10, 25, 48)	(62, 68, 71)	71
2	(8, 18, 25)	(62, 66, 64)	66*
3	(12, 19, 29)	(62, 70, 65)	70
4	(15, 35, 41)	(62, 74, 81)	81
2,1	(18, 33, 56)	(62, 66, 72)	72
2,3	(20, 27, 37)	(71,68,66)	71*
2,4	(23, 43, 49)	(62, 75, 82)	82
2,3,1	(30, 45, 68)	(71, 71, 74)	74*
2,3,4	(53, 55, 61)	(83, 93, 84)	93

The branching and bounding tree for whole procedure is given in the following figure.

Figure 1

Branch & Bounding tree



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