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Notes on (Q, L)-Fuzzy Ideals of A Ring

Mathematics

KEYWORDS : (Q, L)-fuzzy subset, (Q,L)-fuzzy ideal, (Q, L)-anti-fuzzy ideal, pseudo (Q, L)-fuzzy coset.

Mrs.G.Jayanthi — Associate Professor/HOD, Department of Mathematics, TK Govt Arts college, Virudhachalam

Mrs. /Dr. M. SIMARINGA — Department of Mathematics, TK Govt Arts college, Vitrudhachalam

Dr. K. ARJUNAN — Department of Mathematics, H.H. The Rajah's College, Pudukkottai

ABSTRACT

In this paper, we study some of the properties of homomorphism and anti-homomorphism in (Q, L)-fuzzy ideal of a ring and prove some results on these. 2000 AMS SUBJECT CLASSIFICATION: 03F55, 08A72, 20N25.

INTRODUCTION:

After the introdution of fuzzy sets by L.A.Zadeh[19], several researchers explored on the generalization of the notion of fuzzy set. Azriel Rosenfeld[4] defined a fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups and A.Solairaju and R.Nagarajan[16, 17, 18] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of homomorphism and anti-homomorphism in (Q, L)-fuzzy ideal of a ring and established some results.

1.PRELIMINARIES:

1.1 Definition:

Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A **(Q, L)-fuzzy subset** A of X is a function A : X×Q → L.

1.2 Definition:

Let (R, +, ·) be a ring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)**-fuzzy ideal (QLFI)** of R if the following conditions are satisfied:

(i) A(x+y, q) ≥ A(x, q) ∧ A(y, q),

(ii) A(–x, q) ≥ A(x, q),

(iii) A(xy, q) ≥ A(x, q) ∧ A(y, q), for all x and y in R and q in Q.

1.3 Definition:

Let (R, +, ·) and ($R^{|}$, +, ·) be any two rings and Q be a non empty set. Let f : R → $R^{|}$ be any function and A be a (Q, L)-fuzzy ideal in R, V be a (Q, L)-fuzzy ideal in f (R) = $R^{|}$, defined by V(y, q) = $\sup_{x\in f^{-1}(y)}$ A(x, q) , for all x in R and y in $R^{|}$ and q in Q. Then A is called a pre-image of V under f and is denoted by $f^{-1}(V)$.

1.4 Definition:

Let (R, +, ·) be a ring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)**-anti-fuzzy ideal (QLAFI)** of R if the following conditions are satisfied:

(i) A(x+y, q) ≤ A(x, q) ∧ A(y, q),

(ii) A(–x, q) ≤ A(x, q),

(iii) A(xy, q) ≤ A(x, q) ∧A(y, q), for all x and y in R and q in Q.

1.5 Definition:

Let A be a (Q, L)-fuzzy ideal of a ring (R, +, ·) and a in R. Then the **pseudo (Q, L)-fuzzy coset** $(aA)^p$ is defined by ($(aA)^p$)(x, q) = p(a)A(x, q), for every x in R and for some p in P and q in Q.

1.6 Definition:

Let A be a (Q, L)-fuzzy ideal of a ring (R, +, ·). For any a in R, a+A defined by (a+A)(x, q) = A(x–a, q), for every x in R and q in Q, is called a **(Q, L)-fuzzy coset** of R.

2. SOME PROPERTIES:

2.1 Theorem:

Let (R, +, ·) and ($R^{|}$, +, ·) be any two rings and Q be a non-empty set. The homomorphic image of a (Q, L)-fuzzy ideal of R is a (Q, L)-fuzzy ideal of $R^{|}$.

Proof:

Let (R, +, ·) and ($R^{|}$, +, ·) be any two rings and Q be a non-empty set and f : R→$R^{|}$ be a homomorphism. That is f(x+y) = f(x)+f(y), f(xy) = f(x)f(y),for all x and y in R. Let A be a (Q, L)-fuzzy ideal of R. Let V be the homomorphic image of A under f. We have to prove that V is a (Q, L)-fuzzy ideal of f(R) = $R^{|}$. Now, for f(x) and f(y) in $R^{|}$, we have V(f(x)+f(y), q) = V(f(x+y), q) ≥ A(x+y, q) ≥ A(x, q) ∧ A(y, q), which implies that V(f(x)+f(y), q) ≥ V(f(x), q)∧V(f(y), q). For f(x) in $R^{|}$, we have V(–f(x), q) = V(f(–x), q) ≥ A(–x, q) ∧ A(x, q), which implies that V(–f(x), q) ∧ V(f(x), q). And, V(f(x)f(y), q) =V(f(xy), q) ≥ A(xy, q) ≥ A(x, q) ∧ A(y, q), which implies that V(f(x)f(y), q) ≥ V(f(x), q) ∧ V(f(y), q). Hence V is a (Q, L)-fuzzy ideal of a ring $R^{|}$.

2.2 Theorem:

Let (R, +, ·) and ($R^{|}$, +, ·) be any two rings and Q be a non-empty set. The homomorphic pre-image of a (Q, L)-fuzzy ideal of $R^{|}$ is a (Q, L)-fuzzy ideal of R.

Proof:

Let (R, +, ·) and ($R^{|}$, +, ·) be any two rings and Q be a non-empty set and f : R→$R^{|}$ be a homomorphism. That is f(x+y) = f(x)+f(y), f(xy) = f(x)f(y), for all x and y in R. Let V be a (Q, L)-fuzzy ideal of f(R) = $R^{|}$. Let A be the pre-image of V under f. We have to prove that A is a (Q, L)-fuzzy ideal of R. Let x and y in R and q in Q. Then, A(x+y, q) = V(f(x+y), q) = V(f(x)+f(y), q) ∧ V(f(x), q)∧V(f(y), q) = A(x, q) ∧ A(y, q), which implies that A(x+y, q) ≥ A(x, q) ∧ A(y, q), for x and y in R and q in Q. And A(–x, q) = V(f(–x), q) = V(–f(x), q) ∧ V(f(x), q) = A(x, q), which implies that A(–x, q) ∧ A(x, q), for x in R and q in Q. And, A(xy, q) = V(f(xy), q) = V(f(x) f(y), q) ∧ V(f(x), q) ∧ V(f(y), q) = A(x, q) ∧ A(y, q), which implies that A(xy, q) ≥ A(x, q) ∧ A(y, q), for x and y in R and q in Q. Hence A is a (Q, L)-fuzzy ideal of the ring R.

2.3 Theorem:

Let (R, +, ·) and ($R^{|}$, +, ·) be any two rings and Q be a non-empty set. The anti-homomorphic image of a (Q, L)-fuzzy ideal of R is a (Q, L)-fuzzy ideal of $R^{|}$.

Proof:

Let (R, +, ·) and ($R^{|}$, +, ·) be any two rings and Q be a non-empty set and f : R → $R^{|}$ be a anti-homomorphism. That is f(x+y) = f(y)+f(x), f(xy) = f(y)f(x), for all x and y in R and q in Q. Let A be a (Q, L)-fuzzy ideal of R. Let V be the homomorphic image of A under f. We have to prove that V is a (Q, L)-fuzzy ideal of f(R) = $R^{|}$. Now, let f(x) and f(y) in $R^{|}$, we have V(f(x)+f(y), q) =V(f(y+x), q) ≥ A(y+x, q) ≥A(x, q) ∧ A(y, q), which implies that V(f(x) + f(y), q) ≥ V(f(x), q) ∧ V(f(y), q). For x in R and q in Q, V(–f(x), q)= V(f(–x), q) ≥ A(–x, q) ∧ A(x, q), which implies that V(–f(x), q) ∧ V(f(x), q), for x in R and q in Q. And, V(f(x)f(y), q) = V(f(yx), q) ≥ A(yx, q) ≥ A(x, q) ∧ A(y, q), which implies that V(f(x)f(y), q) ≥ V(f(x), q) ∧ V(f(y), q). Hence V is a (Q, L)-fuzzy ideal of $R^{|}$.

2.4 Theorem:
Let (R, +, ·) and (R^l, +, ·) be any two rings and Q be a non-empty set. The anti-homomorphic pre-image of a (Q, L)-fuzzy ideal of R^l is a (Q, L)-fuzzy ideal of R.

Proof: Let (R, +, ·) and (R^l, +, ·) be any two rings and Q be a non-empty set and f : R → R^l be a anti-homomorphism. That is f(x + y) = f(y) + f(x), f(xy) = f(y)f(x), for all x and y in R and q in Q. Let V be a (Q, L)-fuzzy ideal of f(R) = R^l. Let A be the pre-image of V under f. We have to prove that A is a (Q, L)-fuzzy ideal of R. Let x and y in R and q in Q. Now, A(x+y, q)= V(f(x+y), q)= V(f(y)+f(x), q) ∧ V(f(x), q)∧ V(f(y), q) = A(x, q) ∧ A(y, q), which implies that A(x+y, q) ≥ A(x, q)∧A(y, q), for all x and y in R and q in Q. And, A(−x, q) = V(f(−x), q) = V(−f(x), q) ∧ V(f(x), q) = A(x, q), which implies that A(−x, q) ∧ A(x, q) , for x in R and q in Q. Now, A(xy, q) = V(f(xy), q) = V(f(y)f(x), q) ∧ V(f(x), q) ∧ V(f(y), q) = A(x, q) ∧ A(y, q), which implies that A(xy, q) ≥ A(x, q) ∧ A(y, q), for all x and y in R and q in Q. Hence A is a (Q, L)-fuzzy ideal of the ring R.

In the following Theorem ◦ is the composition operation of functions:

2.5 Theorem:
Let A be a (Q, L)-fuzzy ideal of a ring H and f is an isomorphism from a ring R onto H. Then A◦f is a (Q, L)-fuzzy ideal of R.

Proof: Let x and y in R and A be a (Q,L)-fuzzy ideal of the ring H and Q be a non-empty set. Then we have, (A◦f)(x–y, q) = A(f(x–y), q) = A(f(x)–f(y), q) ≥ A(f(x), q) ∧A(f(y), q) ≥ (A◦f)(x, q) ∧ (A◦f)(y, q), which implies that (A◦f)(x–y, q) ≥ (A◦f)(x, q) ∧ (A◦f)(y, q). Then we have, (A◦f)(xy, q)= A(f(xy), q)= A(f(x)f(y), q) ≥ A(f(x), q) ∧ A(f(y), q) ≥ (A◦f)(x, q) ∧ (A◦f)(y, q), which implies that (A◦f)(xy, q) ≥ (A◦f)(x, q) ∧ (A◦f)(y, q). Therefore (A◦f) is a (Q, L)-fuzzy ideal of a ring R.

2.6 Theorem:
Let A be a (Q, L)-fuzzy ideal of a ring H and f is an anti-isomorphism from a ring R onto H. Then A◦f is a (Q, L)-fuzzy ideal of R.

Proof: Let x and y in R and A be a (Q, L)-fuzzy ideal of the ring H and Q be a non-empty set. Then we have, (A◦f)(x–y, q) = A(f(x–y), q) = A (f(y) − f(x), q) ≥ A(f(x), q)∧A(f(y), q) ≥ (A◦f)(x, q)∧(A◦f)(y, q), which implies that (A◦f)(x– y, q) ≥ (A◦f)(x, q)∧(A◦f)(y, q). Then we have, (A◦f)(xy, q) = A(f(xy), q)= A (f(y) f(x), q) ≥ A(f(x), q) ∧ A(f(y), q) ≥ (A◦f)(x, q)∧ (A◦f)(y, q), which implies that (A◦f)(xy, q) ≥ (A◦f)(x, q) ∧ (A◦f)(y, q). Therefore (A◦f) is a (Q, L)-fuzzy ideal of a ring R.

2.7 Theorem:
Let (R, +, .) be a ring and Q be a non-empty set. A is a (Q, L)-fuzzy ideal of R if and only if A^c is a (Q, L)-anti-fuzzy ideal of R.

Proof: Suppose A is a (Q, L)-fuzzy ideal of R. For all x and y in R and q in Q, we have A(x – y, q) ≥ A(x, q) ∧ A(y, q), which implies that 1– A^c(x – y, q) ≥ {1– A^c(x, q)} ∧ {1– A^c (y, q)}, which implies that A^c(x–y, q) ≤ 1–{1–A^c(x, q)∧1–A^c (y, q)}, which implies that A^c(x–y, q)≤ A^c(x)∧A^c (y). Also, A(xy, q) ≥A(x, q) ∧A(y, q), which implies that 1–A^c(xy, q) ≥ {1– A^c(x, q)} ∧{1–A^c(y, q) }, which implies that A^c(xy, q) ≤ 1– {1– A^c(x, q) ∧ 1– A^c (y, q) }, which implies that A^c(xy, q) ≤ A^c(x, q) ∧ A^c (y, q). Thus A^c is a (Q, L)-anti-fuzzy ideal of R. Converse also can be proved similarly.

2.8 Theorem:
Let A be a (Q, L)-fuzzy ideal of a ring R, then the pseudo (Q, L)-fuzzy coset $(aA)^p$ is a (Q, L)-fuzzy ideal of the ring R, for every a in R.

Proof: Let A be a (Q, L)-fuzzy ideal of the ring R. For every x and y in R and q in Q, we have, (($(aA)^p$)(x–y, q) = p(a)A(x–y, q) ≥ p(a) {A(x, q) ∧A(y, q)}= p(a)A(x, q)∧ p(a)A(y, q) = ($(aA)^p$)(x, q)∧($(aA)^p$)(y, q). Therefore, (($(aA)^p$)(x–y, q)≥($(aA)^p$)(x, q)∧ (($(aA)^p$) (y, q), for x, y in R and q in Q. And, (($(aA)^p$)(xy, q) = p(a)A(xy, q) ≥ p(a){A(x, q) ∧A(y, q)}= p(a)A(x, q) ∧ p(a)A(y, q) =($(aA)^p$)(x, q) ∧ (($(aA)^p$)(y, q). Therefore, (($(aA)^p$)(xy, q) ≥ (($(aA)^p$)(x, q) ∧ (($(aA)^p$) (y, q), for x, y in R and q in Q. Hence $(aA)^p$ is a (Q, L)-fuzzy ideal of the ring R.

2.9 Theorem:
Let (R, +, .) be a ring and Q be a non-empty set. If A is a (Q, L)-fuzzy ideal of R, then x+A = y+A if and only if A(x –y, q) = A(0, q), where 0 is the identity element. In that case A(x, q) = A(y, q).

Proof: Given A is a (Q, L)-fuzzy ideal of R. Suppose that x + A = y + A, which implies that (x+A)(x, q) = (y+A)(x, q), which implies that A(x–x, q) = A(x − y, q), which implies that A(0, q) =A(x– y, q). Conversely, assume that A(x– y, q) = A(0, q), then (x +A)(z, q) = A(z– x, q) = A(z– x+y –y, q) ≥ A(z–y, q)∧A(0, q) = A(z–y, q) = (y+A)(z, q), which implies that (x+A)(z, q) ≥ (y+A)(z, q) ------------(1). Now, (y+A)(z, q) = A(z – y, q) = A(z –y+x– x, q) ≥ A(z– x, q)∧A(0, q) = A(z–x, q) = (x+A)(z, q), which implies that (y+A)(z, q) ≥ (x+A)(z, q)--------------------(2). From (1) and (2) we get, x + A = y + A.

2.10 Theorem:
Let (R, +, .) be a ring and Q be a non-empty set. Let A is a (Q, L)-fuzzy ideal and x, y, u, and v be any elements in R, if x+A = u+A and y+A = v+A, then (x+y) + A = (u+v) + A.

Proof: Given A is a (Q, L)-fuzzy ideal of R. By Theorem 2.9, A(x–u, q) = A(y–v, q) = A(0, q). We get, A(x +y – u – v, q) = A(x – u + y – v, q) ≥ A(x – u, q) ∧A(y – v, q) = A(0, q). Again, by Theorem 2.9, (x + y) + A = (u + v) + A.

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