A Study on Path Related Divisor Cordial Graphs

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ABSTRACT
A divisor cordial labeling of a graph G with vertex set V is a bijection from V to {1,2,……,|V|} such that if each edge uv is assigned the label 1 if f(u)/f(v) or f(v)/f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

A graph G is a finite non –empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e=\{u,v\} of vertices in E is called an edge or a line of G in which e is said to join u and v. We write e=uv and say that u and v are adjacent vertices (sometimes denoted as u adj v); vertex u and the edge e are incident with each other, as are v and e. If two distinct edges e_1 and e_2 are incident with a common vertex, then they are called adjacent edges. A graph with p vertices and q edges is called a (p,q) - graph. By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G is denoted by V(G) and E(G) respectively. For graph theoretic terminology we follow [1,2].

Definition: 1.1
Let G be a graph and we define the concept of divisor cordial labeling as follows:
A divisor cordial labeling of a graph G with vertex set V is a bijection from V to \{1,2,……,|V|\} such that if each edge uv is assigned the label 1 if f(u)/f(v) or f(v)/f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

Definition: 1.2
\(S_p(p_m,k_1,n)\) is a graph in which the star \(k_{1,n}\) is attached at one end of the path \(p_m\).

Definition: 1.3
\(H_m \Theta k_{1,n}\) is a graph obtained from a H – graph \(H_m\) by attaching \(k_{1,n}\) at pendant vertex of \(H_m\).

2. Main results
THEOREM 2.1:
Spider \(S_p(p_2,k_{1,n})\) is a divisor cordial graph.

PROOF:
Let V \(S_p(p_2,k_{1,n}) = [u_1,u_2,(v_i : 1 \leq i \leq n)]\)
Let E \(S_p(p_2,k_{1,n}) = [(u_1u_2) \cup \{(u_2v_i) : 1 \leq i \leq n\}]\)
The vertex labeling are defined by
\(f: V(G) \rightarrow \{1,2,........,n+2\}\)
\(f(u_3) = 1\)
\(f(u_2) = 2\)
\( f(v_i) = i + 2 \); \( 1 \leq i \leq n \)

The induced edge labeling are

\[
f(u_1u_2) = 1
\]

\[
f(u_2v_i) = \begin{cases} 
0 & \text{if } i \equiv 1 \mod 2 \\
1 & \text{if } i \equiv 0 \mod 2
\end{cases}
\]

Where \( 1 \leq i \leq n \)

\( e_f(0) = e_f(1) \); if \( n \equiv 1 \mod 2 \)

\( e_f(0) = e_f(1) - 1 \); if \( n \equiv 0 \mod 2 \)

Clearly, it satisfies the condition \( |e_f(0) - e_f(1)| \leq 1 \)

Hence, the induced edge labeling shows that,

\( S_p(p_2,k_1,6) \) is a divisor cordial graph.

For example,

\( S_p(p_2,k_1,5) \) is a divisor cordial graph as shown in the figure 2.2

\( S_p(p_2,k_1,6) \) is a divisor cordial graph as shown in the figure 2.3

\[
\text{Figure 2.3 : } S_p(p_2,k_1,6)
\]

**THEOREM 2.4:**

Spider \( S_p(p_3,k_1,n) \) is a divisor cordial graph.

**PROOF:**

Let \( V[S_p(p_3,k_1,n)] = [u_1,u_2,u_3,(v_i : 1 \leq i \leq n)] \)

Let \( E[S_p(p_2,k_1,n)] = [(u_1u_2) \cup (u_2u_3) \cup \{(u_2v_i) : 1 \leq i \leq n \}] \)

The vertex labeling are defined by

\[
f : V(G) \rightarrow \{1, 2, \ldots \ldots \ldots \ldots \ldots n + 3 \}
\]

\[
f(u_1) = 1
\]

\[
f(u_2) = 3
\]

\[
f(u_3) = 2
\]

\[
f(v_i) = i + 3 \); \( 1 \leq i \leq n \)

The induced edge labeling are

\[
f(u_1u_2) = 1
\]
\[ f(u_2 u_3) = 0 \]

\[ f(u_3 v_i) = \begin{cases} 
0 & i \equiv 1 \mod 2 \\
1 & i \equiv 0 \mod 2 
\end{cases} : 1 \leq i \leq n \]

\[ e_f(0) = e_f(1) \]

\[ e_f(0) = e_f(1) - 1 ; \text{ if } n \equiv 1 \mod 2 \]

Clearly, it satisfies the condition

\[ |e_f(0) - e_f(1)| \leq 1 \]

Hence, the induced edge labeling shows that,

\[ S_p(p_3, k_{1,n}) \] is a divisor cordial graph. For example, \( S_p(p_3, k_{1,4}) \) is a divisor cordial graph as shown in figure 2.5

Figure 2.5 : \( S_p(p_3, k_{1,4}) \)

\( S_p(p_3, k_{1,5}) \) is a divisor cordial graph as shown in figure 2.6

Figure 2.6 : \( S_p(p_3, k_{1,5}) \)

**THEOREM 2.7**:

Spider \( S_p(p_4, k_{1,n}) \) is a divisor cordial graph.

**PROOF**:

Let \( V[S_p(p_4, k_{1,n})] = [u_1, u_2, u_3, u_4, (v_i : 1 \leq i \leq n)] \)

Let \( E[S_p(p_4, k_{1,n})] = [(u_1 u_2) \cup (u_2 u_3) \cup ((u_4 v_i) : 1 \leq i \leq n)] \)

The vertex labeling are given by

\[ f : V(G) \rightarrow \{1, 2, \ldots \ldots \ldots \ldots n + 4 \} \]

\[ f(u_1) = 1 \]
\[ f(u_2) = 3 \]
\[ f(u_3) = 4 \]
\[ f(u_4) = 2 \]
\[ f(v_i) = i + 4 : 1 \leq i \leq n \]

The induced edge labeling are

\[ f(u_1 u_2) = f(u_3 u_4) = 1 ; \]
\[ f(u_2 u_3) = 0 ; \]
\[ f(u_4v_i) = \begin{cases} 
0 & \text{if } i \equiv 1 \mod 2 \\
1 & \text{if } i \equiv 0 \mod 2 
\end{cases} \]

\[ e_f(0) = e_f(1) ; \text{ if } n \equiv 0 \mod 2 \]
\[ e_f(0) = e_f(1) - 1 ; \text{ if } n \equiv 1 \mod 2 \]

Clearly, it satisfies the condition \[ |e_f(0) - e_f(1)| \leq 1 \]

Hence, the induced edge labeling shows that, \( S_p(p_4,k_{1,n}) \) is a divisor cordial graph.

For example, \( S_p(p_4,k_{1,8}) \) is a divisor cordial graph as shown in figure 2.8.

\[ f(u_4v_1) = 2i + 1 \\
f(u_3v_1) = 2n + 1 + 2i \\
f(u_4v_i) = 2(i + 1) \\
f(u_6v_i) = 2n + 3 + 2i \\
f(u_1) = 1 \]

Figure 2.9 : \( S_p(p_4,k_{1,7}) \)

THEOREM 2.10 :

\( H_3 \Theta K_{1,n} \) is a divisor cordial graph.

PROOF:

Let \( V(G) = \{(u_{1i},u_{3i},u_{4i},u_{6i}) ; 1 \leq i \leq n \} \cup \{u_i ; 1 \leq i \leq 6 \} \)

Let \( E(G) = \{(u_1u_{1i}),(u_3u_{3i}),(u_4u_{4i}),(u_6u_{6i}) ; 1 \leq i \leq n \} \)

\[ [(u_1u_2),(u_2u_3),(u_4u_5),(u_5u_6),(u_2u_5)] \]

The vertex labeling are

\[ f(u_{1i}) = 2i + 1 \\
f(u_{3i}) = 2n + 1 + 2i \\
f(u_{4i}) = 2(i + 1) \\
f(u_{6i}) = 2n + 3 + 2i \\
f(u_1) = 1 \]
Clearly, it satisfies the condition
\[ e_f(1) = e_f(0) + 1 \]

Hence, the induced edge labeling shows that,

\[ H_3 \Theta K_{1,n} \] is a divisor cordial graph as shown in Figure 2.11
REFERENCE