Volume : 2 | Issue : 11 | November 2013 • ISSN No 2277 - 8179

Research Paper

The Role of Matrices in Micro Economics

Economics

KEYWORDS : Microeconomics, Mathematical Economics, Matrices, Consumer Behavior, Determinant, Maxima . Minima

Dr. Purushottam V. Deshmukh

Asstant Professor,Department Of Economics ,Dr. Babasheh Ambedkar Marathwada University, Maharashta, (Idia), 431004

ABSTRACT

Mathematical Economics is a combination of Mathematics and Economics. We can't absolute findings since economics is a social science. Correctness is essential for the study of society but it is difficult in economics . The role of mathematics is important in the analysis of various concepts and theories in economics. We can get an idea of effectiveness of mathematics in economics by observing books written by well-known foreign author. The use of mathematical tools is a part and partial of standard literature in economics. Mathematics is very useful to explain the various concepts ,rules , theorems in micro economics. We can able to study and find out results with more accuracy. Economics is a subject which affects government's social, political , economic and other policies. Researcher will helps to the government by understanding various situations in the economy. Mathematics is provide a better circumstances to develop an art of thinking regarding economics phenomena .

Mathematical Economics is a combination of Mathematics and Economics. We can't absolute findings since economics is a social science. Correctness is essential for the study of society but it is difficult in economics . The role of mathematics is important in the analysis of various concepts and theories in economics. We can get an idea of effectiveness of mathematics in economics by observing books written by well-known foreign author. The use of mathematical tools is a part and partial of standard literature in economics. Marginal rate of substitution , technical rate transformation, elasticity, market structure analysis , distribution laws, consumer behavior, functions, multipliers, development models, international trade theories etc can be analyzed with mathematical tools .There is large scope of mathematical tools in micro economics field. Derivative, integration , differential equation , matrices, trigonometry, simple growth, compound growth rate, limit, determinant, ratios, linear programming , game theory, dynamics theory, goal theory, set etc plays pivotal role in micro economics. The use of mathematics in economics has insure the accuracy. Therefore this paper attempts the key role of matrices and its applications in micro economics.

Matrix :-

Matrix is an important technique which helps us to interpretation and analysis in economics. Consumer behavior theory includes various rules and concepts which can explained by using matrices. It is necessary firstly to focus the meaning , types and algebra of matrices.

Meaning-

1.Matrix is a systematic arrangement of objects.

2.Matrix is arrangement of objects according to rows and columns.

3.Matrix is an array of objects in rectangle or square.

We can define matrix 'A' mathematically as:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \dots & a_{23} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{mxn}$$

O(A) =No. of rows x no. of columns.

Types of matrices:-

1.Row Matrix – The matrix having only one row and any of columns is known as row matrix.

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \end{bmatrix}_{1xn}$$

2.Column Matrix:- The matrix having only one column and any number of row is known as column matrix.

$$B=\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ \dots \\ a_{1m} \end{bmatrix}_{mx1}$$

3.Square Matrix:- The matrix in which number of rows and number of columns is called square matrix.

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{32} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{mxn} \text{,here}$$

m=n

4.Zero Matrix:-A matrix having all element are zero is known as zero matrix.

$$o=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$O=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5.Identity Matrix:- The square matrix in which main diagonal elements are +1 and other remaining elements are zero, is called identity matrix. It is denoted by I.

$$I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6.Lower Triangular Matrix:- A square matrix having all elements above the main diagonal elements are zero, is called lower triangular matrix.

$$A=\begin{bmatrix} 2 & 0 & 0 \\ 1 & 5 & 0 \\ -2 & 2 & 1 \end{bmatrix}$$

$$B=\begin{bmatrix} 5 & 0 \\ 3 & 4 \end{bmatrix}$$

7.Upper Triangular Matrix:- A square matrix having all elements below the main diagonal elements are zero, is called lower triangular matrix.

$$A=\begin{bmatrix} 1 & 5 \\ 0 & 4 \end{bmatrix}, B=\begin{bmatrix} 3 & 4 & 0 \\ 0 & 4 & -7 \\ 0 & 0 & 7 \end{bmatrix}$$

8.Diagonal Matrix:-A square in which at least on nonzero element on the main diagonal and other element are zero, is called diagonal matrix.

$$A=\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}, B=\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

9.Scalar Matrix:-A square matrix in which all main diagonal elements are same and remaining elements are zero is known as scalar matrix.

$$A=\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \qquad B=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

10. Transpose Matrix:- A matrix is obtained by interchanging row into column (column into row) is known as transpose matrix. Transpose matrix is denoted by A^T.

11.Symmetric Matrix:- A matrix whose transpose is own matrix , is known as symmetric matrix.i.e. matrix A is said to be symmetric if $A = A^T$. (here A^T is transpose matrix.)

12.Singular Matrix:- A matrix whose determinant is zero, is called singular matrix.

$$A=\begin{bmatrix} 3 & 7 \\ 9 & 21 \end{bmatrix}$$

13.Non-Singular Matrix:- A matrix whose determinant is not zero, is called Non-singular matrix.

$$A=\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

- Algebra of Matrices.

Algebra of matrices is interesting excise in economics. Various terms and lows will be elaborated through matrix operations. The following operations are basic and important

Addition

Subtraction

Multiplication

Inverse.

Addition:- Equality of order for given matrices is basic condition for addition of matrices. If this condition fulfilled the we add relative elements of matrices.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}_{2x2}, B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}_{2x2}$$

$$A + B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$$

We can generalize the above rule for mxn matrix

Subtraction of matrices:-The order should be same of the given matrices for Subtraction. If this condition satisfied the then the difference between corresponding elements.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2x2}, \ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2x2}$$

$$A - B = \begin{bmatrix} a_{11}-b_{11} & a_{12}-b_{12} \\ a_{21}-b_{21} & a_{22}-b_{22} \end{bmatrix}_{2x2}$$

Product of Matrices :- The equality of number of columns of pre matrix and number of rows of post matrix is the essential condition for the existence of product of matrices .If this condition holds then the product of matrices defined as .

$$\text{AB}= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2x2} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{2x2}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Scalar Products :- If we have to multiply some constant (scalar) to matrix the we multiply each element of given matrix by given constant. Suppose we have to multiply constant k to matrix A,

$$\text{KA}= K\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}_{2\times 2}$$

Inverse Matrix:- Inverse matrix is an important technique for solving various problems related to micro economics. There are different types for finding inverse matrix. The cofactor method is comparatively easy for obtaining inverse matrix. There are various steps for obtaining inverse such as determinant, minor, cofactor, adjoint .If A is non singular matrix then inverse of A is defined as

$$A^{-1} = \frac{Adjo\,\text{int}\,of A}{|A|} \quad \text{, here } |A| \neq 0$$

Algebra of matrices can be extended to mxn matrices.

Lows of Matrices:-

Economic interpretation can be easy with following basic rules /laws of matrices .

1.Cummutative Law:-

a) Commutative Law of addition**-**

$$A + B = B + A$$

b) Commutative Law of Multiplication

$$A \times B \neq B \times A$$

2. Associative Law

a) Associative Law of addition

$$A + (B + C) = (A + B) + \text{C}$$

b) Associative Law of multiplication

$$A \times (B \times C) = (A \times B) \times C$$

3.Distributive Law.

$$A(B + C) = AB + AC$$

Determinant :- Determinant is a specific value of matrix. It is denoted by $|A|$. In Micro economics we have to nee 2x2 and 3x3 determinant. The principle of obtaining determinant as below'

$$1)\left|A\right| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \times a_{22} - a_{12} \times a_{21}$$

2)

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

There are various properties of determinants.
Marahall has developed the cardinal utility analysis with the assumption of utility is measurable .He explained the consumer behavior with utility function. Utility function is the technical relationship between utility and consumed quantities q. The utility function shows that the positive relation between utility and quantities. Concave utility function, quasi-concave are important in consumer behavior analysis .The fundamental feature of indifference curve is convexity to origin. We can get an idea of utility function and indifference curve by applying matrices and determinant.

The utility function $U = f(q_1, q_2)$ is said

to be strictly concave if ,F_{11} <0 , F_{22}<0 and Hessian determinant ,

$$\left|H\right| = \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} > 0$$

, The utility function $U = f(q_1, q_2)$ is said

to be strictly concave if ,F_{11} <0 , F_{22}<0 and Borderd Hessian determinant

$$\left|BH\right| = \begin{vmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{21} & f_{22} \end{vmatrix} > 0$$

The basic assumption of consumer theory behavior is the indifference curve convex to origin , that is the underline utility function is quasi-concave .

Utility Maximization :

Utility maximization is the one of main objective of consumer. We can workout maximum utility of consumer with the help of differentiation .It will be better to apply determinant to take decision about function value. The following conditions should be satisfied for utility $U = f(q_1, q_2)$

maximization

1) $\left|f_{11}\right| < 0$

2) $\left|H\right| = \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} < 0$

The following condition for minimization of the function

1) $\left|f_{11}\right| < 0$

2) $\left|H\right| = \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} > 0$

Explanation of Slustky Equation :

Demand is a function of price since demand is affected by price this is not enough expiation of demand function. Price effect is not independent phenomena. Income effect and substitution effect are two component of price effect .This can be describe with indifference curve but application of Slustky Equation is necessary because appropriate and technical interpretation can be done with Slustky Equation. It is very essay to show Price Effect is combination of income and substitution effect with with Slustky equation . The mathematical tools Differentiation ,matrices and determinant are used for the explanation of Slustky Equation . General form of Slustky Equation as below;

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} & \cdots & f_{1n} & -p_1 \\ f_{21} & f_{22} & f_{23} & \cdots & f_{2n} & -p_2 \\ f_{31} & f_{32} & f_{33} & \cdots & f_{3n} & -p_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{n1} & f_{n2} & f_{n3} & \cdots & f_{nn} & -p_n \\ -p_1 & -p_2 & -p_3 & \cdots & -p_n & 0 \end{bmatrix} \begin{bmatrix} dp_1 \\ dp_2 \\ dp_3 \\ \cdots \\ dp_n \\ d\lambda \end{bmatrix}$$

$$= \begin{bmatrix} \lambda dp_1 \\ \lambda dp_2 \\ \lambda dp_3 \\ \cdots \\ \lambda dp_n \\ -dy + \sum_{i=1}^{n} q_1 dp_1 \end{bmatrix}$$

We can solve the above system with the help of Cramer's Rule or inverse matrix method and obtain the following Slutsky equation

$$\frac{\delta q_i}{\delta p_i} = \frac{\delta q_i}{\delta p_i} - q_i \frac{\delta q_i}{\delta y_i}$$ I.e.(Price Effect = Substitution Effect+ Income Effect)

- Game Theory:-

  The theory of game was developed by John Von Neumann and Oskar Morgenstern in their famous work entitled ' Theory of Games and economic Behavior ' in 1944. Game theory is fantastic tool to analysis the strategies of producers or sellers in duopoly market . Game theory can be classified on the basis of number of players and nature of outcomes .One person game, Two person game , n-person game and Zero sum game , Non-Zero sum game etc. are the various types of game. Pay-off matrix has pivotal role in the game theory . Pay –off matrix can be treated as profit matrix and is defined as :

$$A= \begin{bmatrix} X_{11} & X_{12} & X_{13} & \cdots & X_{1n} \\ X_{12} & X_{22} & X_{23} & \cdots & X_{2n} \\ X_{31} & X_{32} & X_{33} & \cdots & X_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ X_{m1} & X_{m2} & X_{m3} & \cdots & X_{mn} \end{bmatrix}_{mxn}$$

- In the above pay-off matrix row represents strategy of first player (seller) and column represents strategy of second player (seller).More study can be done by obtaining saddle point solution including column maxima, row minima and maximin=minimax or constant , non-constant sum game. We con conclude that the is more scope matrices in the analysis of game theory. Game theory is effective tool in microeconomic analysis specially in market structure analysis. The applications Game theory have increasing trends microeconomic analysis

Input-Output Analysis:-

Wassily Leontief Pioneered the empirical investigation of industry general equilibrium theory in 1941 -1951.The relationship between inputs and output is debating issue in micro economics. The various dimensions to concern relationship between inputs and output If we assume each industry producing only one product , there is no joint products then the question is arise how to determine total output in an economy . This question will be solve by applying input-output technique in microeconomics with some assumptions. Leontief's work is remarkable regarding development of input-output model. Leontief's input –output analysis helps us to determine the correct output levels in an economy. Static input-output model, dynamic input-output model, closed input-output model, etc are the various types of input-output models. Leontief has used matrices and determinant in his model.

Transactions Matrix:- The matrix which indicates the inter-industry relationship in an economy. Transaction matrix may be describe as below

P. S	User 's sector I	II	III	..	..	n	F. D
I	X_{11}	X_{12}	X_{13}	$\cdots$	$\cdots$	X_{1n}	f_1
II	X_{21}	X_{22}	X_{23}	..	..	X_{2n}	f_2
III	X_{31}	X_{32}	X_{33}	$\cdots$	$\cdots$	X_{3n}	f_3
..	$\cdots$	$\cdots$	$\cdots$	$\cdots$	..	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
n	X_{n1}	X_{n2}	X_{n3}	..	..	X_{nn}	f_n
P. I	L_1	L_2	L_3	$\cdots$	$\cdots$	L_n	

Here, P.S.= producer's sector, P.I.=primary input , f_n= final demand

Row indicates the sale of output, and column indicates purchase of input .The second concept input-output coefficient matrix is important in input-output model. We can obtain the elements a_{ij} of input-output coefficient matrix by using following formula

$$A = (a_{ij}) = \frac{X_{ij}}{X_j}, \qquad \text{here}$$

X_{ij} are the elements of transactions matrix. i = 1,2,3,4-----n, j=1,2,3,4,……n

Therefore input output coefficient matrix states as below :

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & \dots & a_{nn} \end{bmatrix}$$

Leontief Matrix:- Leontief matrix is a difference between Identity matrix and input coefficient matrix.

Mathematically we can define L.M. as Leontief Matrix=(I - A) =

$$\begin{bmatrix} (1-a_{11}) & -a_{12} & -a_{13} & \dots & \dots & -a_{1n} \\ -a_{21} & (1-a_{22}) & -a_{23} & \dots & \dots & -a_{2n} \\ -a_{31} & -a_{32} & (1-a_{33}) & \dots & \dots & -a_{3n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -a_{n1} & -a_{n2} & -a_{n3} & \dots & \dots & (1-a_{nn}) \end{bmatrix}$$

It is easy to determine the correct output levels by solving following system :

$$[X]_{mx1} = (I-A)^{-1} d =$$

$$\begin{bmatrix} (1-a_{11}) & -a_{12} & -a_{13} & \dots & \dots & -a_{1n} \\ -a_{21} & (1-a_{22)} & -a_{23} & \dots & \dots & -a_{2n} \\ -a_{31} & -a_{32} & (1-a_{33}) & \dots & \dots & -a_{3n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -a_{n1} & -a_{n2} & -a_{n3} & \dots & \dots & (1-a_{nn}) \end{bmatrix}^{-1} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \dots \\ \dots \\ d_n \end{bmatrix}$$

We obtain the vector X by solving inverse of (I-A) elimination method or co-factor method. Where X is mx1 matrix

- Linear Programming:-

Linear programming is a mathematical tool which helps to optimize the linear objective function subject to set of constraints and non-negativity conditions.

Decision regarding production important. The Linear programming is useful in making decision about level of production with optimum utilization of inputs. Mathematically we define LPP,

Optimize F=XV

Subject to, AX>,<,= B

And $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, ...x_n \geq 0$

We can describe the general form of LLP with the help of matrices

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{11} & a_{11} & a_{11} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \dots \\ b_n \end{bmatrix}$$

Here , V is objective function values row matrix and define as

$$V = \begin{bmatrix} v_1 & v_2 & v_3 & .... & v_m \end{bmatrix}$$

We can obtain the mathematical form of LPP by solving product of the matrices A and X.

Conclusion-

Mathematics is very useful to explain the various concepts ,rules , theorems in micro economics. We can able to study and find out results with more accuracy. Economics is a subject which affects government's social, political , economic and other policies. Researcher will helps to government by studding various situations in economy. Therefore it is necessary to accuracy and scientific thinking with policy maker, researcher, producer, consumer etc so that the applications of mathematics in economic is basic condition.

REFERENCE

1. Handerson and Quandt, Microeconomics Theory-a Mathematical Approach, McGraw-Hill Book Company New York | 2. Alpha C. Chiang , Fundamental Methods of Mathematical Economics , McGraw-Hill Book Company New York | 3. Roychowdhury K C (1991), Microeconomics,Tata McGraw-Hill Publishing Company Limited, New Delhi | 4. Archilbald G.C.and Richard G Lipsey(1997), An Introduction to a Mathematical Treatment of Economics,English Language Book Society and Weidenfeld And Nicolson,Londan | 5. Baruah Srinath (2001) , Basic Mathematics and Its Application in Economics , Macmillan,Chennai | 6. Hughes Ann j, Grawiog Dennis (973),Linear Programming , Addison-Wesley Publishing Company , London | 7. Gujarati Damodar N,Basic Econometrics , Tata McGraw Hill Education Private Limited , New Delhi. | 8. Metha B.C.(1987)Mathematical Economics:MicroEconomic Models,Sultan Chand& Sons, New Delhi. | 9. Agarwal H.S.(1976) , Introduction To Econometrics ,Laksimi Narayan Publishing , Agra | 10. Allen R.G.D.(1937), Mathematical Analysis For Economists , English Language Book Society and Weidenfeld And Nicolson,Londan | 11. Martin A.&Norman B.(1966) Mathematics for Economics and Finance,Cambridge, Australia. | |