Research Paper

Volume : 2 | Issue : 11 | November 2013 • ISSN No 2277 - 8179

Fuzzy Sub-Trigroup - Characterstics

Mathematics

KEYWORDS : Trigroup,fuzzy sub-trigroup,fuzzy union,,fuzzy trilevel maximum,fuzzy trilevel minimum

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ABSTRACT

In this work,we give some characteristic description and a kind of representation fuzzy sub-trigroup with respect to operation s '+' , ' . ' and '' introduced.Some important results of fuzzy sub-trigroups are discussed.*

Introduction:

The concepts of fuzzy sets was introduced by Zadeh.Since the paper fuzzy set theory has been considerably developed by zadeh himself and some researchers.The original concept of fuzzy sets was introduced as an extension of cribs (usual) sets, by enlarging the truth value set of "grade of members"from the two value set {0,1} to unit interval {0,1} of real numbers. The study of fuzzy group was started by Rosenfeld.

It was extended by Roventa who have introduced the fuzzy groups operating on fuzzy sets.W.B.Vasantha kandasamy introduced fuzzy sub-bigroup with respect to '+' and '.' with example. W.B.Vasantha kandasamy was the first one to introduce the notion of bigroups in the year 1994.Several mathematicians have followed them in investigating the fuzzy group theory. We now recall the previous and preliminary definitions, and results that are required in our discussion in fuzzy tri-group with respect to operation '+' , '.' and '∗' .

1.Preliminaries:

Definition: 1.1

Let G be a group. A fuzzy subset μ of a group G is called a **fuzzy Subgroup** of the group G if.

i. $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for every $x, y \in G$

ii. $\mu(x^{-1}) = \mu(x)$ for every $x \in G$

Definition: 1.2

Let A be a fuzzy subset of universel set S and $t \varepsilon [0,1]$ the set

$A_t = \{ s \in S / A(x) \geq t \}$ is said tobe **t level Subset** of the fuzzy subset A.

Definition:1.3

Let μ_1 be a fuzzy subset of a set X_1 and μ_2 be a fuzzy subset of a set X_2,then the fuzzy union of the sets μ_1 and μ_2 is defined as a function.

$\mu_1 \cup \mu_2 : X_1 \cup X_2 \rightarrow [0,1]$ given by

$$(\mu_1 \cup \mu_2)(x) = \begin{cases} \max(\mu_1(x), \mu_2(x)) & \text{if } x \in X_1 \cap X_2 \\ \mu_1(x) & \text{if } x \in X_1 \text{ and } x \notin X_2 \\ \mu_2(x) & \text{if } x \in X_2 \text{ and } x \notin X_1 \end{cases}$$

Definition : 1.4

A set (G,+,·) with two binary operation '+' and '·' called a **bigroup** if there exists two proper subsets G_1 and G_2 of G such that

i) $G = G_1 \cup G_2$

ii) $(G_1,+)$ is a group

iii) $(G_2, \cdot)$ is a group

A subset H (≠) of a bigroup (G,+, ·) is called a **subbigroup,**if H itself is a bigroup under '+' and '·' operations defined on G.

2.Trigroup and fuzzy sub-trigroup

Definition : 2.1

A Set (G,+, ·,∗) with three binary operations ,+(addition), · (multiplication),∗ (ab/2) is called a **trigroup** if there exists three proper subsets G_1,G_2 and G_3 of G such that

i) $G = G_1 \cup G_2 \cup G_3$

ii) $(G_1,+)$ is a group

iii) $(G_2, \cdot)$ is a group

iv) $(G_3, *)$ is a group

A subset H (≠) of a triigroup (G,+, ·,∗) is called a **subtriigroup**,if H itself is a triigroup under '+' , '·',and '∗' operations defined on G.

Definition : 2.2

Let (G,+, · , ∗) be a triigroup where $G = G_1 \cup G_2 \cup G_3$ triigroup G is said to be commutative if three $(G_1,+)$,$(G_2, \cdot)$ and $(G_2, *)$ are commutative.

Eg:

Let G = R be a set of all Real numbers G_1,G_2 and G_3 is a subgroup of G with respect to +(addition), · (multiplication),∗ (ab/2) .

$G_1 = \{ 0,\pm1,\pm2,\pm3,\ldots\ldots\ldots\}$

G_2 = Set of all Rational numbers except '0'

G_3 = Set of all real numbers except '0'

i) $G = G_1 \cup G_2 \cup G_3$

ii) $(G_1,+)$ is a group

iii) $(G_2, \cdot)$ is a group

iv) $(G_3, *)$ is a group

$(G,+,\cdot,*)$ is a **trigroup**

Definition : 2.3

Let μ_1 be a fuzzy subset of a set X_1 , μ_2 be a fuzzy subset of a set X_2 and μ_3 be a fuzzy subset of a set X_3 ,then the fuzzy union of the sets μ_1 , μ_2 and μ_3 is defined as a function.

$\mu_1\cup\mu_2\cup\mu_3 : X_1\cup X_2\cup X_3 \rightarrow [0,1]$ given by

$$(\mu_1\cup\mu_2\cup\mu_3)(x) = \begin{cases} \max(\mu_1(x), \mu_2(x), \mu_3(x)) & \text{if } x\in X_1\cap X_2\cap X_3 \\ \max(\mu_1(x), \mu_2(x)) & \text{if } x\in X_1\cap X_2 \text{ and } x\notin X_3 \\ \max(\mu_2(x), \mu_3(x)) & \text{if } x\in X_2\cap X_3 \text{ and } x\notin X_1 \\ \max(\mu_3(x), \mu_2(x)) & \text{if } x\in X_3\cap X_1 \text{ and } x\notin X \\ \mu_1(x) & \text{if} x\in X_1 \text{and } x\notin X_2, x\notin X_3 \\ \mu_2(x) & \text{if } x\in X_2 \text{and} x\notin X_1, x\notin X_3 \\ \mu_3(x) & \text{if } x\in X_3 \text{and} x\notin X_1, x\notin X_2 \end{cases}$$

Eg:

Let $X_1 = \{ 1,2,3,4,5,6,7\}$, $X_2 = \{ 2,4,6,8,10,12,14\}$, $X_3 = \{ 4,8,12,16,20,28\}$ be three sets

Define $\mu_1 : X_1 \rightarrow [0,1]$ by

$$\mu_1(x) = \begin{cases} 0.2 & \text{if } X = 5,7 \\ 0.5 & \text{if } X = 4 \\ 0.6 & \text{if } X = 3 \\ 1 & \text{if } X = 1,2 \end{cases}$$

Define $\mu_2 : X_2 \rightarrow [0,1]$ by

$$\mu_2(x) = \begin{cases} 0.2 & \text{if } X = 10,4 \\ 0.5 & \text{if } X = 8 \\ 0.6 & \text{if } X = 6 \\ 1 & \text{if } X = 2,4 \end{cases}$$

Define $\mu_3 : X_3 \rightarrow [0,1]$ by

$$\mu_3(x) = \begin{cases} 0.2 & \text{if } X = 20,28 \\ 0.5 & \text{if } X = 16 \\ 0.6 & \text{if } X = 12 \\ 1 & \text{if } X = 4,8 \end{cases}$$

Hence

$$(\mu_1\cup\mu_2\cup\mu_3)(x) = \begin{cases} 0.2 & \text{if } X =5,7, 10,14,20,28 \\ 0.5 & \text{if } X = 1 \\ 0.6 & \text{if } X = 3 \\ 1 & \text{if } X =1,2,4,8 \end{cases}$$

Definition : 2.4

Let $G = (G_1\cup G_2\cup G_3,+,\cdot,*)$ be a trigroup.Then $\mu : G \rightarrow [0,1]$ is said to be a **fuzzy sub-trigroup** of the trigroup G if there exists three fuzzy subsets μ_1 of G_1 and μ_2 of G_2, μ_3 of G_3 such that

(i) $(\mu_1,+)$ is a fuzzy subgroup of $(G_1,+)$

(ii) $(\mu_2, \cdot)$ is a fuzzy subgroup of $(G_2, \cdot)$

(iii) $(\mu_3, *)$ is a fuzzy subgroup of $(G_3, *)$

(iv) $\mu = (\mu_1\cup\mu_2 \cup \mu_3)$.

Example

Consider the trigroup

$G = \{\pm i, \pm 0, \pm 1,\pm 2,\pm 3,\pm 4,\ldots\ldots\ldots..\}$

Under the operation '+', '·' and '*' where

$G_1 = \{0, \pm 1,\pm 2,\pm 3,\ldots\ldots\ldots\ldots\ldots\ldots\}$

$G_2 = \{\pm i , \{\pm 1\}, G_3 = \{1,2,4\}$.

Define $\mu: G \rightarrow [0,1]$ by

$$\mu(x) = \begin{cases} 1/3 & \text{if } x = i,-i \\ 1 & \text{if } x \in \{0,\pm 2,\pm 4,\ldots.\} \\ 1/2 & \text{if } x \in \{\pm 1,\pm 2,\pm 3 \end{cases}$$

We can find

Define $\mu_1: G_1 \rightarrow [0,1]$ by

$$\mu_1(x) = \begin{cases} 1 & \text{if } x \in \{0, \pm 2,\pm 4,\ldots\ldots.\} \\ 1/2 & \text{if } x \in \{\pm 1,\pm 3,\pm 5,\ldots\ldots\ldots\} \end{cases}$$

Define $\mu_2: G_2 \rightarrow [0,1]$ by

$$\mu_2(x) = \begin{cases} 1/4 & \text{if } x = \pm i \\ 1/2 & \text{if } x = \pm 1 \end{cases}$$

Define $\mu_3: G_3 \rightarrow [0,1]$ by

$$\mu_3(x) = \begin{cases} 1/5 & \text{if } x = 4 \\ 1/2 & \text{if } x = 2 \\ 1/6 & \text{if } x = 1 \end{cases}$$

Hence there exists two fuzzy subgroups μ_1 of G_1, μ_2 of G_2 and μ_3 of G_3 such that $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$.

Definition : 2.5

Let $(G = G_1 \cup G_2 G_3, +, \cdot, *)$ be a trigroup and $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$ be a fuzzy sub-trigroup of the trigroup G.The **tri level subset** of the fuzzy sub-trigroup μ of the trigroup G is defined as $(G_\mu)^t = (G_{1\mu t})^t \cup (G_{2\mu t})^t \cup (G_{3\mu t})^t$ for every $t \in [0, \min\{\mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$ where e_1, e_2, e_3 are denotes the identity element of the group $(G_1, +)$, $(G_2, \cdot)$, $(G_3, *)$ respectively.A fuzzy subset μ of a group G is said to be a union of three fuzzy sub-groups of the group G if there exists three fuzzy subgroups μ_1,μ_2 and μ_3 of μ ($\mu_1 = \mu$, $\mu_2 = \mu$ and $\mu_3 = \mu$) such that $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$.Here by the term fuzzy subgroup λ of μ we mean that λ is a fuzzy subgroup of the group G and $\lambda \subseteq \mu$ (where μ is also a fuzzy subgroup of G).The condition $t \in [0, \min\{\mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$ is essential for the trilevel subset to be a sub-trigroup for if $t \notin [0, \min\{\mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$ then the trilevel subset need not in general be a sub-trigroup of the trigroup G.

Theorem:2.1

Every t-level subset of a fuzzy sub-trigroup μ of a trigroup G need not in general be a sub-trigroup of the trigroup G but $G_\mu^{1/2}$ is a bigroup with respect to the binary operation $\cdot, *$

Proof :-

We can prove this theorem by an example

Take G = {--1,0,1,2,4} to a trigroup under the operation '+', $\cdot$,*

Where $G_1 = \{0\}$, $G_2 = \{-1,1\}$, $G_3 = \{1,2,4\}$ are groups w.r.to '+' , $\cdot$,*

Define μ: G→ [0,1] by .

$$\mu(x) = \begin{cases} 1 & \text{if } x = 2,4 \\ 1/2 & \text{if } x = -1, 1 \\ 1/4 & \text{if } x = 0 \end{cases}$$

Then clearly $(\mu, +, \cdot, *)$ is a fuzzy sub-trigroup of the trigroup $(G, + \cdot, *)$

Now consider the level subset $G_\mu^{1/2}$ of the fuzzy sub-trigroup μ.

$G_\mu^{1/2} = \{x \in G / \mu(x) \geq 1/2\} = \{-1,1,2,4\}$ is not a sub – trigroup of the trigroup $(G, +, \cdot, *)$.

but $G_\mu^{1/2}$ is a bigroup with respect to the binary operation $\cdot, *$

Hence the t level subset

G_μ^t (of t =1/2) of the fuzzy sub – trigroup μ is not a sub – trigroup $(G, +, \cdot, *)$.

Theorem:2.2

Every trilevel subset of a fuzzy sub – trigroup μ of a trigroup G with respect to the usual addition, multiplication and ab/2 is a sub-trigroup of trigroup G.

Proof :-

Let $\mu = (\mu_1 \cup \mu_2 \cup \mu_3)$ be the fuzzy subgroup of a trigroup $(G = G_1 \cup G_2 \cup G_3, +, \cdot, *)$.

Consider the trilevel subset G_μ^t of the fuzzy sub – trigroup μ for every

$t \in [0, \min\{\mu_1(e_1), \mu_2(e_2), \mu_3(e_3)\}]$

Where e_1, e_2 and e_3 denote the identity elements of the groups G_1,G_2 and G_3 respectively.

Then $G_\mu^t = G_{3\mu 3}^t \cup G_{2\mu 2}^t \cup G_{1\mu 1}^t$

Where $G_{1\mu 1}^t$, $G_{2\mu 2}^t$, and $G_{3\mu 3}^t$ are subgroups of G_1,G_2 and G_3 respectively.

Since $G_{1\mu 1}^t$ is a t-level subset of the group G_1, $G_{2\mu 2}^t$ is a t-level subset of the group G_2 and $G_{3\mu 3}^t$ is a t-level subset of the group G_3.

Hence the sub-trigroup G_μ^t is a sub-trigroup of the trigroup G.

Eg:

G = {0, ±1, ±i,2,4} is a trigroup with repect to the addition, multiplication and ab/2 . Clearly $G_1 = \{0,1\}$,$G_2 = \{\pm i, \pm 1\}$,$G_3 = \{1,2,4\}$ are groups with respect to addition.

Define μ: G → [0,1] by

$$\mu(x) = \begin{cases} 1 & \text{if } x = 0,4 \\ 0.5 & \text{if } x = \pm 1 \\ 0.3 & \text{if } x = \pm i \end{cases}$$

Therefore μ is a fuzzy sub – trigroup of the trigroup G as there exists three fuzzy subgroups

$\mu_1: G_1 \to [0,1]$, $\mu_2: G_2 \to [0,1]$ and $\mu_3: G_3 \to [0,1]$ such that

$\mu = \mu_1 \cup \mu_2 \cup \mu_3$

Where

$$\mu_1(x) = \begin{cases} 0.2 & \text{if } x = 1 \\ 1 & \text{if } x = 0 \end{cases}$$

$$\mu_2(x) = \begin{cases} 0.8 & \text{if } x = \pm i \\ 0.5 & \text{if } x = \pm 1 \end{cases}$$

$$\boldsymbol{\mu_3(x)} = \begin{cases} 0.9 & \text{if } x = 2 \\ 1 & \text{if } x = 4 \\ 0.3 & \text{if } x = \pm i \end{cases}$$

Now we calculate the trilevel subset G_μ^t for

t = 0.5

$G_\mu^t = G_{3\mu 3}^t \cup G_{2\mu 2}^t \cup G_{1\mu 1}^t$

$= \{x \in G_1 / \mu_1(x) \geq t\} \cup \{x \in G_2 / \mu_2(x) \geq t\} \cup \{ x \in G_3 / \mu_3(x) \geq t\}$

$G_\mu^t = \{0\}1 \cup \{\pm i, \pm 1\} \cup \{2,4\}$

$= \{0, \pm 1, \pm i ,2,4\}$

$\therefore$ $G_\mu^t = \{0, \pm 1, \pm i ,2,4\}$ is a sub-trigroup of the trigroup

Conclusion:

In this paper with the basic concepts of group theory,we define related concepts of fuzzy sub-trigroup with respect binary operation addition, multiplication and ab/2 and further we proved properties of fuzzy sub-trigroup.

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