

Noise Reduction using Low Pass Filter and Wavelet Transform



Engineering

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ABSTRACT

Wavelet transform (WT) is a powerful tool for removing noise from a variety of signals. In this experiment, wavelet transform is used along with low pass filter. The results of Wavelet transform and combination of Low pass filter with WT are compared. Error between original signal and reconstructed signal is found to be less in combination of Low pass filter with WT.

I. INTRODUCTION

In a real world data signals do not exist without noise. It must be removed from the data in order to proceed with further data analysis. There are many approaches in the literature to remove noise. This can be roughly divided into two categories: denoising in the original signal domain (e.g., time or space) and denoising in the transform domain (e.g., Fourier or WT). Traditional Fourier Transform (FT) is suitable for stationary signals. But, often times the information that cannot be easily seen in the time-domain can be seen in the frequency domain. [1] For non-stationary signals it is important to find the time-frequency characteristics of the signals and WT is particularly suitable for the applications of non-stationary signals. The wavelet transform (WT) is a powerful tool for signal and image processing. It has been successfully used in various research fields like signal processing, image compression, pattern recognition etc. Researchers are focusing on continuous wavelet transform (CWT) that gives more reliable and detailed time-scale information as compare to classical short time Fourier transform (STFT). [2]

II. why wavelet transform?

FT and WT are reversible transforms, that is, one can get back raw information from processed (transformed) signals. However, only either of them is available at any given time. That is, no frequency information is available in the time-domain signal, and no time information is available in the Fourier transformed signal. FT gives information about each frequency that exists in the signal, but it does not tell us timing of these frequency components. This information is not required when the signal is stationary signal.

Mathematically, FT equation is:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \dots (1)$$

$$\text{and } x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df \dots (2)$$

The signal $x(t)$, is multiplied with an exponential term, at some certain frequency "f", and then integrated from $-\infty$ to $+\infty$. If the signal has a same component of frequency "f", then that component term will coincide, and the product of them will give a (relatively) large value. But, if frequencies do not match then product will give zero value. So, Fourier transform is not suitable for time varying frequency, i.e. the signal is non-stationary.

The continuous wavelet transform is defined as follows:

$$CWT_x^{\psi}(\tau, s) = \Psi_s^{\psi}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \Psi^*\left(\frac{t-\tau}{s}\right) dt \dots (3)$$

Here, the transformed signal is a function of two variables, τ and s , the translation and scale parameters, respectively. $\Psi(t)$ is the transforming function, and it is called the mother wavelet. Here, if the signal has a spectral component that cor-

responds to the current value of s the product of the wavelet with the signal at the location where this spectral component exists gives a relatively large value. If the spectral component that corresponds to the current value of s is not present in the signal, the product value will be relatively small, or zero.

As shown in fig. 1(a) and 1(b) original signal contain 10, 25, 50 and 100 Hz. FT of signal is given in 1(b) which gives information about frequency components in signal. But, it is not giving any information about timing of different frequency components. While fig 1(c) WT of signal is showing not only frequency components, but also timing of different frequency components.

Fig: 1-a Signal of 50Hz along with other frequency are 100Hz, 25Hz and 10Hz

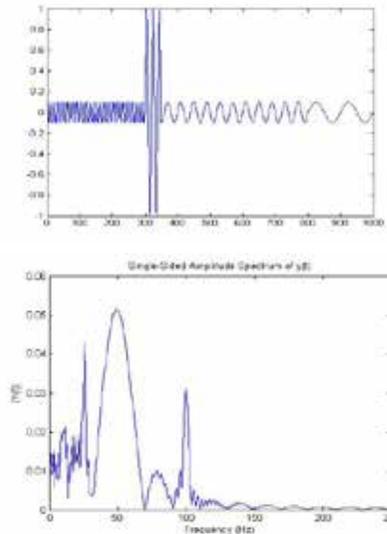


Fig: 1-b FT spectrum of signal 1(a)

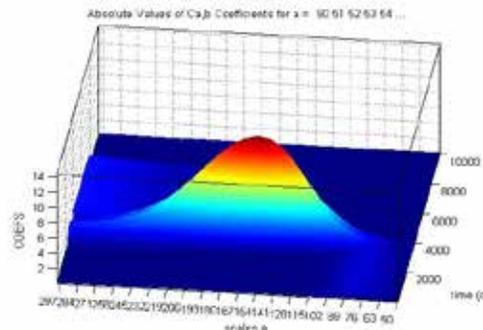


Fig. 1-c: CWT of signal 1(a)

III. Experimental setup

Programs were written in MATLAB 2011a. A standard signal fig. 1(a) was generated and Gaussian noise was introduced in it fig. 2(a). Then noisy signal was passed through first order low pass filter. Frequency spectrum of WT of filtered noisy signal and frequency spectrum of WT of noisy signal was compared with frequency spectrum of original signal for 100 cases.

IV. Results and discussion

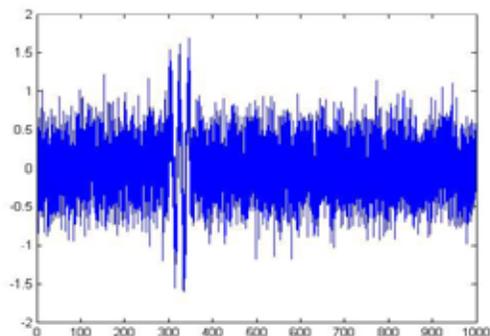


Fig. 2-a: Signal with White Gaussian noise of SNR= 10db

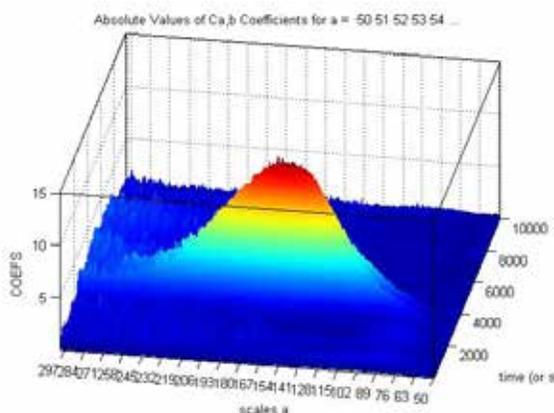


Fig. 2(b): CWT of with signal noise

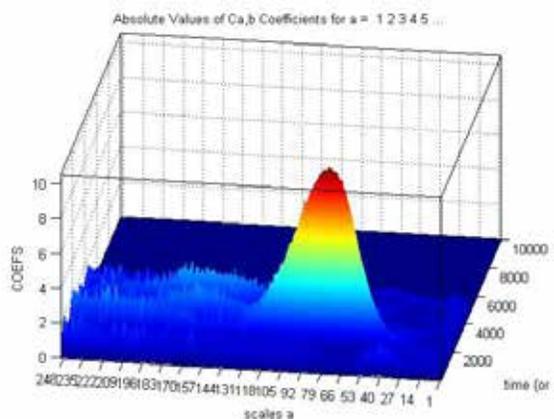


Fig. 2(c): CWT of LPF output of noise signal

Below table shows the error between the spectrum of original signal vs. spectrum of WT of noisy signal with and without LPF. Combination of WT and LPF increases signal to noise ratio i.e. it removes noise effectively as compare to WT alone.

Sr. No.	SNR in db	Error below 2%		Error below 3%	
		WT	WT +LPF	WT	WT+LPF
1	-2	0	5	0	36
2	-1	0	7	0	52
3	-0.5	0	10	0	64
4	0.5	0	14	0	74
5	1	0	28	0	87

Table: Error between spectrums with WT vs. LPF+WT.

The results of WT and WT+LPF combination are tabulated for different values of noise. Combination of WT and LPF results approaches exact values of WT of signal without noise. Here, error means the difference between spectrum of original signal and spectrum of noisy signal with and without LPF.

V. Conclusion

Above results clearly indicate that the combination of LPF and WT gives much better signal to noise ratio than WT alone.

REFERENCE

[1] Robi Polikar, The Wavelet Tutorial | [2] Burhan Ergen, Signal and Image Denoising Using Wavelet Transform pg.495 |