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Solving Fully Fuzzy Multi-Objective Linear Programming Problems

Mathematics

KEYWORDS :

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ABSTRACT

In this paper, a new method namely total objective-segregation method for finding a properly fuzzy efficient solution to fully fuzzy multi-objective linear programming problems is proposed. The proposed method is only based on linear programming technique in which fuzzy ranking functions are not used. Numerical example is used to illustrate the proposed method.

1. Introduction

The classical Linear Programming (LP) problem is used to find an optimal solution for single objective function whereas many problems in real world may involve multiple objectives with conflicting nature. This type of problems can be formulated as Multi-Objective Linear Programming (MOLP) problems. In the MOLP problem, 'k' different linear objective functions are optimized subject to a set of linear constraints where $k \geq 2$. Optimizing all objective functions in the MOLP at the same time is not possible because of the conflicting nature of the objectives. The concept of optimality in the MOLP problem is replaced with that of efficiency (Pareto optimality) / proper efficiency. Pandian [7] proposed a new approach namely, Sum of Objectives (SO) method for finding a properly efficient solution to MOLP problem.

Using the concept of decision making in fuzzy environment given by Bellman and Zadeh [1], Tanaka et al. [12] proposed a method for solving fuzzy mathematical programming problems. Jayalakshmi and Pandian [5] proposed bound and decomposition method to solve fully fuzzy linear programming (FFLP) problems. Many authors (Buckley and Feuring [1], Zhang et al. [17], Thakre et al. [13], Xue - quan Li et al. [15] and Pandian [8]) have developed various approaches for solving fuzzy linear programming (FLP) problems using MOLP technique.

MOLP problems occurring with real world problems require handling and evaluation of fuzzy data for decision making. Wu et al. [14] solved Fuzzy Multi-Objective Linear Programming (FMOLP) problems by converting them into LP, which are then solved by simplex method. De and Bharti Yadav [3] presented two algorithms to give efficient solutions as well as an optimal compromise solution for FMOLP problems. Sanjaya Kumar Behera et al. [10] solved FMOLP problems by using α–cut and Zimmermann's method. Mohanaselvi and Ganesan [6] proposed an algorithm to find the fuzzy Pareto-optimal solution for the given FFMOLP problem. Purnima Pandit [9] proposed a method to solve FMOLP problems by converting them into equivalent crisp MOLP problems which were solved then using Pareto's optimality technique. Sophia Porchelvi and Vasanthi [11] introduced a method for solving FMOLP problems directly using linear ranking functions.

In this paper, the implementation of total objective-segregation method for fully fuzzy multi-objective linear programming (FFMOLP) problems was discussed. A properly fuzzy efficient solution procedure to MOLP problem is proposed in which costs of the objective functions, coefficients of the constraints and decision variables are triangular fuzzy numbers. In this method, the FFMOLP problem is transformed into an equivalent FFLP problem by using fuzzy arithmetic and SO method, then the fuzzy optimal solution to the above problem is obtained using bound and decomposition method introduced by Jayalakshmi and Pandian [5] and then, we derive that the optimal solution of the FFLP yields a properly fuzzy efficient solution to the given FFMOLP problem. The procedure followed in the proposed method is illustrated through a numerical example.

2 Preliminaries

We need the following mathematical orientated definitions of fuzzy set, fuzzy number and membership function, which can be found in Zadeh [16].

Definition 2.1 Let A be a classical set and $\mu_A(x)$ be a real valued function defined from R into [0,1]. A fuzzy set A^* with the function $\mu_A(x)$ is defined by $A^* = \{ (x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1] \}$. The function $\mu_A(x)$ is known as the membership function of A^*.

Definition 2.2 A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1) & \text{for } a_1 \le x \le a_2 \\ (a_3 - x)/(a_3 - a_2) & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

We need the following definitions of the basic arithmetic operations on fuzzy triangular numbers based on the function principle which can be found in George J. Klir and Bo Yuan [4].

Definition 2.3 Let (a_1, a_2, a_3) and (b_1, b_2, b_3) be two triangular fuzzy numbers. Then

(i) $(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

(ii) $(a_1, a_2, a_3) \, \Theta \, (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

(iii) $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$, for $k \ge 0$.

(iv) $k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$, for $k < 0$.

(v) $(a_1, a_2, a_3) \otimes (b_1, b_2, b_3)$

$$= \begin{cases} (a_1 b_1, a_2 b_2, a_3 b_3), & a_1 \ge 0, \\ (a_1 b_3, a_2 b_2, a_3 b_3), & a_1 < 0, a_3 \ge 0, \\ (a_1 b_3, a_2 b_2, a_3 b_1), & a_3 < 0. \end{cases}$$

Let $F(R)$ be the set of all real triangular fuzzy numbers.

Definition 2.4 Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be in $F(R)$, then

(i) $\tilde{A} \approx \tilde{B}$ iff $a_i = b_i, i = 1, 2, 3$;

(ii) $\tilde{A} \preceq \tilde{B}$ iff $a_i \le b_i, i = 1,2,3$;

(iii) $\tilde{A} \succeq \tilde{B}$ iff $a_i \ge b_i, i = 1,2,3$ and $\tilde{A} \succeq \tilde{0}$ iff $a_i \ge 0, i = 1,2,3$.

Consider the following fully fuzzy linear programming problems:

(FFLP) Maximize $Z = \tilde{C}^T \otimes \tilde{X}$

subject to

$$\tilde{A} \otimes \tilde{X} \{ \preceq, =, \succeq \} \tilde{B}; \qquad (2.1)$$

$$\tilde{X} \succeq \tilde{0}, \qquad (2.2)$$

where $\tilde{C}^T = (\tilde{c}_j)_{1\times n}$, $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, $\tilde{X} = (\tilde{x}_j)_{nx1}$, $\tilde{B} = (\tilde{b}_i)_{mx1}$ and $\tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i \in F(R)$, for all $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$.

Definition 2.5 A real fuzzy vector $\tilde{B} = (\tilde{b}_i)_{m\times 1}$ is called non-negative and denoted by $\tilde{B} \succeq \tilde{0}$ if each element of $\tilde{B}$ is a non-negative real fuzzy number, that is $\tilde{b}_i \succeq \tilde{0}$, for all $i = 1,2,..., m$.

Definition 2.6 A non-negative fuzzy vector $\tilde{X}$ is said to be a fuzzy feasible solution of the problem (FFLP) if $\tilde{X}$ satisfies (2.1) and (2.2).

Definition 2.7 A fuzzy feasible solution $\tilde{X}$ is said to be a fuzzy optimal solution of the problem (FFLP) if there exists no fuzzy feasible solution $\tilde{U}$ of (FFLP) such that $\tilde{C} \otimes \tilde{U} \succ \tilde{C} \otimes \tilde{X}$.

3. Fully Fuzzy Multi-Objective Linear Programming Problems

Consider the fully fuzzy multi-objective linear programming (FFMOLP) problems as follows:

(FFMOLP) Maximize $\tilde{Z} = \left(\tilde{Z}_1, \tilde{Z}_2, ..., \tilde{Z}_k\right)$

subject to $\tilde{A} \otimes \tilde{X} \ \{\preceq, \approx, \succeq\} \ \tilde{B}$;

$$\tilde{X} \succeq \tilde{0},$$

where each $\tilde{Z}_i : F(R) \to F(R) \quad (i = 1,2,...,k)$ $\tilde{A} = \left(\tilde{a}_{ij}\right)_{m\times n}$, $\tilde{X} = (\tilde{x}_j)_{n\times 1}$, $\tilde{B} = (\tilde{b}_i)_{m\times 1}$ and $\tilde{Z}_i, \tilde{a}_{ij}, \tilde{x}_j, \tilde{b}_i \in F(R)$, for all $1 \le i \le m$ and $1 \le j \le n$.

Let $FP = \left\{\tilde{X} \in F(R) : \tilde{A} \otimes \tilde{X} \{\preceq, \approx, \succeq\} \tilde{B} \text{ and } \tilde{X} \succeq \tilde{0}\right\}$ be the set of all fuzzy feasible solution to the problem (FFMOLP).

From the problem (FFMOLP), we obtain the following fully fuzzy single objective linear programming problem, (FFSOLP):

(FFSOLP) Maximize $\tilde{W} = \sum_{i=1}^{k} \tilde{Z}_i$

subject to $\tilde{A} \otimes \tilde{X} \ \{\preceq, \approx, \succeq\} \ \tilde{B}$;

$$\tilde{X} \succeq \tilde{0},$$

Based on the definition properly efficient solution which can be found in De and Bharti Yadav [3], we define the following:

Definition 3.1 A fuzzy efficient solution $\tilde{X}^\circ \in FP$ is said to be a properly fuzzy efficient solution for (FFMOLP) if there exists a fuzzy scalar $\tilde{M} \succ \tilde{0}$ such that, for each $i \in \{1,2,\ldots,k\}$ and for all feasible $\tilde{X} \in FP$ of (FFMOLP) satisfying $\tilde{Z}_i(\tilde{X}^\circ) \prec \tilde{Z}_i(\tilde{X})$, we have $\tilde{Z}_i(\tilde{X}) - \tilde{Z}_i(\tilde{X}^\circ) \preceq \tilde{M} \otimes \left(\tilde{Z}_r(\tilde{X}^\circ) - \tilde{Z}_r(\tilde{X})\right)$, for some $r \in \{1, 2, ..., k\}$ such that $\tilde{Z}_r(\tilde{X}) \prec \tilde{Z}_r(\tilde{X}^\circ)$.

Now, we derive a relation between an optimal solution to the problem (FFSOLP) and a properly fuzzy efficient solution to the problem (FFMOLP).

Theorem 3.1 Let $\tilde{X}^\circ = \left\{\left(x_j^\circ, y_j^\circ, t_j^\circ\right); j = 1,2,...m\right\}$ be a fuzzy optimal solution to the problem (FFSOLP). Then, $\tilde{X}^\circ = \left\{\left(x_j^\circ, y_j^\circ, t_j^\circ\right); j = 1,2,...m\right\}$ is a properly fuzzy efficient solution to the problem (FFMOLP).

Proof: Now, since $\tilde{X}^\circ = \left\{\left(x_j^\circ, y_j^\circ, t_j^\circ\right); j = 1,2,...m\right\}$ is a fuzzy optimal solution to the problem (FFSOLP), $\tilde{X}^\circ = \left\{\left(x_j^\circ, y_j^\circ, t_j^\circ\right); j = 1,2,...m\right\}$ is a feasible solution to the problem (FFMOLP).

Assume that $\tilde{X}^\circ = \left\{\left(x_j^\circ, y_j^\circ, t_j^\circ\right); j = 1,2,...m\right\}$ is not a fuzzy efficient solution to the problem (FFMOLP). Then, there exists a feasible solution $\tilde{X} = \left\{\left(x_j, y_j, t_j\right); j = 1,2,...m\right\}$ of (FFMOLP) such that $\tilde{Z}_i\left(\tilde{X}\right) \succeq \tilde{Z}_i\left(\tilde{X}^\circ\right)$, $i = 1, 2, ..., k$ and $\tilde{Z}_r\left(\tilde{X}\right) \succ \tilde{Z}_r\left(\tilde{X}^\circ\right)$, for some $r \in \{1, 2, ..., k\}$. This implies that $\tilde{W}(\tilde{X}) \succ \tilde{W}(\tilde{X}^\circ)$ which contradicts the optimality of $\tilde{X}^\circ = \left\{\left(x_j^\circ, y_j^\circ, t_j^\circ\right); j = 1,2,...m\right\}$ to the problem (FFSOLP). Therefore, $\tilde{X}^\circ = \left\{\left(x_j^\circ, y_j^\circ, t_j^\circ\right); j = 1,2,...m\right\}$ is a fuzzy efficient solution to (FFMOLP).

Assume that $\tilde{X}^\circ = \left\{\left(x_j^\circ, y_j^\circ, t_j^\circ\right); j = 1,2,...m\right\}$ is not a properly fuzzy efficient solution to the problem (FFMOLP). Then, for every $\tilde{M} \succ \tilde{0}$, there exists a feasible solution $\tilde{X} = \{ (x_j, y_j, t_j); j = 1,2,...m \}$ of (FFMOLP) and an index $i = 1, 2, ..., k$ such that $\tilde{Z}_i(\tilde{X}) - \tilde{Z}_i(\tilde{X}^\circ) \succ \tilde{M} \otimes (\tilde{Z}_r(\tilde{X}^\circ) - \tilde{Z}_r(\tilde{X}))$ for some $r \in \{1, 2, ..., k\}$ satisfying $\tilde{Z}_r(\tilde{X}^\circ) \succ \tilde{Z}_r(\tilde{X})$ whenever $\tilde{Z}_i(\tilde{X}) \succ \tilde{Z}_i(\tilde{X}^\circ)$.

This means that $\tilde{Z}_i(\tilde{X}) - \tilde{Z}_i(\tilde{X}^\circ)$ can be made arbitrarily very large. It leads to $\sum_{i=1}^{k} \tilde{Z}_i(\tilde{X}) \succ \sum_{i=1}^{k} \tilde{Z}_i(\tilde{X}^\circ)$, that is, $\tilde{W}(\tilde{X}) \succ \tilde{W}(\tilde{X}^\circ)$ which contradicts the optimality of $\tilde{X}^\circ = \{ (x_j^\circ, y_j^\circ, t_j^\circ); j = 1,2,...m \}$ of (FFSOLP).

Thus, $\tilde{X}^\circ = \{ (x_j^\circ, y_j^\circ, t_j^\circ); j = 1,2,...m \}$ is a properly efficient solution to the problem (FFMOLP).

Hence the theorem is proved.

4. Total Objective-Segregation Method

Note 4.1 If at least one of the problems in the Bound and Decomposition method has no solution, then the FFMOLP problem has no fuzzy solution.

The above said method is illustrated by the following example.

Example 4.1 Consider the following fully fuzzy multi-objective linear programming problem:

Maximize $\tilde{Z}_1 \approx (1, 2, 3) \otimes \tilde{x}_1 \oplus (2, 4, 5) \otimes \tilde{x}_2$;

Maximize $\tilde{Z}_2 \approx (2, 3, 4) \otimes \tilde{x}_1 \oplus (3, 4, 5) \otimes \tilde{x}_2$

The proposed method namely, total objective-segregation method for finding a properly fuzzy efficient solution to the FFMOLP problem which is mainly based on FFLP problem and the ordinary simplex method.

The step involved in the total objective-segregation method is as follows:

STEP 1: Construct a FFLP problem from the given FFMOLP problem by using fuzzy arithmetic operations and SO method (Pandian [7]).

STEP 2: Find the fuzzy optimal solution to the FFLP problem obtained in step 1 by using Bound and Decomposition method (Jayalakshmi and Pandian [5]).

STEP 3: The fuzzy optimal solution obtained in step 2 to the FFLP problem yields a properly fuzzy efficient solution to FFMOLP problem and find the fuzzy objective values of FFMOLP problem by substituting the fuzzy solution (by Theorem 3.1).

subject to

$$(0, 1, 2) \otimes \tilde{x}_1 \oplus (1, 2, 3) \otimes \tilde{x}_2 \preceq (1, 10, 27);$$

$$(1, 2, 3) \otimes \tilde{x}_1 \oplus (0, 1, 2) \otimes \tilde{x}_2 \preceq (2, 11, 28);$$

$\tilde{x}_1$ and $\tilde{x}_2 \succeq 0$.

Let $\tilde{x}_1 = (x_1, y_1, t_1)$, $\tilde{x}_2 = (x_2, y_2, t_2)$ and $\tilde{Z} = (Z_1, Z_2, Z_3)$ be a triangular fuzzy number, then the above FFMOLP problem can be written as:

Maximize

$\tilde{Z}_1 \approx (1, 2, 3) \otimes (x_1, y_1, t_1) \oplus (2, 4, 5) \otimes (x_2, y_2, t_2)$

Maximize

$\tilde{Z}_2 \approx (2, 3, 4) \otimes (x_1, y_1, t_1) \oplus (3, 4, 5) \otimes (x_2, y_2, t_2)$

subject to

$$(0, 1, 2) \otimes (x_1, y_1, t_1) \oplus (1, 2, 3) \otimes (x_1, y_1, t_1) \preceq (1, 10, 27)$$

$$(1, 2, 3) \otimes (x_1, y_1, t_1) \oplus (0, 1, 2) \otimes (x_2, y_2, t_2) \preceq (2, 11, 28)$$

$x_1, x_2, y_1, y_2, t_1, t_2 \geq 0$.

Now, by using fuzzy arithmetic operations on the triangular fuzzy numbers and SO method, the FFLP problem is obtained from the above FFMOLP problem as:

Maximize

$$\tilde{Z} \approx (3x_1 + 5x_2, 5y_1 + 8y_2, 7t_1 + 10t_2)$$

subject to

$(0x_1 + x_2, y_1 + 2y_2, 2t_1 + 3t_2) \preceq (1, 10, 27)$;

$(x_1 + 0x_2, 2y_1 + y_2, 3t_1 + 2t_2) \preceq (2, 11, 28)$;

$x_1, x_2, y_1, y_2, t_1, t_2 \geq 0$.

Now, by bound-decomposition method, the optimal solution to FFLP problem is $\tilde{x}_1 \approx (2, 4, 6)$, $\tilde{x}_2 \approx (1, 3, 5)$ and the maximum value of $\tilde{Z} \approx (11, 44, 92)$.

Thus, ($x_1 = 2$; $x_2 = 1$; $y_1 = 4$; $y_2 = 3$; $t_1 = 6$; $t_2 = 5$) is a properly fuzzy efficient solution to the given FFMOLP problem and Maximum $\tilde{Z}_1 \approx (4, 20, 43)$;
Maximum $\tilde{Z}_2 \approx (7, 24, 49)$.

5. Conclusion

The present work described the application of total objective-segregation method to find a properly fuzzy efficient solution to FFMOLP problem by using LP technique. In this new method, fuzzy ranking functions are not used and there was no restriction on the elements of the fuzzy cost matrix and fuzzy coefficient matrix. Also, the computation of the solution using this method is very easy and simple because of the LP technique. Due to these advantages the proposed new method will be an efficient mathematical tool to solve FFMOLP. In addition, FFMOLP problems can be solved using the existing LP solver.

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