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An Arithmetic Edge Graceful Graph

Mathematics

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ABSTRACT

In 1985, Lo [3] introduced the notion of edge graceful graphs. Acharya [1] introduced (k,d) arithmetic labeling of graphs. In this paper we extended the concept of edge graceful labeling to (k,d) arithmetic edge graceful labeling of path, cycle, complete graph K_4 and fan graph F_3. Also obtained the relation between sum of the edge labels and the Estrada Index of these graphs.

1. Introduction

For all terminology and notation in graph theory we follow [6].

Given an integer $k \geq 0$, a graph G = (V,E) with p vertices and q edges is called k – edge graceful, if there exists a bijection $f : E \rightarrow$

{k, k+1,...,k+(q-1)} such that the induced vertex sum $f^+ : V \rightarrow \{0,1,...(p-1)\}$, where $f^+(u) = [\Sigma f(uv) : uvE]$ (mod p) is also a bijection. E](mod p) is also a bijection. Such a labeling is called k edge graceful labeling [3,4,5].

Given non-zero positive integers k, and d a graph G = (V,E) is defined to be (k,d) arithmetic edge graceful, if there exists a bijection $f : E \rightarrow \{k, k + d, \ldots, k + (q - 1)d\}$ such that the induced vertex sum $f^+ : V \rightarrow \{0,1,...(p-1)\}$, where $f^+(u) = [[\Sigma f(uv):uvE]$ (mod p) is also a bijection. Such a labeling is called (k,d) arithmetic edge graceful labeling. The set of all k and d are called the (k,d) arithmetic edge graceful spectrum of G. A good account on graceful labeling problems can be found in the dynamic survey of Gillian[2] .

The eigenvalues of the adjacency matrix A(G) of G are called the eigen- values of G, denoted by $\lambda_1, \lambda_2, \ldots, \lambda_n$. The Estrada index of a graph G is defined as

$$EI(G) = \sum_{i=1}^{n} e^{\lambda i}$$

This concept was proposed in [7,8].

2. Main Result

Here we obtained the (k,d) arithmetic edge graceful labeling of path P_n, cycle C_n (where n is odd), complete graph K_4, and the fan graph with four vertices.

Theorem 2.1 The path P_4 is (k,d) arithmetic edge graceful labeling.

Proof.

Le v_1, v_2, v_3 and v_4 be the four vertice and e_1, e_2 and e_3 be the t hree edges of a path P_4.

Case (i): $k \equiv 1, 3 (\text{mod } 4)$ and $d \equiv 2 (\text{mod } 4)$.

We define $f(e_1) = k$, $f(e_2) = k+2d$ and

$f(e_3) = k + d$.

Case (ii): $k \equiv 0, 2 \ (\text{mod } 4)$ and $d \equiv 1, 3 (\text{mod } 4)$.

We define $f(e_1) = k$, $f(e_2) = k + d$ and

$f(e_3) = k + 2d$. Hence the path P_4 is (k,d) arithmetic edge graceful labeling.

Theorem 2.2. The path P_n (where $n \geq 3$ is a prime number) is (k,d) arithmetic edge graceful labeling.

Proof.

Let the path P_n ,where $n \geq 3$ is a prime number, be the graph with vertex set $\{v_1, v_2, \ldots, v_n\}$ and the edge set $\{e_1, e_2, \ldots, e_{n-1}\}$.

If $k, d \equiv r \ (\text{mod } n)$, $1 \leq r < n$ and

$k(\text{mod } n) = d(\text{mod } n)$ then we define

$f(e_i) = k + (i - 1)d$, $1 \leq i \leq (n - 1)$. Clearly P_n, where $n \geq 3$ is a prime number, is (k,d) arithmetic edge graceful graph.

Theorem 2.3. The path P_n (where n is an odd number other than prime) is (k,d) arithmetic edge graceful labeling.

Proof.

Consider the path P_n (where n is an odd number other than prime), with vertex set

$\{v_1, v_2, \ldots, v_n\}$ and the edge set $\{e_1, e_2, \ldots, e_{n-1}\}$.

If $k, d \equiv 2 \ (\text{mod } n)$ then we define

$f(e_i) = k + (i - 1)d$, $1 \leq i \leq (n - 1)$. We get arithmetic edge graceful labeling.

Theorem 2.4. The cycle C_n (where n is an odd number), is (k,d) arithmetic edge graceful labeling.

Proof.

The cycle C_n , where n is an odd number, with vertex set $\{v_1, v_2, \ldots, v_n\}$ and the edge set $\{e_1, e_2, \ldots, e_n\}$. The edge $e_i = (v_i, v_{i+1})$, $1 \leq i \leq (n - 1)$ and $e_n = (v_n, v_1)$. Here we consider two cases for n.

Case (i): If $k, d \equiv r \ (\text{mod } n)$, $0 \leq r < n$ and n is a prime number then $f(e_i) = k + id$, $1 \leq i \leq (n-1)$ and $f(e_n) = k$.

Case (ii): If $k \equiv 0, 1, \ldots (n - 1) \ (\text{mod } n)$,

$d \equiv 1, 2, \ldots (n - 1) \ (\text{mod } n)$, n is odd number other than prime and d is not divisible by n. then $f(e_i) = k + id$, $1 \leq i \leq (n - 1)$ and

$f(e_n) = k$.

Theorem 2.5 The fan F_3 is (k, d) arithmetic edge graceful graph.

Proof.

Let $\{v, v_1, v_2, v_3\}$ be the vertices of F_3 with v as the center vertex and

$\{e_1, e_2, e_3, e_4, e_5\}$ be the set of edges of F_3 defined by $e_1 = (v_1, v_2)$, $e_2 = (v_2, v_3)$,

$e_3 = (v, v_3)$, $e_4 = (v, v_2)$ and $e_5 = (v, v_1)$.

We label the edges as follows:
Case (i) : $f(e_i) = k + (i-1)d$, if $k \equiv 1 \pmod 4$ and $d \equiv 1, 2 \pmod 4$. (Fig 1)

Case (ii): If $k \equiv 1 \pmod 4$ and $d \equiv 3 \pmod 4$ then $f(e_1) = k + d$, $f(e_2) = k$,

$f(e_3) = k + 2d$, $f(e_4) = k + 3d$ and

$f(e_5) = k + 4d$.

Then the vertex labels as {0, 1, 2, 3}. (Fig 2).

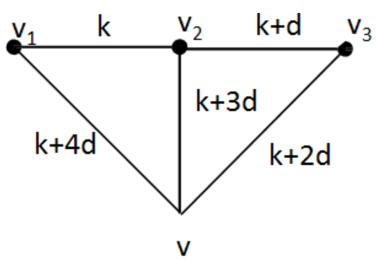

Fig 1

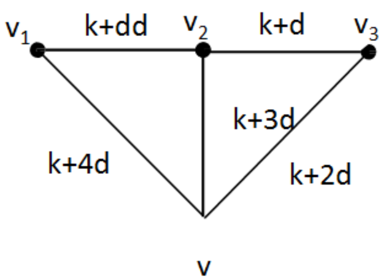

Fig 2

Theorem 2.6. The complete graph K_4 is (k,d) arithmetic edge graceful for all k, d 1, k #d, d is an odd integer.

Proof.
Let the complete graph K_4 be the graph with vertex set $\{v_1, v_2, v_3, v_4\}$ and the edge set

$\{e_1, e_2, \ldots, e_6\}$.Let v_1 be the center vertex. Suppose that the edges $e_1 = (v_1, v_2)$,

$e_2 = (v_2, v_3)$, $e_3 = (v_1, v_3)$, $e_4 = (v_1, v_4)$, e_5 (v_3, v_4) and $e_6 = (v_4, v_2)$.

We label the edges as follows $f(e_i) = k + (i-1)d$, $1 \le i \le 6$. Then the vertex labels as{ 0, 1, 2, 3 }. (Fig 3).

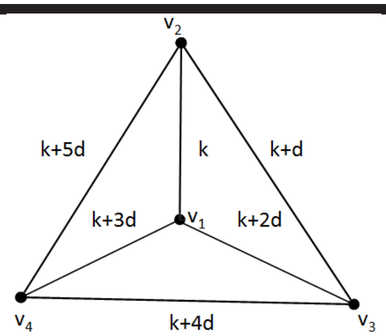

Fig 3

Result 2.7.
The eigenvalues of the path Pn consists of the numbers 2cos $\left(\frac{\pi i}{n+1}\right)$, i = 1, 2, . . . , n.

The Estrada index of the path is

$$EI(P_n) = \sum_{i=1}^{n} e^{2\cos\left(\frac{\pi i}{n+1}\right)}$$

If P_i, $1 \le i \le 4$ then the Sum of the edge labels of the path is less than the Estrada index. If P_i, $5 \le i \le n$,then the Sum of the edge labels of the path is greater than the Estrada index.

Result 2.8.
The eigen values of the Cycle Cn consists of the numbers 2cos $\left(\frac{2\pi i}{n+1}\right)$, i = 1, 2, . . . ,(n-1)

The Estrada index of the path is

$$EI(P_n) = \sum_{i=1}^{n-1} e^{2\cos\left(\frac{2\pi i}{n}\right)}$$

The Sum of the edge labels of the cycles C_3 and C_5 are less than the Estrada index. If C_i, $6 \le i \le n$,then the Sum of the edge labels of the cycle is greater than the Estrada index.

Result 2.9.
The eigenvalues of the complete graph K_4 and the fan graph F_3 are the numbers −1,−1,−1 and 3. The Estrada index of the above mentioned graphs is $3 \times e^{-1} + e^{3} = 21.19$ The Sum of the edge labels of both the graphs K_4 and F_3 is 15, which is less than the Estrada index.

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