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Research Paper

A Study on Path Related Divisor Cordial Graphs

Mathematics

KEYWORDS : Divisor cordial labeling,Divisor cordial graph

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ABSTRACT

A divisor cordial labeling of a graph G with vertex set V is a bijection from V to {1,2,....,|V|} such that if each edge uv is assigned the label 1 if f(u) / f(v) or f(v)/f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.
A graph which admits divisor cordial labeling is the divisor cordial graph.
In this paper ,it is proved that $S_p(p_2, k_{1,n})$, $S_p(p_3,k_{1},_n)$,$S_p(p_4,k_1,n)$, $H\_{}_3\theta K\_{}_{(1,n)}$ are divisor cordial graphs.

A graph G is a finite non –empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges.Each pair e={u,v} of vertices in E is called an edge or a line of G in which e is said to join u and v.We write e=uv and say that u and v are adjacent vertices (sometimes denoted as u adj v);vertex u and the edge e are incident with each other,as are v and e.If two distinct edges e_1 and e_2 are incident with a common vertex,then they are called adjacent edges.A graph with p vertices and q edges is called (p,q)-graph. By a graph ,we mean a finite simple and undirected graph.The vertex set and edge set of a graph G denoted by V(G) and E(G) respectively.For graph theoretic terminology we follow [1,2].

Definition:1.1

Let G be a graph and we define the concept of divisor cordial labeling as follows:

A divisor cordial labeling of a graph G with vertex set V is a bijection from V to {1,2,....,|V|} such that if each edge uv is assigned the label 1 if f(u)/f(v) or f(v)/f(u) and 0 otherwise, then the number of edges labeled with 0 with 1 differ by atmost 1.

A graph which admits divisor cordial labeling is the divisor cordial graph.

Definition:1.2

$S_p\left(p_m, k_{1,n}\right)$ is a graph in which the star $k_{1,n}$ is attached at one end of the path p_m.

Definition: 1.3

$H_m \ominus k_{1,n}$ is a graph obtained from a H – graph H_m by attaching $k_{1,n}$ at pendant vertex of H_m .

2.Main results

THEOREM 2.1 :

Spider $S_p\left(p_2, k_{1,n}\right)$ is a divisor cordial graph.

PROOF :

Let $V\left[S_p\left(p_2, k_{1,n}\right)\right] =$

$$[u_1, u_2, (v_i \ : \ 1 \leq i \leq n)]$$

Let $E\left[S_p\left(p_2, k_{1,n}\right)\right] =$

$$[(u_1 u_2) \cup \{(u_2 v_i) : 1 \leq i \leq n\}]$$

The vertex labeling are defined by

$$f: V(G) \to \{1, 2, \dots\dots\dots. n+2\}$$

$$f(u_1) = 1$$

$$f(u_2) = 2$$

$f(v_i) = i + 2 \; ; \; 1 \leq i \leq n$

The induced edge labeling are

$f(u_1 u_2) = 1$

$$f(u_2 v_i) = \begin{cases} 0 \; if \; i \equiv 1 \; mod \; 2 \\ 1 \; if \; i \equiv 0 \; mod \; 2 \end{cases}$$

Where $1 \leq i \leq n$

$e_f(0) = e_f(1) \quad ; \quad if \; n \equiv 1 \; mod \; 2$

$e_f(0) = e_f(1) - 1 \quad ; \quad if \; n \equiv 0 \; mod \; 2$

Clearly , it satsisfies the condition

$|e_f(0) - e_f(1)| \leq 1$

Hence , the induced edge labeling shows that ,

$S_p\left(p_2, k_{1,n}\right)$ is a divisor cordial graph.

For example ,

$S_p\left(p_2, k_{1,5}\right)$ is a divisor cordial graph as shown in the figure 2.2

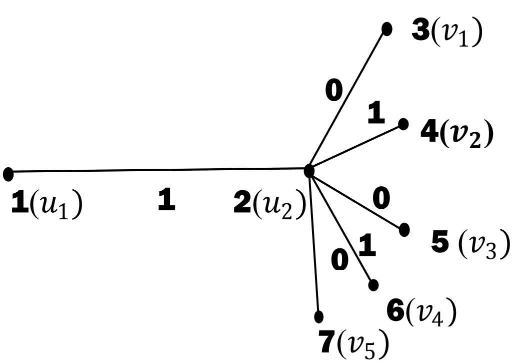

Figure 2.2 : $S_p\left(p_2, k_{1,5}\right)$

$S_p\left(p_2, k_{1,6}\right)$ is a divisor cordial graph as shown in the figure 2.3

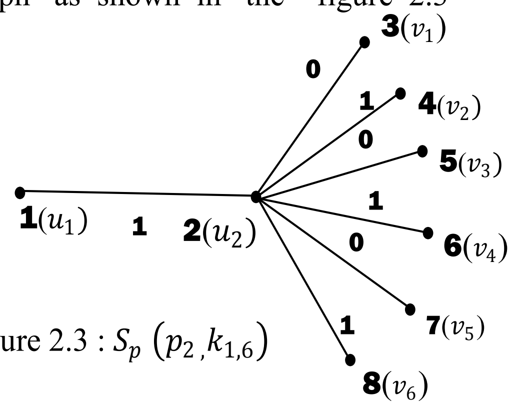

Figure 2.3 : $S_p\left(p_2, k_{1,6}\right)$

THEOREM 2.4 :

Spider $S_p\left(p_3, k_{1,n}\right)$ is a divisor cordial graph.

PROOF:

Let $V\left[S_p\left(p_3, k_{1,n}\right)\right] = [u_1, u_2, u_3, (v_i \; : \; 1 \leq i \leq n)]$

Let $E\left[S_p\left(p_2, k_{1,n}\right)\right] = [(u_1 u_2) \cup (u_2 u_3) \cup \{(u_2 v_i) : 1 \leq i \leq n\}]$

The vertex labeling are defined by

$f : V(G) \rightarrow \{1, 2, \ldots\ldots\ldots\ldots\ldots.. n+3\}$

$f(u_1) = 1$

$f(u_2) = 3$

$f(u_3) = 2$

$f(v_i) = i + 3 \; ; \; 1 \leq i \leq n$

The induced edge labeling are

$f(u_1 u_2) = 1$

$f(u_2 u_3) = 0$

$$f(u_3 v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases}$$

$: \; 1 \leq i \leq n$

$e_f(0) = e_f(1)$

$; \; if \; n \equiv 0 \bmod 2$

$e_f(0) = e_f(1) - 1 \; ; \; if \; n \equiv 1 \bmod 2$

Clearly, it satisfies the condition

$|e_f(0) - e_f(1)| \leq 1$

Hence , the induced edge labeling shows that,

$S_p\ (p_3, k_{1,n})$ is a divisor cordial graph. For example, $S_p\ (p_3, k_{1,4})$ is a divisor cordial graph as shown in figure 2.5

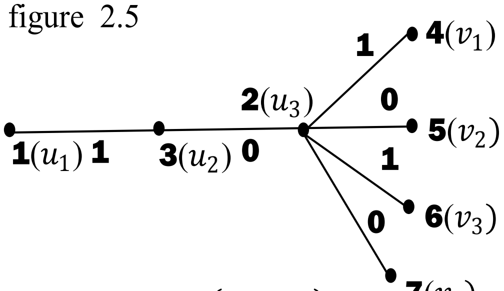

Figure 2.5 : $S_p\ (p_3, k_{1,4})$

$S_p\ (p_3, k_{1,5})$ is a divisor cordial graph as shown in figure 2.6

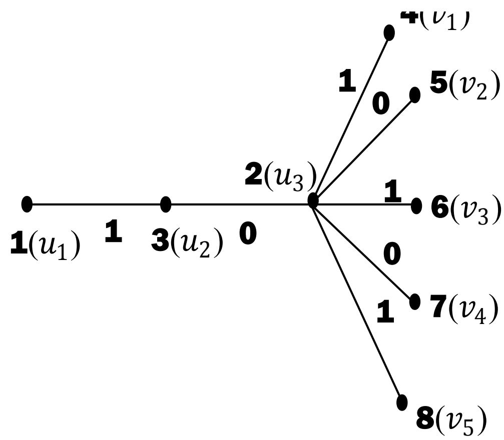

Figure 2.6 : $S_p\ (p_3, k_{1,5})$

<u>THEOREM 2.7 :</u>

Spider $S_p\ (p_4, k_{1,n})$ is a divisor cordial graph.

<u>PROOF:</u>

Let $V[S_p\ (p_4, k_{1,n})] =$

$[u_1, u_2, u_3, u_4, (v_i : 1 \leq i \leq n)]$

Let $E[S_p\ (p_4, k_{1,n})] = [(u_1 u_2) \cup (u_2 u_3) \cup \{(u_4 v_i) : 1 \leq i \leq n\}]$

The vertex labeling are given by

$f : V(G) \to \{1, 2, \ldots \ldots \ldots \ldots . n+4\}$

$f(u_1) = 1$

$f(u_2) = 3$

$f(u_3) = 4$

$f(u_4) = 2$

$f(v_i) = i + 4 \quad : \quad 1 \leq i \leq n$

The induced edge labeling are

$f(u_1 u_2) = f(u_3 u_4) = 1 \; ;$

$f(u_2 u_3) = 0 \; ;$

$$f(u_4 v_i) = \begin{cases} 0 \ \ if \ \ i \equiv 1 \ mod \ 2 \\ 1 \ \ if \ \ i \equiv 0 \ mod \ 2 \end{cases}$$

$$e_f(0) = e_f(1) \quad ; \quad if \ n \equiv 0 \ mod \ 2$$

$$e_f(0) = e_f(1) - 1 \quad ; \quad if \quad n \equiv 1 \ mod \ 2$$

Clearly, it satisfies the condition

$$|e_f(0) - e_f(1)| \leq 1$$

Hence ,the induced edge labeling shows that ,

$S_p\,(p_{4}, k_{1,n})$ is a divisor cordial graph.

For example ,

$S_p\,(p_{4}, k_{1,8})$ is a divisor cordial graph as shown in figure 2.8

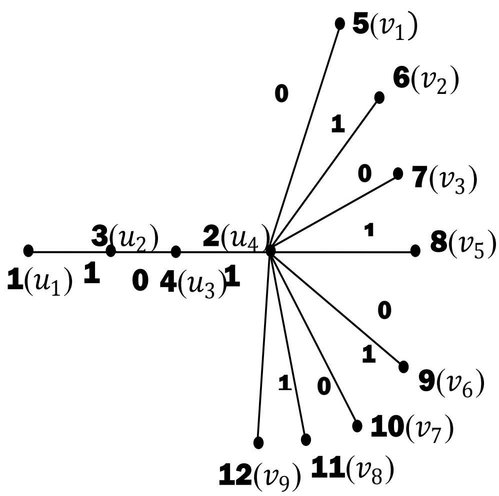

Figure 2.8 : $S_p\,(p_{4}, k_{1,8})$

$S_p\,(p_{4}, k_{1,7})$ is a divisor cardial graph as shown in figure 2.9

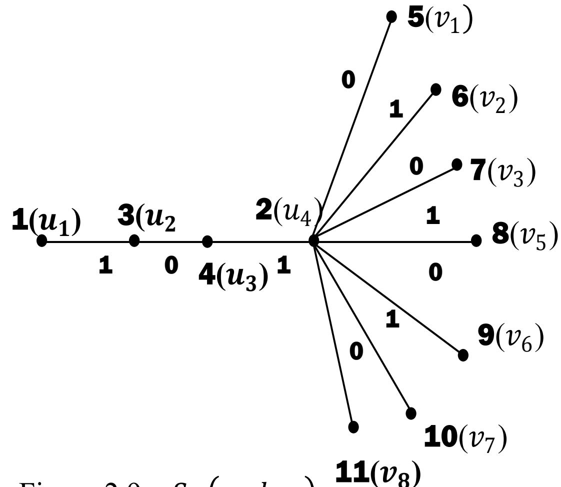

Figure 2.9 : $S_p\,(p_{4}, k_{1,7})$

THEOREM 2.10 :

$H_3 \Theta\, K_{1,n}$ is a divisor cordial graph.

PROOF:

Let $V(G) = \left\{ \begin{matrix} (u_{1i}, u_{3i}, u_{4i}, u_{6i}): \\ 1 \leq i \leq n \end{matrix} \right\} \cup \{u_i : 1 \leq i \leq 6\}$

Let $E(G) =$

$$\left\{ \begin{matrix} (u_1 u_{1i}), (u_3 u_{3i}), (u_4 u_{4i}), (u_6 u_{6i}) \\ : 1 \leq i \leq n \end{matrix} \right\}$$

$$[(u_1 u_2), (u_2 u_3), (u_4 u_5), (u_5 u_6), (u_2 u_5)]$$

The vertex labeling are

$$f(u_{1i}) = 2i + 1$$

$$f(u_{3i}) = 2n + 1 + 2i$$

$$f(u_{4i}) = 2(i + 1)$$

$$f(u_{6i}) = 2n + 3 + 2i$$

$$f(u_1) = 1$$

$f(u_2) = 4n + 5$

$f(u_3) = f(u_2) - 2$

$f(u_4) = 2$

$f(u_5) = 4n + 6$

$f(u_6) = f(u_5) - 2$

The induced edge labeling are

$f(u_{1i}) = 1 : 1 \leq i \leq n$

$f(u_{4i}) = 1 : 1 \leq i \leq n$

$f(u_1 u_2) = 1$

$f(u_4 u_5) = 1$

$f(u_{3i}) = 0 \quad : 1 \leq i \leq n$

$f(u_{6i}) = 0 \quad : 1 \leq i \leq n$

$f(u_2 u_5) = 0$

$f(u_2 u_3) = 0$

$f(u_5 u_6) = 0$

$e_f(1) = e_f(0) + 1$

Clearly, it satisfies the condition

$|e_f(0) - e_f(1)| \leq 1$

Hence, the induced edge labeling shows that,

$H_3 \Theta K_{1,n}$ is a divisor cordial graph

$H_3 \Theta K_{1,3}$ is a divisor cordial graph as shown in figure 2.11

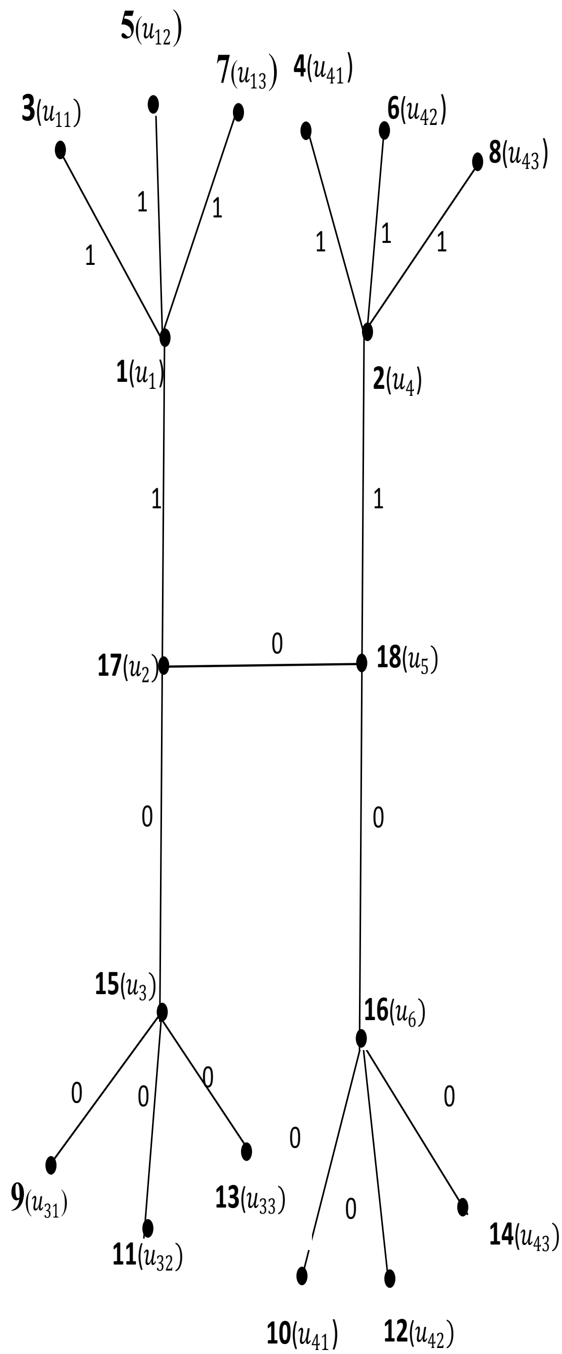

Figure 2.11 : $H_3 \Theta K_{1,3}$

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