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Super Resolution Imaging – Wavelet based Interpolation using Hidden Markov Tree Model and Image Restoration by Cycle Spinning

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ABSTRACT

High resolution images are desirable in many applications such as medical diagnosis, remote sensing, monitoring and surveillance. A brief survey of spatial domain super resolution enhancement methods that create high resolution beyond diffraction limit using several low resolution images is presented. Several low resolution images of the same scene at different sub pixel shifts are taken and registered on a common reference frame. A wavelet transform based interpolation is used. The image function is expanded in terms of a sealing function at a particular resolution jo and wavelet functions at different resolutions more than jo. The computation of wavelet coefficient is made easier by employing image pyramid. As wavelet coefficients show probability distribution relation of a Markov process and the statistical behavior of image signals can be suitably modeled using hidden Markov tree. Using Markov tree property we predict wavelet coefficient at finer scales or at higher resolution. This interpolation algorithm produces sharper images. For restoration of images we use cycle spinning and we get well defined edges and textures with high clarity.

INTRODUCTION

Super resolution imaging is the process of enhancing the resolution of a digital image even beyond the diffraction limit [1,2] by the recovery of spatial frequency. This information is achieved from a single low resolution image or from a multiple low resolution images of the same object. The process of attaining super resolution is achieved in three steps.

Registration: Several low resolution images captured from the same scene [3,4] are super imposed on a common reference frame. Theses L.R. images represent different looks of the same scene. These images are sub sampled and shifted with sub pixel displacements. Since these images have arbitrary sub pixel displacements. Since these images have arbitrary sub pixel shifts, they will give entirely different details of the same scene. For registration of these images, the sub pixel shift consequent to relative motion between image capturing device and the scene has to be accurately estimated with precision. The earliest such formulation was first made by Tsai and Huang[5]. Observed images are modeled as under sampled versions of the unchanging scene undergoing global translation and rotation. If (x,y) refer to the coordinates of a pixel before relative motion and (x',y') refer to coordinates

after motion, then these are related as $x' = x\cos\theta - y\sin\theta + h$, $y' = x\sin\theta + y\cos\theta + k$, where h and k are translation along X and Y axes, θ angle of rotation about Z axis. By this motion estimation, we have done registration in spatial domain. But our algorithm is in frequency domain and two shifted images differ by a phase shift depending upon the motion parameter. We have iteratively shifted each image by a small angle and continue this iteration until we get the resultant correlation. The second step of attaining super resolution is interpolation[6].

Interpolation is the re-sampling in imaging method to increase the number of pixels in a digital image to include more image details. Interpolation transforms a discrete matrix into a continuous image and we fit the data with a continuous function and re-samples the functions at finer intervals. So in interpolation we estimate image value at a location between image pixels. The common interpolation techniques are (1) Nearest neighbour interpolation (2) Bilinear interpolation (3) Bicubic interpolation (4) Polynomial interpolation.

NEAREST NIEGHBOUR INTERPOLATION

In this interpolation method, the image value of a point is the corresponding value of the nearest pixel of the original image. If you enlarge a pixel to a (2×2) area of the pixel by this interpolation, area of new pixel increases to area of four pixels of the original image. So this interpolation is useful for enlarging a digital image.

BILINEAR INTERPOLATION

The image value of the position of the new pixel is the weighted average of the corresponding values of four pixels in the nearest 2×2neighbourhood of the pixel in the original image. So it produces relatively smoother edges.

BICUBIC INTERPOLATION

In this method, image value at a new pixel is the weighted average of the corresponding values of 16 pixels in the nearest 4×4 neighbourhood of the pixel in the original image. This interpolation produces smoother and sharper edges.

POLYNOMIAL INTERPOLATION

In polynomial interpolation, the image value at a new pixel 'x' is estimated by an interpolation function $\hat{f}(x)$ given by $\hat{f}(x) = \sum_{k=-\infty}^{\infty} c_k \beta(x - x_k)$, β(x) is the interpolation kernel, x_k the discrete spatial coordinates of the k_{th} pixel c_k – interpolation constant. Since $\hat{f}(x)$ is band limited to (-n,n), the above interpolation function is approximated as

$$\hat{f}(x) = f_{x_{k-1}}\left(\frac{-s^3+2s^2-s}{2}\right) + f(x_k)\left(\frac{3s^3-5s^2+2}{2}\right) + f_{x_{k+1}}\left(\frac{-s^3+4s^2-s}{2}\right) + f_{x_{k+2}}\left(\frac{s^3-s^2}{2}\right),$$

s being the distance of pixel x to the neighboring pixel. The sample points are used for evaluating interpolation coefficients. By this method sharper images can be obtained. All the above traditional methods of interpolation cause loss in high frequency information and introduce blurring effect. Linear interpolation imposes continuity of image function; bicubic interpolation assumes the continuity of signal and its first derivative. Polynomial interpolation has continuous higher derivatives. The limitations of the above traditional methods of interpolation are overcome in wavelet based interpolation, since it is possible to add the detail information by prediction technique. In this paper we present wavelet based interpolation using hidden Markov tree model. Final step of attaining super resolution images of high clarity to free the image from blurring effect and in our paper we present a restoration stage using cycle spinning technique.

III. WAVELET BASED INTERPOLATION – MODIFIED VERSION USING HIDDEN MARKOV TREE JUSTIFICATION OF ADOPTING WAVELET BASED INTERPOLATION

Studies on human visual system observes that spatial frequency band in human visual system is responsive approximately to one octave of spatial frequency and the central frequency of each band differs from the next neighbor by a factor of 2 and there are five to seven bands spanning the spatial frequency range of the whole visual system. There is evidence that the spatial frequency information in a particular band is processed independently of the information in any other band. The spatial frequency information in a particular band is processed independently of the information in any other band. Since the each wavelet has finite spatial range and frequency width, collecting information in each wavelet space can be done independently of another wavelet space. Almost all traditional methods of interpolation methods are based on the assumption that there exists interpixel redundancy and interpixel correlation in all directions. Those assumptions are not entirely true and hence the results of those interpolations introduce blurring effect as a result of loss in high frequency information. In fact most of the natural images have correlation among neighbouring pixels but in specific directions. These local features which are inherent in natural images can be identified by wavelet transform.

WAVELET TRANSFORM OF AN IMAGE FUNCTION (7,8)

The image function in one dimension can be decomposed into a coarser approximation at a particular resolution and a succession of fluctuations at intermediate scales of resolutions, i.e. $\phi = \sum_k c_{j_0,k}\phi_{j_0,k} + \sum_{j_0 \le j \le J} \sum_k d_{j,k}\psi_{j,k}$

Where $c_{j_0,k} = \int f(x)\ \phi_{j_0,k}\,\mathrm{d}x$ and $d_{j,k} = \int f(x)\ \psi_{j,k}\ \mathrm{d}x$

Φ is the called scaling function and ψ called wavelet, $\phi_{j_0,k}$ and $\psi_{j,k}$ are generated from translation and dilations of ψ. The expansion functions are composed of integer translations and binary scaling of the real square integrable function Φ(x) i.e. $\phi_{j_0,k}$ (x) =$2^{j_0/2}$ Φ ($2^{j_0} x - k$)

Where j_0,k ∈ 2 and Φ(x) ∈ L^2, k determines the position of $\phi_{j_0,k}$ along X axis, j_0 determines the width of $\phi_{j_0,k}$. $2^{\frac{j_0}{2}}$ control the amplitude of the function and the shape of function changes with j_0. The set of scaling function $\{\phi_{j_0,k}\}$ for all values of k span the subspace V_{j_0}. The scaling function obeys the following requirements.

The scaling function is orthogonal to its integer translates. If the scaling function has support $[0,1]$, whenever the value of the scaling function is 1, its integer translate is 0 so that the product of the two is zero.

The subspace spanned by the scaling function at low scales is nested within those spanned at higher scales. i.e. $V_{-\infty} < \ldots. < V_{-1} < V_0 < V_1 \ldots. < V_{\infty}$

If the scaling function $\phi(x) \in V_j$, then $\phi(2x) \in V_{j+1}$

The only function common to all V_j is $\phi(x)$ = 0. If we consider the coarsest possible expansion function i.e. j = -∞, the only representable function is that with no information i.e. $V_{-\infty} = \{0\}$.

Any function can be represented with arbitrary precision i.e. as resolution j $\to \infty$, $V_{\infty} = \{L^2(R)\}$.

An approximation of a signal at a resolution 2^j can be characterized by 2^j samples per unit length. When the resolution increases towards $+\infty$, the approximation signal converges towards the original signal and when the resolution decreases to zero, the approximated signal contains less and less information and should ultimately converges to zero.

Since the approximated signal at a resolution 2^j is equal to the orthogonal projection on a space V_j and as j increases the space V_j should ultimately cover almost all the initial vector space L^2(R) and conversely as j $\to 0$, it shrinks to the vector space $\{0\}$. The set of vector spaces $\{V_j\}_{j \in Z}$ satisfying the above properties is known as a multi resolution vector space sequence. The one dimensional transforms can be easily extended to two dimensions. In two dimension, a two dimensional scaling function $\phi(x, y)$ and three two dimensional wavelets $\psi^H(x, y)$, $\psi^V(x, y)$ and $\psi^D(x, y)$. Each is a product of two one dimensional functions. ψ^H corresponds to horizontal orientation, ψ^V corresponds to vertical orientation and ψ^D corresponds to diagonal representation.

$$\phi(x,y)=\phi(x)\phi(y), \quad \psi^H(x,y)=\psi(x)\phi(y)$$

$$\psi^V(x,y)=\phi(x)\psi(y), \; \psi^D(x,y)=\psi(x)\psi(y).$$

These wavelet measure intensity variations along different orientations. The scaled and translated basis functions are expressed as

$$\phi_{j_0,m,n}(x,y)=2^{\frac{j_0}{2}}\phi(2^j x-m,2^j y-n)$$

$$\psi_{j,m,n}(x,y)=2^{\frac{j}{2}}\psi^i(2^j x-m,2^j y-n)$$

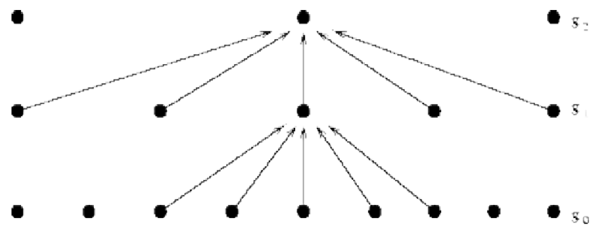
LAPLACIAN PYRAMID AS A COMPACT IMAGE CODE AND EVALUTION OF WAVELET COEFFICIENTS[9,10].

In a digital image the neighbouring pixels are highly correlated and hence representation of image in terms of the pixel value is inefficient owing to redundancy of encoded information [13]. So image pixels are decorrelated and this is achieved through predictive and transform technique. Burt [4] has described a non casual technique for removing correlation which combines features of predictive and transform methods. The predictive value for each pixel is computed as a local weighted average using a weighting function centered on the pixels. These predicted values for all pixels are obtained by convolving these weighting functions with the image.

Burt pyramid is an efficient algorithm used to separate an image into a set of contiguous spatial frequency bands. The original image which may be 512x512 pixels is filtered by applying low pass filter and this passes only half the frequency bands. Since the maximum frequency in the new image is half of the original, the image is resampled by eliminating half the samples in both directions. The new image will have 256X256 pixels. This process is called reduction operation and can be repeated. The original image denoted by go has C columns and R rows of pixels. This can be labeled as 0 level. Each pixel has a light intensity of integer value I between 0 and k-1, k being number of gray levels.

The reduction technique can be summarized as: First step is to low pass filter the original image g_0 to obtain g_1 corresponding to level 1. Each value within level 1 is computed as a weighted average of values in level 0 within a 5x5 window.

Gaussian pyramid



g_0 = Image $\quad g_1$ = Reduce (g_{L-1})

Then low pass filter g_1 to obtain image g_2 at level 2. Each value within level 2 is obtained from values within level 1 by the same pattern of weights. Level to level average process is performed by a reduce operation expressed as

$$g_l(i,j)=\sum_{m=-2}^{2}\sum_{m=-2}^{2}w(m,n)g_{l-1}(2i+m,2j+m)$$

Levels are related as 0<l<N, and C_l, R_l are number of columns and rows of l^{th} level. For all samples I and j, we have $0<i<R_l$ and $0<j<C_l$. w(m,n) is a two dimensional window function, constrained by the relations.

w(m,n)=w(m) x w(n) splitting to one dimension.

w(i)=w(-i) filter symmetry

$$\sum_{i=-2}^{2}w(i)=1 \qquad \text{normalization}$$

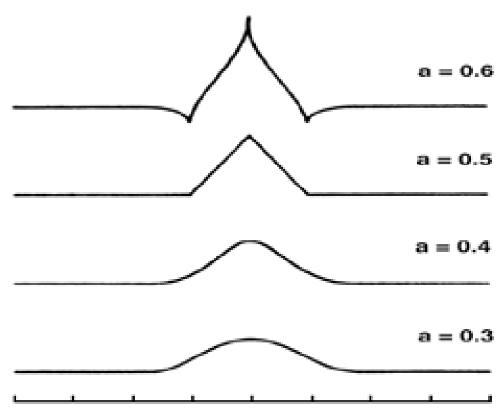
From these relations we deduce that

$$w(0)+2\sum_{i=even}w(i)=2\sum_{i=odd}w(c)$$

If w(0)=a, w(1)= b = 0.25= w(-1)

W(2)=c = 0.25-a/2= w(-2)

The shape of the equivalent weighting function converges to a characteristic form with successive higher levels of the image pyramid so that only its scale changes. The shape of the weighting function depends on the choice of w(0)= a value when a =0.6, the central positive mode is sharp peaked. When a= 0.5, the shape is triangular when a=0.4, the shape is Gaussian-like and when a=0.3, it is flatter. The shape fore sighting function is shown in figure.



The shape of equivalent weighting function.

We can define an operation(function). Expand reverse of operation(function) reduce. The effect to expand an $(\mu+1)\times(N+1)$ array into a $(2\mu+1)\times(2N+1)$ array. If $g_{l,n-1}$ is the result of expanding g_l n times.

$g_{l,n}$ = Expand ($g_{l,n-1}$)

$$g_{l,n}(i,j) = 4\sum_{m=-2}^{2}\sum_{m=-2}^{2} w(m,n)g_{l,n-1}\left(\frac{i-m}{2},\frac{j-n}{2}\right)$$

for which $\frac{i-m}{2},\frac{j-n}{2}$ are integers included.

Laplacian pyramid

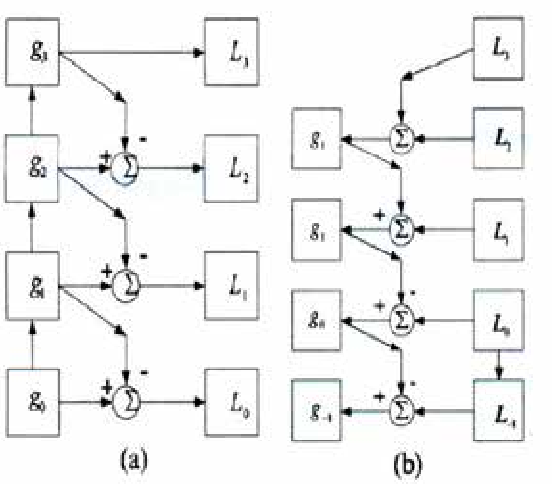
As a result Of an appropriate low pass filter we down sample image go to obtain level 1 image g_1 . Up sampling g_1 we get predicted value of g_0 . The prediction error

$L_0 = g_0$ – up sampling g_1

And similarly

$L_1 = g_1$ – up sampling g_1 .

Thus we get by repeated use of down sampling operations we get a series of two dimensional arrays $g_0,g_1,g_2......g_n$ and by correction operations also , we get a series of two dimensional arrays $L_0,L_1,L_2......L_n$ .In our implementations each is smaller than its predecessor by a scale factor of 2. The low pass filtered images $g_0,g_1,g_2......g_n$ form Gaussian pyramid and $L_0,L_1,L_2......L_n$ form laplacian pyramid. Since there is no image g_{n+1} is serve as prediction image, $L_n = g_n$. We can also have laplacian pyramid reconstruction by reversing steps in laplacian pyramid regeneration.



(a) (b)

Prediction error at K^{th} level is given by

$L_k = g_k$ – up sampling g_{k+1} .

Our aim is to get higher resolution image than g_0

ie , $g_{-1} = L_{-1}$ – up sampling g_0 .

There are different methods to get prediction error L_{-1}

Suppose the pixel value of L_{-1} = 0. Then the obtained HR image is the result of interpolation for LR image.

Suppose L_{-1} = up sampling L_0. Then g_{-1} = up sampling L_{-1} + g_0.

L_{-1} can be predicted by residual pyramid method. If we look on $L_0,L_1,L_2,....L_n$

As a new image pyramid, we get a new residual pyramid $L_0,L_1,L_2,....L_n$ through the same

method as obtaining $L_0,L_1,L_2,....L_n$ by the process of pyramid reconstruction, i.e., up sampling L_{n-1} and add it to L_{n-2} then up sampling the new image and add to L_{n-3}. Repeating we get L_{n-4}...... and finally L_{-1}.

d) Interpolation of image by hidden Markov free model[14]

Image interpolation, which we call prediction of image detail can be done by modelling the wavelet coefficients as nodes in a Markov free with a certain hidden state. In a Markov process, its transition probability depends only on its current state. On a hidden Markov free model, we do not know explicitly the state of the process. The state is associated with other probability distribution which is the one we observe. The parent of a particular nodal would be the coefficient of the next coarsest scale directly before it. Markov frame work effectively models the persistence property of wavelet since relationship between coefficients across scale is captured. The non-Gaussian distribution of coefficients is done by the mixture probability of the hidden states. The probability densities were chosen to be Gaussian with different means and variance according to the state. The result of such a mixture probability is that the eventual probability distribution of the wavelet coefficient looks like laplacian distribution.

The expectation Maximisation algorithm (EM algorithm) is used to train the Markov free model. The algorithm essentially works by finding the set of parameters which would most likely result in the set of observed wavelet coefficients. In this particular implementation the algorithm takes as input the wavelet coefficients and produces the state transition probabilities and the means and variance for each coefficient. The parameters are enough to model the hidden Markov model and we get additional information.i.e., state probabilities for wavelet coefficients. The EM algorithm works by successively iterating model parameters until a specified error is observed.

The problem of image interpolation was formulated as one of predicting the HL, LH and HH bands of an image and then taking the converse wavelet transform, resulting in a picture twice the resolution.

One method of predicting coefficient in finest band in our hidden Markov model is done by using the probity distribution relation given by

$$P(w_c^k) = \frac{1}{\sigma_{c,m}^k\sqrt{2\pi}}e^{\frac{(w_c^k-\mu_{i,m}^k)^2}{2\sigma_{c,m}^k}}$$

where w_c^k are the wavelet coefficient at the finest scale k which we try to predict. Variables $\sigma^{k}_{c,m}, \mu^{k}_{c,m}$ are obtained from the training set.

To interpolate the image, we adopt following steps Create a hidden Markov free of the image statistics of a relatively similar image. The model will be consisting of state transition probabilities state mean and variance and state probabilities. Training should be done by tying within scale. Obtain information about all sign changes occurring in the training set wavelet coefficients from parent to child in the hidden Markov model. Take the wavelet transform of the image to be interpolated. Iterate the EM algorithm for a single iteration for the Markov free found during training. Use the appropriate equation according to the method chosen and find the state the coefficients in the finest scale. Use the Gaussian probability distribution to randomly generate a value for wavelet coefficients. Use the probabilities of sign change and check the coefficients and make appropriate changes.

Use a Gaussian filter followed a sharpening mask to remove any mask. The above interpolation method is superior to other interpolation methods. The image formed by this interpolation method has clear edges delineated by smooth images.

IV. RESTORATION OF IMAGE USING CYCLE SPINNING

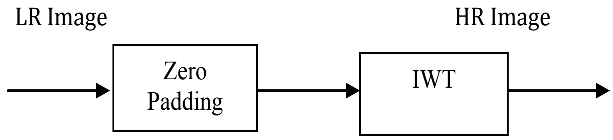
In SR image reconstruction algorithm, the mathematical model that relates the LR observation to the required HR image is given by

$Y_k = B_k x + n_k$ and $B_k = A H_k D_k$

Where A is the N*PN down sampling matrix, H_k is the PN*PN blurring matrix D_k is PN*PN warping matrix. So effects of down sampling, blurring and warping can be combined into a single N*PN system matrix B_k. x is HR image Y_k the set of LR image k can take values from 1 to L, if we have L low resolution images N is the number of pixels of the HR image x and PN number of pixels in LR image. n_t is N*1 acquisition noise. The HR image x will not obtained from LR image by a more matrix inversion process. This is because it is an ill posed problem. Regularisation is the main method of solving this kind of problem. A large number of iterations are made to make the noise part minimum and after regularisation, using inverse matrix operation we obtain HR image. the computational complexity of this SR image reconstruction is very high. So this paper we use a restoration of image based on wavelet transform and cycle spinning.

Our method consists mainly of two steps

An initial HR approximation is generated by using wavelet domain zero padding. In all wavelet super resolution reconstruction it is assumed that the LR images to be enhanced are the corresponding low pass filtered sub bands of decimated wavelet transform for HR images. So mostly the high pass filtered sub bands of a decimated wavelet transform are used for HR image and using inverse wavelet transform, we obtain HR image, a simple approach is wavelet domain zero padding (W = P). The process is shown in the following figure.



Using a given LR image of size m*n, the unknown HR image x is reconstructed by using zero padding of high frequency bands. Set all the elements of sub bands to zero followed by inverse wavelet transform. It is expressed as $\hat{X} = W^{-1} \begin{bmatrix} y & \vartheta_{mn} \\ \vartheta_{mn} & \vartheta_{mn} \end{bmatrix}$ Where ϑ_{mn} is an all zero sub band matrix of dimension m x n and W^{-1} is the inverse discrete wavelet transform.

Cycle spinning. The decimated wavelet transform is not shift invariant and as a result, a distortion of wavelet coefficient in non-exact estimation of high frequency coefficients in resolution enhancement application introduces cyclostationarity into the image and these manifests as ring in the neighbourhood of discontinuities. Cycle spinning can be used effectively to eliminate ringing and increasing the perceptual quality of the image. CS method produces approximate shift invariant statistics by averaging cycle stationarity (15, 16). Cycle spinning method is conceived in following stages

First number of LR images Y_k are generated from HR image x by special shifting. Subject to wavelet transform. Discard the high frequency coefficients $Y_{jk} = D\ W\ S_{jk}\ x$, where D represents discarding of high frequency , W denotes wavelet transform S_{jk} is the shift operator applying horizontal and vertical shift finally these intermediate HR images are re-aligned and averaged to give the final HR reconstructed image.

V. EXPERIMENTAL RESULTS AND CONCLUSIONS

In this paper, we have pointed out those three steps in obtaining HR images from several low resolution images.

Registration – All the low resolution images are aligned in the same coordinate system by the Registration phase. We compute accurately relative motion between the image capturing system and the object. Motion estimation is used to estimate the pixel position of LR image with respect to the reference frame.

Interpolation – It is the process of defining a spatially continuous image from a set of discrete sample. In this paper we presented a wavelet based interpolation method. We used Laplace pyramid structure to estimate wavelet

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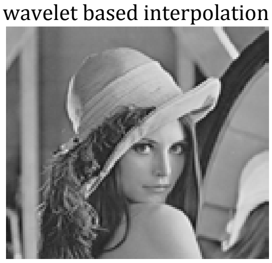
coefficient. These wavelet coefficients are modelled as nodes of hidden Markov free. Using this model, wavelet coefficients at finer scales can be predicted. A large wavelet coefficient indicates the presence of a singularity edges in general manifests themselves in the wavelet domain as chains of large coefficients propagating across scales. Also wavelet coefficients have experimentally decaying magnitudes at finer scales. This type of wavelet based interpolation produces sharper images with well-defined edges.

Restoration of image – After smoothening noise effects, using inverse operator, we restore the HR image. In our paper we make use of cycle spinning to reduce ringing effect and produce sharper and well defined images of high resolution.

After experiments using our proposed algorithm for wavelet based interpolation using hidden Markov tree model, it is seen that image is several times sharper with well defined edges. Original Leena image image using wavelet based interpolation using Markov tree model




Bilinear interpolation wavelet based interpolation using Markov tree model


A typical L.R. image algorithm using cycle spinning.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