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Research Paper

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Bayesian Estimation Of Posterior Risk Of Tuberculosis Using Different Loss Functions

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ABSTRACT

The Bayesian estimation approach is a non-classical device in the estimation part of statistical inference which is very useful in real world situation. In Bayesian estimation loss function, prior distribution and posterior distribution are the most important ingredients. The conjugate prior, uniform prior and the Jeffrey's prior are considered for finding the Bayes estimator. The objective of this paper is to examine the effects of TB incidence in the 30 districts of Tamilnadu in the year 2011-2012 using different priors. The relative risk for each sub region of the study area has been estimated. The posterior probabilities have been calculated to find the risk of the disease throughout the state of Tamilnadu and the Bayes shrinkage estimates have also been calculated. Bayes estimators of the parameters are studied under different symmetric and asymmetric loss functions.

INTRODUCTION

The Poisson distribution is a discrete probability distribution and is used to model the number of occurences of rare events occuring randomly through time or space at a constant rate during a fixed time interval. The Poisson distribution has several unique features. It has only one parameter, namely the average frequency of the event. The Poisson distribution is not symmetrical; it is skewed toward the infinity end. The conditional model for Poisson distributed number of occurrences is

$$P(x_i/\theta) = \frac{\theta^x e^{-\theta}}{x!},\quad x = 0,1,2,\ldots\ldots \text{ for rate parameter } \theta > 0$$

Bayesian prediction plays an important role in different areas of applied statistics. We consider the Bayesian prediction of the unknown observable parameters based on the present sample. Miller (1980) used the conjugate priors and showed that the Bayes estimates can be obtained only through numerical integration. David P M Scollnik(1998) estimated the parameters of the truncated Poisson distribution using Bayesian method. Recently, Son and Oh (2006) consider the Poisson model and compute the Bayes estimates of the unknown parameters using Gibbs sampling procedure, under the vague priors, and compare their performances with the maximum likelihood estimators (MLEs) and the modified moment estimators. Studies have been done vigorously in the literature to determine the best method in estimating its parameters. Recently, much attention has been given to the Bayesian estimation approach for parameters estimation under different loss functions which is in contention with other estimation method. Loss functions lie in the heart of statistical decision theory, which is commonly recognized as a coherent foundational framework for inferential problems and comparison of procedures.

This work intent to quantify the incidence of tuberculosis in the different districts of the state of Tamilnadu. Considering the continual return of tuberculosis in the state of Tamilnadu and the need of estimating the true incidence of disease at the different districts of the state provides the backdrop of such a study. It is important to find the relative risk for such disease in each sub region which will throw more light on the risk in that region. The study can help the corresponding public health authorities of the states to adjust their programs, leading to the reduction of tuberculosis in Tamilnadu. The estimation of the relative risk will give some information on how much risk is present in a sub region. The relative risk of observed cases for the i^{th} district can be modeled as a Poisson model. The Bayes estimators of the parameter of the Poisson model are studied for different loss functions under different priors.

MODEL DESCRIPTION

Let θ_i be the unknown relative risk for the region i, with probability distribution function f (θ_i). The incident rate is a common measure of relative risk. The incident rate is defined as $\theta_i = \frac{o_i}{n_i}$ where o_i is the observed number of cases and n_i is the expected number of cases for region i. Let the probability for an individual to get the disease be θ. The relative risk of observed cases for the i^{th} district can be modeled as a Poisson model. Thus the relative risk for the i^{th} district has Poisson (θ_i) with means $n_i\theta_i$. (i=1, 2, 3……….n).

Likelihood function and posterior distribution

The likelihood is the joint probability function of the data, but viewed as parameters, treating the observed data as fixed quantities. The likelihood function is given by,

$$L(x_i,\theta) = \prod_{i=1}^{n} P(x_i/\theta)$$

$$= \prod_{i=1}^{n} \frac{\theta^x e^{-\theta}}{x!}$$

$$\propto e^{-n\theta}\theta^m, \text{ where } m = \sum x_i$$

Posterior distribution using conjugate prior

A flexible choice of conjugate prior distribution for a Poisson probability is a Gamma distribution with parameters α and β. The probability function is given by

$$P(\theta/\alpha,\beta) = \frac{\beta^\alpha e^{-\beta}\,\theta^{\alpha-1}}{\Gamma(\alpha)} \text{ with mean}$$

$E(\theta) = \frac{\alpha}{\beta}$ and $Var(\theta) = \frac{\alpha}{\beta^2}$.

The joint pdf $H(x_1,x_2,\ldots,x_n,\theta)$ is given by $H(x_1,x_2,\ldots,x_n,\theta) = L(x_1,x_2,\ldots,x_n,\theta)\ g(\theta)$

where $L(x_1,x_2,\ldots,x_n,\theta)$ is the likelihood function of $P(x_i/\theta)$ and $g(\theta)$ is the prior distribution.

Hence the joint pdf is

$$H(x_1,x_2,\ldots,x_n,\theta) = e^{-n\theta}\theta^m \frac{\beta^\alpha e^{-\beta}\,\theta^{\alpha-1}}{\Gamma(\alpha)}$$

$$\propto \theta^{m+\alpha-1}e^{-\theta(\beta+n)}$$

The marginal pdf of $x_1,x_2,\ldots,x_n$ is given by

$$P(x_1,x_2,\ldots,x_n) = \int H(x_1,x_2,\ldots,x_n,\theta)\,d\theta$$

$$\propto \int \theta^{m+\alpha-1}e^{-\theta(\beta+n)}\,d\theta$$

The posterior pdf $P(\theta/x_i)$ of θ is given by

$$P(\theta/x_1,x_2,\ldots,x_n) = \frac{H(x_1,x_2,\ldots,x_n,\theta)}{P(x_1,x_2,\ldots,x_n)}$$

$$= \frac{\theta^{m+\alpha-1} e^{-\theta(\beta+n)}}{\int \theta^{m+\alpha-1} e^{-\theta(\beta+n)} d\theta}$$

which is proportional to a Gamma ($\alpha + m, \beta + n$) density.

The posterior mean and variance are

$$E(\theta/x) = \frac{\alpha+m}{\beta+n}, \; Var(\theta/x) = \frac{\alpha+m}{(\beta+n)^2}$$

The posterior mean is a weighted average of the prior mean and the MLE.

Once the posterior distribution has been determined, inferential conclusions can be summarized with an appropriate analysis. The posterior mean and mode are used to compute the point estimates for the parameter. The Posterior mean and mode are respectively $\frac{\alpha+m}{\beta+n}$ and $\frac{\alpha+m-1}{\beta+n}$.

Also, the shrinkage estimator can be used to improve the estimation by reducing the mean squared error towards zero. The shrinkage estimator is given by $B_i = \frac{\alpha}{\alpha+n_i}$.

Posterior distribution using uniform prior

The uniform prior distribution for a Poisson probability is

$$g(\theta) \propto 1$$

Hence the joint pdf is

$$H(x_1, x_2, \ldots, x_n, \theta) = e^{-n\theta}\theta^m$$

The marginal pdf of $x_1, x_2, \ldots, x_n$ is given by

$$P(x_1, x_2, \ldots, x_n) = \int H(x_1, x_2, \ldots, x_n, \theta)\, d\theta$$

$$\propto \int e^{-n\theta}\theta^m d\theta$$

The posterior pdf $P(\theta/x_i)$ of θ is given by

$$P(\theta/x_1, x_2, \ldots, x_n) = \frac{H(x_1, x_2, \ldots, x_n, \theta)}{P(x_1, x_2, \ldots, x_n)}$$

$$= \frac{e^{-n\theta}\theta^m}{\int e^{-n\theta}\theta^m\, d\theta}$$

which is proportional to a Gamma ($m+1, n$) density.

The posterior mean and variance are

$$E(\theta/x) = \frac{m+1}{n}, \; Var(\theta/x) = \frac{m+1}{n^2}$$

Posterior distribution using Jeffrey's prior

$$g(\theta) \propto \frac{1}{\sqrt{\theta}}$$

Hence the joint pdf is

$$H(x_1, x_2, \ldots, x_n, \theta) = e^{-n\theta}\theta^{m-\frac{1}{2}}$$

The marginal pdf of $x_1, x_2, \ldots, x_n$ is given by

$$P(x_1, x_2, \ldots, x_n) = \int H(x_1, x_2, \ldots, x_n, \theta)\, d\theta$$

$$\propto \int e^{-n\theta}\theta^{m-\frac{1}{2}} d\theta$$

The posterior pdf $P(\theta/x_i)$ of θ is given by

$$P(\theta/x_1, x_2, \ldots, x_n) = \frac{H(x_1, x_2, \ldots, x_n, \theta)}{P(x_1, x_2, \ldots, x_n)}$$

$$= \frac{e^{-n\theta}\theta^{m-\frac{1}{2}}}{\int e^{-n\theta}\theta^{m-\frac{1}{2}} d\theta}$$

which is proportional to a Gamma ($m+\frac{1}{2}, n$) density.

The posterior mean and variance are

$$E(\theta/x) = \frac{m+\frac{1}{2}}{n}, \; Var(\theta/x) = \frac{m+\frac{1}{2}}{n^2}$$

Parameter estimation using different loss functions
Asymmetric Loss Functions

The linear exponential (LINEX) loss function is an asymmetric loss function. It is under the assumption that the minimal loss occurs at $\hat{\theta} = \theta$ and is expressed as

$L(\Delta) \propto \exp(a\Delta) - (a\Delta) - 1;\ a \neq 0$ ---------------- (1)

with $\Delta = (\hat{\theta} - \theta)$, where $\hat{\theta}$ is an estimate of θ. The sign and magnitude of the shape parameter a α represents the direction and degree of symmetry, respectively. There is overestimation if a > 0 and underestimation if a < 0 but when $a \cong 0$, the LINEX loss function is approximately the squared error loss function. The posterior expectation of the LINEX loss function, according is

$$E_\theta[L(\hat{\theta} - \theta)] \propto \exp(a\hat{\theta})\, E_\theta[\exp(-a\theta)] - a(\hat{\theta} - E_\theta(\theta)) - 1$$

------------------ (2)

The Bayes estimator of θ θ, represented by $\hat{\theta}_L$ under LINEX loss function, is the value of $\hat{\theta}$ which minimizes (1) and is given as

$$\hat{\theta}_L = -\frac{1}{a} \ln E_\theta[\exp(-a\theta)]$$

------------------- (3)

Provided that $E_\theta[\exp(-a\theta)]$ exists and is finite. The Bayes estimator $\hat{u}_L$ of a function

$u = [\exp(-a\alpha)\, \exp(-a\beta)]$ is given as

$$\hat{u}_L = E[\exp(-a\alpha)\, \exp(-a\beta)/y]$$

$$= \frac{\int u[\exp(-a\alpha)\, \exp(-a\beta)]\, \pi(\alpha,\beta)\, d\alpha\, d\beta}{\int \pi(\alpha,\beta)\, d\alpha\, d\beta}$$

------------------ (4)

From equation (4), it can be observed that it contains a ratio of integrals which cannot be solved analytically, and for that we employ Linley's approximation procedure to estimate the parameters. Lindley considered an approximation for the ratio of integrals for evaluating the posterior expectation of an arbitrary function $\hat{u}(\theta)$ as

$$E[u(\theta)/x] = \frac{\int u(\theta)v(\theta)\exp[L(\theta)]\, d\theta}{\int v(\theta)\exp[L(\theta)]\, d\theta}$$

Lindley's expansion can be approximated asymptotically by

$$\hat{\theta} = u + \frac{1}{2}[\, u_{11}\ \delta_1\ ) + (u_{22}\ \delta_2\ ] + u_1\rho_1\delta_1 + u_2\rho_2\delta_2 + \frac{1}{2}[\, L_{30}\ u_1\delta_1^2)(L_{03}\ u_2\delta_2^2\ ]$$

where L is the log- likelihood function, and

$$u(\alpha) = \exp(-a\alpha)\,,\quad u_1 = \frac{\partial u}{\partial \alpha} = -a\exp(-a\alpha)$$

$$u_{11} = \frac{\partial^2 u}{\partial \alpha^2} = -a^2\exp(-a\alpha)$$

$$u(\beta) = \exp(-a\beta)\,,\quad u_2 = \frac{\partial u}{\partial \beta} = -a\exp(-a\beta)$$

$$u_{22} = \frac{\partial^2 u}{\partial \beta^2} = -a^2\exp(-a\beta)$$

ρ = log of prior

$$\rho_1 = \frac{\partial \rho}{\partial \alpha}, \rho_2 = \frac{\partial \rho}{\partial \beta}$$

$$\delta_{11} = (-L_{20})^{-1}, \delta_{22} = (-L_{02})^{-1}$$

Entropy Loss Function

Another useful asymmetric loss function is a entropy loss function (ENLF) which is given as

$$L(\hat{\theta} - \theta) \propto \left(\frac{\hat{\theta}}{\theta}\right)^k - k\,\mathrm{h}\left(\frac{\hat{\theta}}{\theta}\right) - 1$$

The Bayes estimator $\hat{\theta}_G$ of θ under the entropy loss function is

$$\hat{\theta}_G = [E_\theta(\theta^{-k}]^{-\frac{1}{k}}$$

provided $E_\theta(\theta^{-k})\, E_\theta\,(\theta^{-k})$ exists and is finite. The Bayes estimator for this loss function is

$$\hat{u}_G = E\{u[(\alpha)^{-k},(\beta)^{-k}]/t\} = \frac{\int u[(\alpha)^{-k},(\beta)^{-k}]\pi(\alpha,\beta)\,d\alpha\, d\beta}{\int \pi(\alpha,\beta)\,d\alpha\, d\beta}$$

Similar Lindley approach is used for the general entropy loss function as in the LINEX loss but here the Lindley approximation procedure as stated in (3), where u_1,u_{11} and u_2,u_{22} are the first and second derivatives for α and β, respectively, and are given as

$$u = (\alpha)^{-k}\,,\quad u_1 = \frac{\partial u}{\partial \alpha} = -k(\alpha)^{-k-1}$$

$$u_{11} = \frac{\partial^2 u}{\partial \alpha^2} = -(-k^2 - k)(\alpha)^{-k-2},\ u_2 = u_{22} = 0,$$

$$u = (\beta)^{-k},\quad u_2 = \frac{\partial u}{\partial \beta} = -k(\beta)^{-k-1}$$

$$u_{22} = \frac{\partial^2 u}{\partial \beta^2} = -(-k^2 - k)(\beta)^{-k-2},\ u_1 = u_{11} = 0$$

Symmetric Loss Functions

The squared error loss function is symmetric in nature and is given by

$l(\hat{\theta} - \theta) = (\hat{\theta} - \theta)^2$. The Bayes estimator of a function u = u(α,β) of the unknown parameters under squared error loss function is the posterior mean.

where $\hat{u} = E[u(\alpha,\beta)/t] = \dfrac{\int u(\alpha,\beta)\pi^*(\alpha,\beta)\,d\alpha\, d\beta}{\pi^*(\alpha,\beta)\,d\alpha\, d\beta}$

Applying the same Lindley approach here with u_1, u_{11} and u_2, u_{22} being the first and second derivatives for α and β, respectively, we have

$$u = \alpha \qquad u_1 = \frac{\partial u}{\partial \alpha} = 1$$

$$u_{11} = u_2 = u_{22} = 0$$

$$u = \beta \qquad u_2 = 1$$

$$u_{11} = u_1 = u_{22} = 0$$

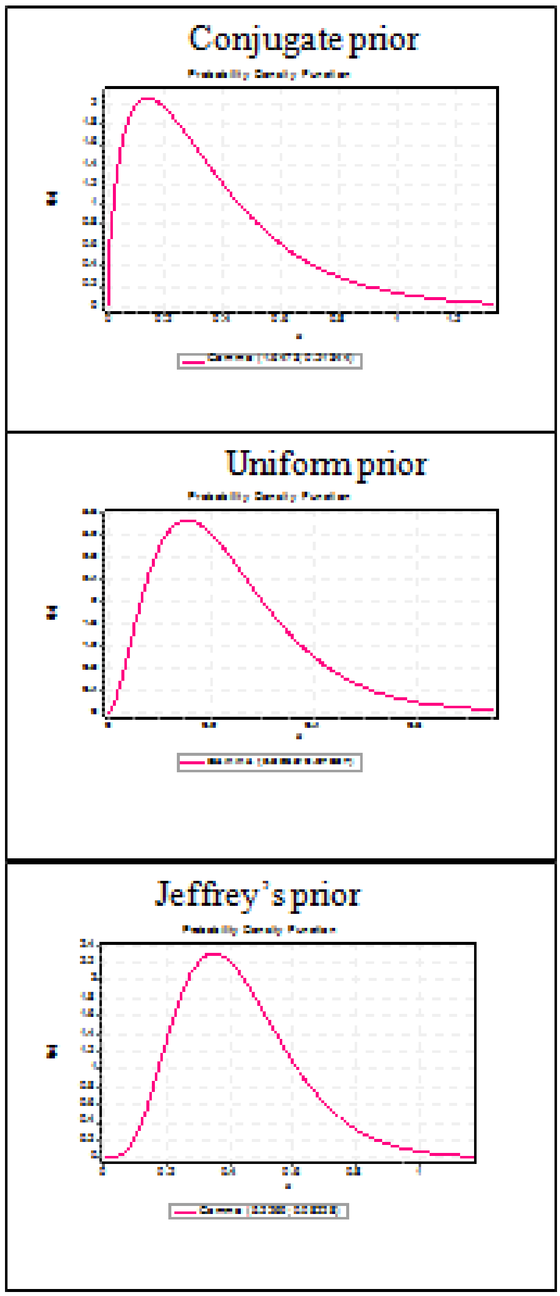
Absolute error loss function

Another symmetric loss function is the absolute error loss function and is given by $l(\hat{g}(\theta)\ g(\theta)\ = |\hat{g}(\theta) - g(\theta)|$. Under this loss function the Bayes estimate of g (θ) will be the median of the posterior distribution of g(θ)

Data Analysis

The data required for this study is based on the data obtained from the NTBC from the fourth quarter of 2010 to third quarter of 2011. The report provides district wise suspected TB cases and the smear positive TB cases. The estimates for each of the states are examined through a Bayesian analysis. All the calculations have been done using the SPSS software. For the TB problem, the posterior distribution using different priors such as conjugate prior, uniform prior and the Jeffrey's prior have been found. The graph of the probability function of the posterior distribution with different priors is given in Figure 1

Figure 1:



Also the Posterior Summary using the real data with different priors is given in Table.1

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Posterior Summary using the real data with different priors

TABLE - 1
Posterior Summary using the real data with different priors

	CP	UP	JP
Mean	0.23375	0.24154	0.3516
Mode	0.15389	0.1357	0.13816
Variance	0.01867	0.02557	0.07504
Skewness	1.169	1.324	1.5583
Kurtosis	2.05	2.6293	3.6423

The above results reveal that due to the smaller values of posterior distribution under conjugate prior, it is more compatible for the unknown parameters of the distribution using the real life data.

Simulation Study
In our simulation study, we chose a sample size of n= 50, 75 and 100 to represent small, medium and large data set. The Bayesian estimation of the parameter θ have been estimated for the Poisson distribution using different priors such as Jeffrey's prior, uniform prior and the conjugate prior with the four loss functions such as the linex loss function, the squared error loss function, the entropy loss function and the absolute error loss function. It is shown that in Table 3 –Table 6, the comparison of Bayes posterior risk under different loss function using different priors has been made through which we conclude that within each loss function the conjugate prior provides less Bayes posterior risk so it is more suitable for the class of lifetime distributions and amongst loss functions, LINEX loss function, is more preferable as compared to all other loss functions which are provided here because under this loss function Bayes posterior risk is small for each and every value of parameter .

Discussion
Among the districts in Tamilnadu, maximum TB cases are recorded in Chennai. The Districts of Karur, Krishnagiri, Nagapattinam, Namakkal, Pudukottai, Sivaganga, Thanjavur, Tirunelveli, Tiruchirappalli, Tiruppur and Tiruvanamalai are areas with high risk from the results in the Table.1. Also it is found that there is no district with very low risk. The shrinkage estimates also supports the results obtained using the model. Since the risk factor is high almost in all the districts, necessary steps should be taken to control the deaths due to TB. Early identification of all infectious TB cases, improved integration with the general health system, and committed field staff for home-based case finding**,** tracing patients already diagnosed through notification from all sources, improved referral for treatment mechanisms, etc are some of the factors needed to control the deaths due to TB

TABLE - 2
Posterior probability under different priors and the shrinkage estimation

	Districts	o_i / n_i	Posterior probability			Shrinkage estimation
			CP	UP	JP	
	Chennai	.085536	.5297910	.3171036	.5530809	.0000230
	Coimbatore	.090213	.6064544	.3502021	.5867994	.0000639
	Cuddalore	.040384	.0093238	.0530008	.1749013	.0001074
	Dharmapuri	.058618	.1115463	.1398118	.3316486	.0001937
	Dindigul	.101429	.7619676	.4290102	.6604935	.0000554
	Erode	.102943	.7793112	.4394711	.6696525	.0000518
	Kanchipuram	.089470	.5946466	.3449401	.5815576	.0001042
	Kanyakumari	.056139	.0874624	.1258640	.3098747	.0001041
	Karur	.079100	.4183597	.2719668	.5039789	.0002985
	Krishnagiri	.063960	.1752413	.1717015	.3782548	.0001808
	Madurai	.105791	.8094937	.4589770	.6863740	.0000466
	Nagapattinam	.075093	.3487240	.2443915	.4719533	.0001605
	Namakkal	.074165	.3328683	.2380880	.4643910	.0001867
	Perambalur	.089826	.6003237	.3474614	.5840746	.0001897
	Pudukottai	.069417	.2548985	.2064556	.4249318	.0001615
	Ramanathapuram	.051295	.0505768	.1003911	.2673965	.0002092
	Salem	.102816	.7778911	.4385959	.6688914	.0000677
	Sivaganga	.077732	.3944487	.2624936	.4931638	.0001601
	Thanjavur	.062176	.1522453	.1607929	.3627638	.0000771
	The Nilgiris	.030926	.0009941	.0245414	.1036225	.0006505
	Theni	.082216	.4727784	.2937242	.5281284	.0001256
	Thiruvallur	.046386	.0258589	.0772473	.2249490	.0000892
	Thiruvarur	.095199	.6812046	.3854434	.6208267	.0001854
	Tiruchirappalli	.062148	.1518977	.1606236	.3625200	.0000694
	Tirunelveli	.075949	.3634689	.2502360	.4788820	.0000736
	Tiruppur	.076748	.3773148	.2557155	.4853074	.0001838
	Tiruvanamalai	.074645	.3410510	.2413441	.4683091	.0000788
	Toothukudi	.095306	.6827155	.3861972	.6215348	.0001079
	Vellore	.037714	.0054258	.0437366	.1536444	.0000478
	Ariyalur	.0318341	.0012872	.0267465	.1099208	.0005605

TABLE - 3 Bayes estimates and respective posterior risk under LINEX loss function

θ	0.5			1.0			1.5		
n	CP	UP	JP	CP	UP	JP	CP	UP	JP
50	0.748772 (0.01584)	0.964558 (0.04887)	0.771794 (0.02199)	0.971272 (0.02601)	0.998487 (0.04976)	0.983225 (0.03915)	0.997324 (0.01796)	0.99999 (0.08291)	0.999833 (0.03832)
75	0.761035 (0.01506)	0.965089 (0.05813)	0.738693 (0.02982)	0.977328 (0.01927)	0.999658 (0.08091)	0.991252 (0.03222)	0.998317 (0.01565)	0.99999 (0.13830)	0.999962 (0.02761)
100	0.782050 (0.01788)	0.954983 (0.06525)	0.741533 (0.02184)	0.981833 (0.01498)	0.999928 (0.06781)	0.995509 (0.03103)	0.998821 (0.01260)	0.99999 (0.06335)	0.999971 (0.02998)

TABLE - 4 Bayes estimates and respective posterior risk under squared error loss function

θ	0.5			1.0			1.5		
n	CP	UP	JP	CP	UP	JP	CP	UP	JP
50	0.34548 (0.07676)	0.22832 (0.07574)	0.45345 (0.06621)	0.34423 (0.09105)	0.28289 (0.08742)	0.49188 (0.09903)	0.38805 (0.17533)	0.26023 (0.13904)	0.44030 (0.12039)
75	0.36100 (0.07338)	0.23574 (0.06561)	0.40746 (0.06007)	0.37946 (0.09772)	0.25177 (0.09101)	0.47108 (0.09630)	0.44881 (0.12890)	0.23752 (0.15086)	0.43944 (0.13981)
100	0.35194 (0.06763)	0.23493 (0.06756)	0.41645 (0.07150)	0.35838 (0.08764)	0.23913 (0.09981)	0.44806 (0.09214)	0.33692 (0.16849)	0.20563 (0.15729)	0.44337 (0.14536)

TABLE - 5 Bayes estimates and respective posterior risk under entropy error loss function

θ	0.5			1.0			1.5		
n	CP	UP	JP	CP	UP	JP	CP	UP	JP
50	0.738671 (0.181870)	0.954457 (0.187490)	0.761693 (0.326290)	0.961171 (0.037750)	0.988386 (0. .082890)	0.973124 (0.037010)	0.987223 (0.04030)	0.989889 (0.058170)	0.989732 (0.073030)
75	0.750934 (0.221070)	0.954988 (0.325450)	0.728592 (0.515290)	0.967227 (0.026730)	0.989557 (0.052070)	0.981151 (0.030730)	0.988216 (0.045350)	0.989889 (0.035790)	0.989861 (0.040210)
100	0.771949 (0.269810)	0.944882 (0.277430)	0.731432 (0.215670)	0.971732 (0.051210)	0.989827 (0.024310)	0.985408 (0.021570)	0.988720 (0.003870)	0.989889 (0.022210)	0.989870 (0.034230)

TABLE-6 Bayes estimates and respective posterior risk under absolute error loss function

θ	0.5			1.0			1.5		
n	CP	UP	JP	CP	UP	JP	CP	UP	JP
50	0.29958 (0.065766)	0.20565 (0.064747)	0.43550 (0.055220)	0.29736 (0.080059)	0.25289 (0.076430)	0.46563 (0.088038)	0.31936 (0.164338)	0.23763 (0.128049)	0.41167 (0.109397)
75	0.30887 (0.062387)	0.21483 (0.054613)	0.38336 (0.049077)	0.31095 (0.086727)	0.22676 (0.080012)	0.44850 (0.085308)	0.35059 (0.117905)	0.21596 (0.139864)	0.41615 (0.128815)
100	0.29223 (0.056638)	0.21010 (0.056560)	0.39912 (0.060505)	0.29745 (0.076641)	0.21860 (0.088818)	0.42771 (0.081145)	0.28574 (0.157490)	0.18560 (0.146300)	0.42106 (0.134369)

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