Research Paper

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An Alternative Method to Construct Turyn Type Sequences of Length 34

Science

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ABSTRACT

The Hadamard conjecture is that Hadamard matrices exist for all orders 1,2,4t where t≥1 is an integer. The most compatible way of constructing Hadamard matrices is to use the Kronecker product whenever there exist a Hadamard matrix of order t above. Otherwise, the most convenient way to develop a Hadamard matrix is to use certain sequences of zeros and ones both positive and negative. In this research, the focus is to discuss an alternative method to construct a set of sequences of ones, both positive and negative called Turyn type sequences which can be used to form a set of sequences with zeros and ones such as Base sequences and T-sequences, which are useful in constructing certain Hadamard matrices. Turyn type sequences,TT(n) are quadruples of {±1}sequences,(A,B,C,D) of length n,n,n,n-1 respectively,where the sum of non-periodic auto-correlation function of A,B and twice that of C,D vanishes everywhere except at zero.The proposed procedure consists of segmentation algorithms of constructing TT(34) that consist of three sequences of length 34 and a sequence of length 33 together with an algorithm to verify the non-periodic auto-correlation condition and the properties of Turyn type sequences. These algorithms are created treating each sequence as a binary number and try all the possible sets of sequences that satisfy the non-periodic auto-correlation condition. Thereupon, by checking the properties of the Turyn type sequences using C/C++ computer program will lead to the desired set of sequences.

INTRODUCTION

From the Hadamard conjecture, for some those that are not a multiple of 4,there does not exist an Hadamard matrix of order ,but there may be a Hadamard matrix of order (J. Seberry and M. Yamada, 1992).In such problems, the use of a set of sequences of zeros and ones, both positive and negative will reach to the required result. For an instance, the construction of the Hadamard matrix of order 428 used TT(36)(] H. Kharaghani and B. Tayfeh-Rezaie, 2005). Turyn type sequences can be acquainted as the opening of constructing those sequences with zeros and ones(J. Seberry and M. Yamada, 1992).Hence the intention is to discuss a general procedure to find a quadruple of binary sequences.

The orthogonal designs are useful in constructing Hadamard matrices. An orthogonal design was first found by Baumert-Hall and by Weltch.(L.D.Baumert,MHJ 1965) Since we are concerned with orthogonal designs, we will consider sequences of low correlation values. Early work of Golay (M.J.E.Golay 1961) was concerned with two sequences with zero auto-correlation function, but Welti (G.R.Welti 1960) approached with two orthonormal vectors with waveforms. Later work, including Turyn's (R.J.Turyn 1972) used four or more sequences.

Definition 1: (Non-periodic autocorrelation function)
Given A={a_0,a_1,...,a_(n-1) } be a sequences of length n,the non-periodic autocorrelation function N_A is defined by,

$$N_A(k) = \begin{cases} \sum_{i=0}^{n-k} a_i a_{i+k} \ for\ k = 0,1,\dots,n-1 \\ N_A = 0\ for\ k \geq n \end{cases}$$

For such sequences we associate the polynomial

$$A(x) = a_0 + a_1 x + \cdots + a_{n-1} x^{n-2} \quad (1)$$

$$= \sum_{i=0}^{n-1} a_i x^{i-1}$$

and refer to the Laurent polynomial,

$$N(A) = A(x)A(x^{-1}) \quad (2)$$

$$= N_A(0) + \sum_{k=1}^{n-1} N_A(k)\,(x^k + x^{-k}) \quad (3)$$

as the norm of . (Earl Glen Whitehead,J 1978)

Further, we associate a real function f_A, defined by $f_A(\theta)=|A(e^{i\theta})|^2$ (Earl Glen Whitehead,J 1978) and f_A is a non-negative periodic function with a period .From it is easy to see that,

$$f_A(\theta) = N_A(0) + 2\sum_{k=1}^{n-1} N_A(k)\cos(k\theta) \quad (4)$$

For a particular sequence, if the value of N_A (k) or f_A (θ) is zero or small, then it guarantees that the sequence is orthogonal to other sequences.

Definition 2: (Turyn type sequences) (R.J.Turyn 1974)
A set of four {-1,1} sequences A,B,C,D with lengths n,n,n,n-1 is defineds to be of Turyn type if

$$(N_A+N_B+2N_C+2N_D)(k)=0, \text{for } k\geq 1 \quad (5)$$

Using equation and we have,

$$N(A)+N(B)+2N(C)+2N(D)=6n-2 \quad (6)$$

By setting x=1 in (2),the equation (8) reduces to the sum of squares as follows:

$$A(1)^2+B(1)^2+2C(1)^2+2D(1)^2=6n-2$$

In order to have a better understanding of the structure of Turyn type sequences,it is essential to classify them for as many values of as possible.(Kharaghani and C. Koukouvinos,2007) Assume that (A,B,C,D) is a TT(n).From (5),we have

$$(N_A+N_B+2N_C+2N_D)(k)=0, \text{for } k\geq 1$$

Turyn (R.J.Turyn 1974)found these sequences for and Robinson(P.J.Robinson,J.Seberry,1976) extended this results to and proved that they cannot exist for .s

In this research, the main goal is to provide a search method to discover the Turyn type sequence of length 34,TT(34).Thus, in the next section ,the search method of these sequences will be explained briefly.

METHODOLOGY

To construct the sequences emphasized,three main algorithms were implemented using C/C++ programming languages.

Step 1: Generating 34 number sequences (Type 1 sequences)
First, sequences of 34 numbers which consists only +1s and -1s have to be generated. Since, the numbers are only composed of

+1 and -1s,they are just like binary numbers with 0s and 1s.This property was used in writing the computer program using C/ C++ programming languages in generating the sequences of length 34 . Since, there are 34 bits in one sequence, all together there is a total of 234 (=17179869184) number of sequences. Therefore, the number of sequences is very large. So, keeping all the sequences in the main memory will be inefficient and it causes the tendency to a memory dump . Therefore, each number that is generated from the program, is stored in an external file.(External file 1)

Step 2: Generating 34 number sequences with the final bit of '0' (Type 2 sequences)
The only difference of these sequences with the above sequences is that their final bit is zero. Therefore there will only be 233 (=8589934592) number of sequences. So,first 233 number of sequences of length 33 were generated using a C/C++ program which uses the same method explained above. But,a '0' bit was appended to each of the above generated sequences as the last bit,in order to convert them to 34 number sequences. Since,a large number of sequences are generated,they are also stored in an external file.(External file 2)

Step 3: Filtering the sequences with specified sum of entries
As it is mentioned above, the values were selected for respectively such that ,where are the sums of entries of the sequences above.Then,all the possible solutions for each sequence with above selected values are filtered and each of the results were saved in four separate files(External files A,B,C,D)

Step 4: Filtering the last two sequences.
From these sequences, all sequences with sum of entries equal to and for which for all are found and save proper sequences with their identical entries in File C. The same process is done for the set of sequences with the sum of the entries and save the result in File D. Now, every possible combination from the recently filtered two sequences are found so that, and save the final result in another File CD.

Step 5: Selecting 2 sequences.
First, two sequences must be selected from the external files A and B which contain the sequences of sum of entries and the correlation value of each pair of sequence was determined and the results were saved in another file,File A . Then, each of these couples of sequences in File AB must be combined with each of the results obtained in step 4 and the non periodic auto-correlation condition was checked.

The flow of the steps of the above mentioned algorithm is depicted in Figure 04.

RESULTS AND DISCUSSION
From the first part of the algorithm, it generated 2333606220 number of sequences of length 34 with 0 sum. Approximately 14795seconds were taken by the algorithm to generate the results displayed in Figure 02.

Figure 01-The figure that includes the details of generating sequences of length 34 with zero sum of the entries.

In the next part of the process, a set of sequences of length 34 whose sum of the elements being 10 and which satisfy the equation was compensated. For that process, approximately 7026 seconds were consumed and resulted 486925510 number of sequences.

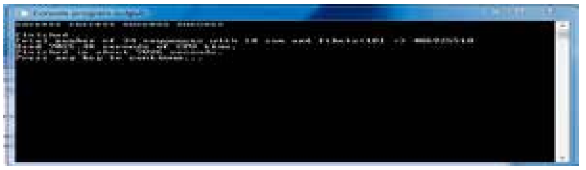


Figure 02-The figure that includes the details of generating sequences of length 34 which has 10 as the sum of the entries.

Then, after a calculation for 11513 seconds ,the algorithm generated 1002322287 number of sequences of length 33,whose entries add up to 1 and that satisfy the equation given above.

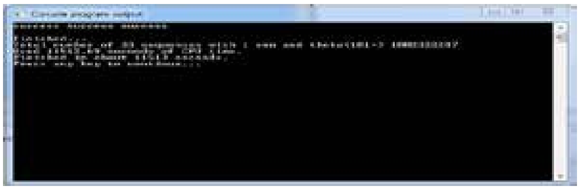


Figure 03-The figure that includes the details of generating sequences of length 33 whose sum of the entries is 1.

After generating these basic sets of sequences, the rest of the process was continued for about 6 months and obtained the following Turyn Type sequences of length 34.

Here the chosen values are a=0,b=0,c=10,d=1, that is the sum of the entries in sequences A and B is 0 and 10 and 1 are the sums of the entries in sequences C and D. Further,in each of the following sets of sequences + refers to +1 and - refers to -1.

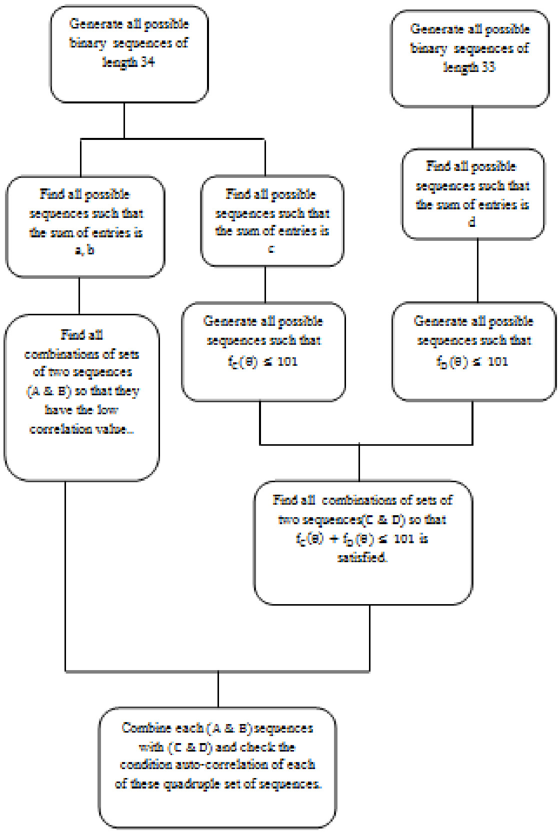
A= +-+-+-+-++--+--+-+++++---++---++--

B= +++----+++--++-+++--+-+++-+-+-----

C= +--++-+-+---++++-++-+-+-++-+++++++

D= ----+-++-+++++--++++--+---++-++--

CONCLUSION
According to the above facts which have been proved in(Timothy Vis,2006),there are base sequences of lengths and T-sequences of length 101.The existence of T-sequences of length 101 concludes that there are an infinite class of Hadamard matrices(Chistos Koukouvinos,Stratis Kounias,1989) with the first one being the order 404.



04: Flow of the steps of the sequence generating algorithm.

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