Research Paper

Volume : 2 | Issue : 9 | September 2013 • ISSN No 2277 - 8179

# P-Fuzzy Sub- Bigroup and its Properties

Mathematics

**KEYWORDS :** P-fuzzy subgroup, fuzzy sub-bigroup, Bilevel subset, P-fuzzy sub-bigroups

**G. Nirmala** Associate Professor, PG and Research, Department of Mathematics, Kunthavai Nachiar Government Arts College (W) Autonomous, Thanjavur.

**S. Priyadarshini** Assistant Professor, Department of Mathematics and Statistics, J.J College of Arts and Science, Pudukkottai

## ABSTRACT

*In this paper we introduce P-Fuzzy sub-bigroup, its definition and properties. The t-level subset of P-fuzzy sub-bigroup is compared with sub-bigroup and P-fuzzy groups.*

## 1. Introduction

As it well known, in the "Classical" fuzzy theory established by L.A.Zadeh in 1965, a fuzzy set A is defined as a map from A to the real unit interval I = [0,1]. The set of all fuzzy set on A is usually denoted by $I_{A.}$

In its trajectory of stupendous growth, it has also come to include the theory of fuzzy algebra. Several researchers have been working on concepts like fuzzy groups, fuzzy rings etc.

Wenxiang Gu and LuTu introduced the notion of fuzzy algebras over fuzzy fields and obtained some fundamental results. This paper gave the definition of P-fuzzy sub-bigroups and also offers some properties of the P-fuzzy sub-bigroups.

## 2. Preliminaries

**Definition 2.1**

A P-fuzzy set μ€PA is called a P-fuzzy algebra on the algebra A, if

- For any n-ary(n≥1) operation f€F

μ(f(x1x2……….xn)) ≥ μ(x1)* μ(x2)*………….* μ(xn)

- For any constant (nullary operation) C

μ(c)≥ μ(x) for all x€A.

**Definition 2.2**

Let G = (G1UG2, +, .) be a bigroup, then μ:G→[0,1] is said to be P-fuzzy sub-bigroup of the bigroup G if therexist two fuzzy subset μ1 of G1, μ2 of G2 such that,

(i) (μ1, + ) is a P-fuzzy subgroup of (G1 , +)

(ii) (μ2, . ) is a P-fuzzy subgroup of (G2 , .)

(iii) μ=μ1Uμ2

**Definition 2.3**

Let G = (G1UG2, +, .) be a bigroup and μ=μ1Uμ2 be a P-fuzzy sub-bigroup of the bigroup G. The bilevel subset of the P-fuzzy sub-bigroup μ of the bigroup G is defined as Gtμ= Gt 1 μ1U Gt 2 μ2 for every t€[0, min{(e1)U(e1)} where e1 denotes the identity element of the group (G1 , +) and e2 denotes the identity element of the group (G2 , .)

**Definition 2.4**

A fuzzy subset μ of a group G is said to be a fuzzy sub-bigroup of the group G if therexist two fuzzy subgroups μ1 and μ2 of μ (μ1 ≠μ and μ2 ≠μ) such that μ= μ1Uμ2. Here by the term fuzzy subgroup λ of μ. We mean that λ is a fuzzy subgroup of the group G and λ C μ.

## 3.Properties of P-Fuzzy Sub-bigroup

Theorem 3.1: Every t-level subset of a P-fuzzy sub-bigroup μ of a bigroup G need not in general be a sub-bigroup of the bigroup G.

Let G={a1, a2,a3} to be a bigroup under the binary operation + and . where G1={ a2} and G2={a1,a3} are groups respectively with respect to usual addition and usual multiplication.

Define μ: G→ [0,1]

μ(x) = b1 if x= a1,a3

b2 if x=a2

Then clearly (μ, +, . ) is a P-fuzzy sub-bigroup of the bigroup (G, +, .)

Now consider the level subset Gb1μ of the P-fuzzy sub-bigroup μ

Gb1μ={x€G/μ(x) ≥b1}={ a1,a3}

{ a1,a3} is not a sub-bigroup of the bigroup (G1 , +, .)

Gμt (for t=b1) of the P-fuzzy sub-bigroup μ is not a sub-bigroup of the bigroup

(G1, +, .)

Example: G={-1,0,1} be a bigroup under the binary operation + and . where G1={0} and

G2={-1, 1} are groups respectively with respect to usual addition and multiplication.

Define: μ: G→ [0,1] by

μ(x) = 1/2 if x= -1,1

1/4 if x=0

Then clearly (μ, +, .) is a P-fuzzy sub-bigroup of the bigroup (G, +, .). Now consider the level subset Gμ1/2 = {x€G / μ(x)≥1/2} = {-1, 1}

It is easy to verify that {-1, 1} is not a sub-bigroup of the bigroup (G, +, .). Hence the t-level subset Gμt (for t=1/2) fo the P-fuzzy sub-bigroup μ is not a sub-bigroup of the bigroup (G, +, .).

Theorem 3.2 Every bilevel subset of a P-fuzzy sub-bigroup μ of a bigroup G is a sub-bigroup of the bigroup G.

Let μ=μ1Uμ2 be the P-fuzzy subgroup of a bigroup (G = G1UG2, +, .). Consider the bilevel subset Gtμ of the P-fuzzy sub-bigroup for every t€[0, min{(e1)U(e1)} where e1 denotes the identity element of the group (G1 , +) and e2 denotes the identity element of the group (G2 , .), then Gtμ= Gt 1 μ1U Gt 2 μ2 where Gt 1 μ1 and Gt 2 μ2 are subgroups of G1 and G2 respectively (since Gt 1 μ1 is a t-level subset of the group G1 and Gt 2 μ2 is a t-level subset of G2)

Example: G={0, ±1, ±i} is a bigroup with respect to addition modulo 2 and ·. Clearly G1={0,1} and G2={±1, ±i} are group with respect to addition modulo and · respectively.

Define μ:G→[0,1] by

1 if x=0

μ(x)= 0.5 if x=±1

0.2 if x=±i

Since μ is a P-fuzzy sub-bigroup of the bigroup G as therexist two fuzzy subgroups μ1:G→[0,1] and μ2:G→[0,1] such that μ=μ1Uμ2 where

μ1(x) = 1 if x=0

0.5 if x=1 and

μ2(x)= 0.5 if x=±1

0.2 if x=±i

Now we calculate the bilevel subset Gtμ for t=0.5

Gtμ= Gt 1 μ1U Gt 2 μ2

= {xЄG1/ μ1(x)≥t} U {xЄG2/ μ2(x)≥t}

= {0}U{±1}

Gtμ ={0, ±1}

Theorem 3.3 Let μ=μ1Uμ2 be a P-fuzzy sub-bigroup of a group G, where μ1and μ2 are P-fuzzy subgroups of the group G, for tЄ[0, min{μ1(e), μ2(e)}] the level subset Gtμ of μ can be represented as the union of two subgroups of the group G ., i.e Gt 1 μ1U Gt 2 μ2.

Let μ=μ1Uμ2 be a P-fuzzy sub-bigroup of a group G and for tЄ[0, min{μ1(e), μ2(e)}]. This implies that therexist P-fuzzy subgroups μ1 and μ2 of the group G1 such that μ=μ1Uμ2.

Let Gtμ be the level subset of μ then

we have xЄ Gtμ⇔μ(x)≥t

⇔max{ μ1(x), μ2(x)} ≥ t

⇔ μ1(x)≥t or μ2(x)≥t

⇔xЄ Gt 1μ1 or xЄGt 2μ2

If and only if x Є Gt 1μ1UGt 2μ2

Hence Gtμ=Gt 1μ1UGt 2μ2

Theorem 3.4: Every P-fuzzy sub-bigroup of a group G is a P-fuzzy subgroup of the group but not conversely.

Every P-fuzzy sub-bigroup of a group G is a P-fuzzy subgroup of the group G.

Using the definition of P-fuzzy sub-bigroup, G = (G1UG2, +, .) be a bigroup, then μ:G→[0,1] is said to be P-fuzzy sub-bigroup of the bigroup G if therexist two fuzzy subset μ1 of G1, μ2 of G2 such that,

(i) (μ1, + ) is a P-fuzzy subgroup of (G1 , +)

(ii) (μ2, . ) is a P-fuzzy subgroup of (G2 , .)

(iii) μ=μ1Uμ2

Therefore every subset G is a fuzzy subgroup of the group G. Conversely, if μ is a P-fuzzy subgroup of the group G and there does not exist two P-fuzzy subgroups μ1 and μ2 of μ (μ1 ≠μ and μ2 ≠μ) such that μ= μ1Uμ2

- μ is a P-fuzzy sub-bigroup of the group G.

Example: Consider the Group G={ 1, -1, i, -i} under the usual multiplication.

Define μ:G→[0,1] by

μ(x)= 0 if x=-i, i

1 if x=-1, 1

Since μ is a P-fuzzy subgroup of the group G as all of its level subset are subgroups of G. Further O(im(μ))=2

If μk is a P-Fuzzy subgroup of μ such that μk≤μ (μk≠μ) then μk takes the following form

μk= 0 if x= -i, i

αi if x=-1, 1

with 0≤ αk<1 for every k in the index set I.

Therefore μj U μk≠μ for any j,kЄI. Thus there does not exist two P-fuzzy subgroups μ1 and μ2 of

μ (μ1 ≠μ and μ2 ≠μ) such that μ= μ1Uμ2.

**3.Conclusion**

In this paper we studied some special properties in P-fuzzy sub-bigroup using P-fuzzy subgroup. Also we compared P-fuzzy sub-bigroup with fuzzy subgroup using bilevel subset.

## REFERENCE

[1] Dr.G.Nirmala and S.Suganthi, "Fuzzy sub-Trigroup-Trilevel properties" International Journal of Scientific and Research Publications, Vol 2, Issue 6, June 2012 ISSN 2250-3153. | [2] Dr.G.Nirmala and S.Priyadarshini, "P-Fuzzy Congruence class" International Journal of Scientific and Research Publications, Vol 11, Nov 2012 ISSN 2250-3153. | [3] Dr.G.Nirmala and S.Priyadarshini, "P-Fuzzy Algebra and its properties" International Journal of Scientific and Research Publications, Vol 2, Issue 6, June 2012 ISSN 2250-3153. | [4] Dr.G.Nirmala and S.Priyadarshini, "P-Fuzzy Normal subgroup" Journal of Periyar Maniammai university. | [5] Dr.G.Nirmala and S.Priyadarshini, "P-Fuzzy Algebra with database to facilitate uncertainity management" Aryabhatta J. of Maths and Info. Vol 5, No. 1, Jan-June 2013. | [6] Dr.G.Nirmala and S.Priyadarshini,"Selection of books from the table of preference with | p-fuzzy database model" International Review of Fuzzy Mathematics, to be published on Jul- Dec 2013. | [7] Dr.G.Nirmala and S.Suganthi (Dec. 2012) "Innovative Characteristics and properties of fuzzy sub-tri ring tri-ideals" Aryabhatta J. of Maths and Info. Vol 4, No. 2, pp357-362. |