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Research Paper

Channel Estimation Using LS and MMSE Estimators

Engineering

KEYWORDS : OFDM; channel estimation; pilot based channel estimation

Shakuntala Chouhan

Contract Lecturer, Department of Bio-Medical Engineering, Samrat Ashok Technological Institute Vidisha(M.P.)

ABSTRACT

This paper focuses on the channel estimation techniques that are based on the block-type pilot arrangement. Once we establish reliable and practical channel estimation schemes for this type of pilot arrangement. In wireless OFDM transmission using pilot carriers and compare the bit error rate performance by using MMSE and LSE algorithms. Thus comb type pilots provide better resistance to fast fading channels. Also comb type pilot arrangement is sensitive to frequency when comparing to block type arrangement.

INTRODUCTION

Multimedia wireless services require high data-rate transmission over mobile radio channels. Orthogonal Frequency Division Multiplexing (OFDM) is widely considered as a promising choice for future wireless communications systems due to its high-data-rate transmission capability with high bandwidth efficiency. In OFDM, the entire channel is divided into many narrow sub-channels, converting a frequency-selective channel into a collection of frequency-flat channels. Moreover, inter-symbol interference (ISI) is avoided by the use of cyclic prefix (CP), which is achieved by extending an OFDM symbol with some portion of its head or tail [1]. In fact, OFDM has been adopted in digital audio broadcasting (DAB), digital video broadcasting (DVB), digital subscriber line (DSL), and wireless local area network (WLAN) standards such as the IEEE 802.11a/b/g/n [2-5]. It has also been adopted for wireless broadband access standards such as the IEEE 802.16e, and as the core technique for the fourth-generation (4G) wireless mobile Communications [6]. To eliminate the need for channel estimation and tracking, differential phase-shift keying (DPSK) can be used in OFDM systems. However, this result in a 3 dB loss in signal-to-noise ratio (SNR) compared with coherent demodulation such as phase-shift keying (PSK) [7].The performance of OFDM systems can be improved by allowing for coherent demodulation when an accurate channel estimation technique is used.

CHANNEL ESTIMATION IN OFDM SYSTEMS

Channel estimation techniques for OFDM systems can be grouped into two categories: blind and non-blind. The blind channel estimation method exploits the statistical behavior of the received signals, while the non-blind channel estimation method utilizes some or all portions of the transmitted signals, i.e., pilot tones or training sequences, which are available to the receiver to be used for the channel estimation.

BLOCK TYPE

In this case pilots are inserted in a dense fashion in a few chosen symbols (typically the preamble symbol in most standards) with the following symbols running pilot-free. The pilot-dense symbols are expected to yield excellent channel estimates which are then, typically fed in to a channel tracker, which yields estimates on the pilot-less symbols. This paradigm is often termed semi-blind method of channel estimation. Clearly, the temporal spacing *it* between two consecutive pilot-rich symbols is limited by the Doppler spread of the environment. Hence, this type of pilot arrangement is often seen in static and low-Doppler environments such as wire-line transmissions or fixed wireless communication systems [13].

COMB TYPE

Contrary to the block-type case, specific sub-carriers are chosen over which pilots are inserted in every symbol. This allows for simple 1-D (frequency domain) interpolation of the channel at the non-pilot locations and is suited for vehicular and high-Doppler environments. The frequency spacing of pilots D_f in this case limits the maximum delay spread of the channel τ_{max}. Practically, we see that an OFDM frame is peppered with pilots in a pattern which achieves a

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perfect trade-off between estimation accuracy, Doppler robustness and pilot-overhead. Schemes such as hopping pilots have been suggested to exploit maximum frequency diversity.

OFDM

OFDM is a modulation technique in that it modulates data onto equally spaced sub-carriers. The information is modulated onto the sub-carrier by varying the phase, amplitude, or both. Each sub-carrier then combined together by using the inverse fast Fourier transform to yield the time domain waveform that is to be transmitted. To obtain a high spectral efficiency the frequency response of each of the sub-carriers are overlapping and orthogonal. This orthogonality prevents interference between the sub-carriers (ICI) and is preserved even when the signal passes through a multipath channel by introducing a Cyclic Prefix, which prevents Inter-symbol Interference (ISI) on the carriers. This makes OFDM especially suited to wireless communications applications.

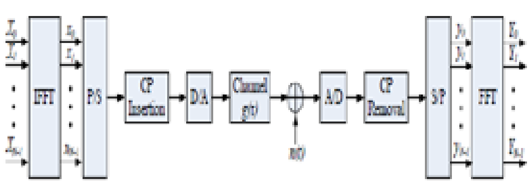

Figure - Baseband OFDM

ADVANTAGE / DISADVANTAGES

OFDM has the following advantages:

OFDM is an efficient way to deal with multipath; implementation complexity is significantly lower than single carrier with equalizer.

1. In relatively slow time-varying channels, performance can be enhanced by the adaptability of the data rate according to the SNR ratio of that sub-carrier.

2. OFDM is robust against narrowband interference, because such interference affects only a small number of sub-carriers.

3. OFDM makes single-frequency networks possible, which is especially attractive for broadcasting applications.

OFDM has the following disadvantages compared to single-carrier modulation:

1. OFDM is more sensitive to frequency offsets and phase noise.

2. OFDM has a relatively large peak-to-average power ratio, which reduces the power efficiency of the RF amplifier.

ORTHOGONALITY

Two periodic signals are orthogonal when the integral of their product over one period is equal to zero.

For the case of continuous time:

$$\int_0^T cos(2\pi n f_0 f_0 t) \cos (2\pi m f_0 t)\, dt = 0 \, , (m \neq n) \quad ...(2.1)$$

For the case of discrete time:

$$\sum_{K=0}^{N-1} \int_0^T cos(\frac{2\pi kn}{N}) \cos (\frac{2\pi km}{N})\, dt = 0 \, , (m \neq n) \quad ...(2.2)$$

To maintain orthogonality between sub-carriers, it is necessary to ensure that the symbol time contains one or more multiple cycles of each sinusoidal carrier waveform. In the case of OFDM, the sinusoids of our sub-carriers will satisfy this requirement since each is a multiple of a fundamental frequency. Orthogonality is critical since it prevents inter-carrier interference (ICI). ICI occurs when the integral of the carrier products are no longer zero over the integration period, so signal components from one sub-carrier causes interference to neighboring sub-carriers. As such, OFDM is highly sensitive to frequency dispersion caused by Doppler shifts, which results in loss of orthogonality between sub-carriers.

CHANNEL ESTIMATION TECHNIQUES

CHANNEL ESTIMATION While evaluating OFDM system performance in previous sections, we assumed perfect knowledge of the channel for equalization. While perfect channel knowledge can be used to find the upper limit of OFDM system performance, such perfect channel knowledge is not available in real-life and needs to be estimated. Channel estimation can be done in various ways: with or with the help of a parametric model, with the use of frequency and/or time correlation properties of the wireless channel, blind or pilot (training) based, adaptive or not.

Adaptive channel estimation methods are typically used for rapidly time-varying channel.

SYSTEM ENVIRONMENT

WIRELESS: The system environment we will be considering in this thesis will be wireless indoor and urban areas, where the path between transmitter and receiver is blocked by various objects and obstacles. For example, an indoor environment has walls and furniture, while the outdoor environment contains buildings and trees. This can be characterized by the impulse response in a wireless environment.

MULTIPATH FADING

Most indoor and urban areas do not have direct line of sight propagation between the transmitter and receiver. Multi-path occurs as a result of reflections and diffractions by objects of the transmitted signal in a wireless environment. These objects can be such things as buildings and trees. The reflected signals arrive with random phase offsets as each reflection follows a different path to the receiver. The signal power of the waves also decreases as the distance increases. The result is random signal fading as these reflections destructively and constructively superimpose on each other. The degree of fading will depend on the delay spread (or phase offset) and their relative signal power.

FADING EFFECT DUE TO MULTIPATH FADING

Time dispersion due to multi-path leads to either flat fading or frequency selective fading:

- Flat fading occurs when the delay is less than the symbol period and affects all frequencies equally. This type of fading changes the gain of the signal but not the spectrum. This is known as amplitude varying channels or narrowband channels, since the bandwidth of the applied signal is narrow compared to the channel bandwidth.
- Frequency selective fading occurs when the delay is larger than the symbol period. In the frequency domain, certain frequencies will have greater gain than others frequencies.

WHITE NOISE

In wireless environments, random changes in the physical environment resulting in thermal noise and unwanted interference from many other sources can cause the signal to be corrupted. Since it is not possible to take into account all of these sources, we assume that they produce a single random signal with uniform distributions across all frequencies. This is known as white noise.

CHANNEL MODEL

This section will show how the channel will become diagonalised from a cyclic convolution due to the insertion of the Cyclic Prefix. If the Cyclic Prefix is longer than the impulse response of the channel, we can show that the OFDM channel can be viewed as a set of parallel Gaussian channel (a complex gain followed by Additive White Gaussian noise) that is free of ISI and ICI.

First, let our QAM signaling symbols be expressed as

$$X = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix} \ldots (3.1)$$

After we apply the IFFT , our OFDM symbol becomes

$$X = F^H \mathrm{X} = \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} \ldots (3.2)$$

where the matrix F is the DFT matrix. For the channel impulse response

$$h = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{m-1} \end{bmatrix} \ldots (3.3)$$

To simplify our derivation, we will choose $N = 4$ sub-carriers and $m = 2$ tap impulse response, but this proof will generally apply as long as m satisfies the above condition. So after passing through the multi-path channel, the received OFDM symbol can be expressed as a convolution, h $\otimes$x. In matrix form, this becomes.

$$y = \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 \\ 0 & h_1 & h_0 & 0 \\ 0 & 0 & h_1 & h_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} \ldots (3.4)$$

If we insert the cyclic prefix before sending across the channel, this convolution becomes

$$y = \begin{bmatrix} Y_{CP} \\ Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 & 0 \\ 0 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & h_1 & h_0 & 0 \\ 0 & 0 & 0 & h_1 & h_0 \end{bmatrix} \begin{bmatrix} X_3 \\ X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} \ldots (3.5)$$

And after removing the cyclic prefix at the receiver, we can express this as

$$y = \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 \\ 0 & h_1 & h_0 & 0 \\ 0 & 0 & h_1 & h_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} \leftrightarrow Y = HX' = H_C X \ \ldots$$
(3.6)

This is equivalent to a circular convolution. The channel matrix H_C is now a circulant matrix and X' is the cyclically extended symbol.

We can now use the property of circular convolution on finite length sequences,

Y=h⊗ x ↔ Y[m] = H[m] X[m], m= (0,...,N-1) ...(3.7)

This property means every circulant matrix H_C is diagonalised by the DFT matrix F.

$$X = FY = FH_C\, X = FH_C F^H X = \begin{bmatrix} H[0] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H[N-1] \end{bmatrix} \ldots$$
(3.8)

So our multi-path fading channel can be written as:

$$y = H_C X + n \quad ..(3.9)$$

where y is the received vector of signalling points, x is the transmitted signaling points,$H_C$is the diagonalised channel attenuation vector, and n is a vector of complex, zero mean, Gaussian noise with variance σ_n^2.

The attenuation on each tone is given by

$$H[m] = G\left(\frac{m}{NT_S}\right) \quad , m = 0, \ldots, N-1 \quad \ldots$$
(3.10)

Where G(.) is the frequency response of the channel during the current OFDM symbol and T_S is the sampling period of the system.

The impulse response of the channel can be expressed as

$$g(\tau) = \sum_{m=0}^{M-1} \propto_m \delta(\tau - \tau_m T_S) \ldots (3.11)$$

Where $\propto_m$ are independent zero mean, complex Gaussian random variables, and $\tau_m$is the delay of the mth impulse. The next few sections will talk about our considerations regarding some issues on the wireless channel and the justifications for our channel model.

POWER DELAY PROFILE

The Power Delay Profile (PDP) describes the envelope of the impulse response as a function of the delay. The PDP of an urban and indoor environment is generally described by an exponential, since each delayed impulse usually has less power than the previous ones. Thus, our PDP is described by the following equation

$$\emptyset(\tau) \approx (-\frac{\tau}{\tau_{rms}}) \ldots (3.12)$$

AWGN

When the signal passes through the channel, it is corrupted by white noise. This is modeled by the addition of white Gaussian noise (AWGN). AWGN is a random process with power spectral density as follows:

$$\emptyset(f) = \frac{1}{2} N_0 [W/H_Z] \ldots (3.13)$$

Where ,N_0 is a constant and often called the noise power density.

TIME-FREQUENCY INTERPRETATION AND PILOT ALLOCATION

The OFDM transmission can also be described using a two-dimensional lattice in the time-frequency domain. Figure 3.3 shows such a time-frequency domain representation where EUTRAN OFDM symbols are transmitted sequentially. As shown in the Figure 3.3, an OFDM symbol consists of N_C sub-carriers, symmetrically arranged around the carrier frequency (or DC). Note that not only N_0 out of N_C sub-carriers are used for data

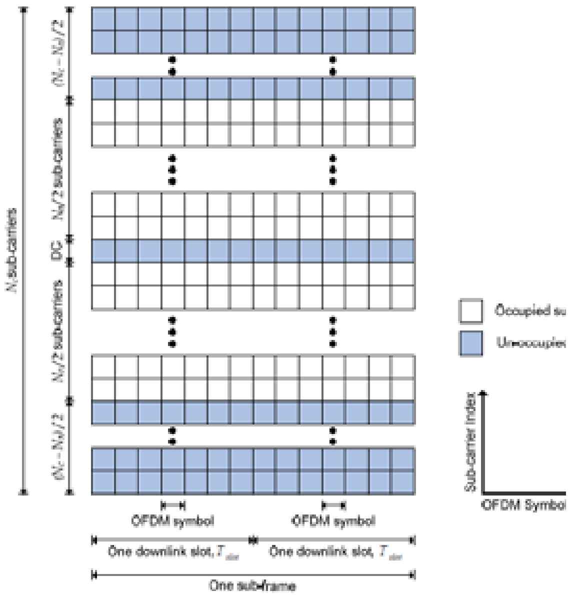

Figure-Time-Frequency lattice for OFDM transmission (generic frame structure, normal cyclic prefix)

CHANNEL ESTIMATION BASED ON BLOCK-TYPE PILOT ARRANGEMENT

In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted periodically, in which all sub-carriers are used as pilots.

The task here is to estimate the channel conditions (specified by$\overline{H}$ or$\bar{g}$) given the pilot signals (specified by matrix or vector) and received signals (specified by), with or without using certain knowledge of the channel statistics. The receiver uses the estimated channel conditions to decode the received data inside the block until the next pilot symbol arrives. The estimation can be based on least square (LS), minimum mean-square error (MMSE), and modified MMSE

If the channel is constant during the block, there will be no channel estimation error since the pilotsare sent at all carriers. The estimation can be performed by using either LSE or MMSE .If inter symbol interference is eliminated by the guard interval, we write in matrix notation:

Y=XFh+ W

= XH +W…(4.1)

Where

X=diag{X(0),X(1),…,X(N-1)}

$$Y=[Y(0, Y(1), \ldots Y(N-1)]^T$$

$$W=[W(0, W(1), \ldots W(N-1)]^T$$

$$H=[H_P(0), H_P(1), \ldots, H_P(N_P-1)]^T$$

$$F=\begin{bmatrix} TW_N^{00} & \cdots & TW_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ TW_N^{(N-1)0} & \cdots & TW_N^{(N-1)(N-1)} \end{bmatrix}$$

And $TW_N^{nk} = \frac{1}{\sqrt{N}} e^{-j2\pi ik/N}$is called as the twidle factor matrix.

LS ESTIMATOR

The LS estimator minimizes the parameter $(\overline{Y} - \underline{X}\overline{H})^H(\overline{Y} - \underline{X}\overline{H})$,where $(.)^H$ means the conjugate transpose operation. It is shown that the LS estimator of $\overline{H}$is given by .

$$\hat{H}_{LS} = \underline{X}^{-1}\overline{Y} = [(X_k/Y_k)]^T \qquad (k=0,\ldots,N-1)$$
… (4.2)

Without using any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity, but they suffer from a high mean-square error.

MMSE ESTIMATOR

The minimum mean-square error is widely used in the OFDM channel estimation since it is optimum in terms of mean square error (MSE) in the presence of AWGN . In fact, it is observed in that many channel estimation

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techniques are indeed a subset of MMSE channel estimation techniqueThe MMSE estimator employs the second-order statistics of the channel conditions to minimize the mean-square error.

Denote by $\underline{R}_{gg}$, $\underline{R}_{HH}$ and $\underline{R}_{YY}$ the autocovariance matrix of $\overline{g}, \overline{H}$ and $\overline{Y}$,respectively, and by $\underline{R}_{gy}$ the cross covariance matrix between $\overline{g}$ and $\overline{Y}$ .Also denote by σ_N^2 the noise variance $E\{\overline{N}^2\}$. Assume the channel vecto $\overline{g}$ and the noise $\overline{N}$

$$\underline{R}_{HH} = E\overline{H}\overline{H}^H = E\{(\underline{F}\,\overline{g})(\underline{F}\,\overline{g})^H\} = \underline{F}\underline{R}_{gg}\underline{F}^H \quad ...(4.3)$$

$$\underline{R}_{gY} = E\{\overline{g}\overline{Y}^H\} = E\{\overline{g}(\underline{X}\underline{F}\overline{g} + \overline{N})^H\} = \underline{R}_{gg}\underline{F}^H\underline{X}^H$$

...(4.4)

$$\underline{R}_{YY} = E\{\overline{Y}\overline{Y}^H\} = \underline{X}\underline{F}\underline{R}_{gg}\underline{F}^H\underline{X}^H + \sigma_N^2 I_N$$

...(4.5)

Assume $\underline{R}_{gg}$ and σ_N^2 are known at the receiver in advance, the MMSE estimator of $\overline{g}$ is given by $\hat{g}_{MMSE} = \underline{R}_{gy}\underline{R}_{YY}^{-1}\overline{Y}^{HH}$ Note that if $\overline{g}$ is not Gaussian $\hat{g}_{MMSE}$ is not necessarily a minimum mean-square error estimator, but it is still the best linear estimator in the mean-square error sense. At last, it is calculated that

$$\hat{H}_{MMSE} = \underline{F}\hat{g}_{MMSE} = \underline{F}[(\underline{F}^H\underline{X}^H)^{-1}\underline{R}_{gg}^{-1}\sigma_N^2 + \underline{X}\underline{F}]^{-1}\overline{Y}$$

...(4.6)

$$= \underline{F}\underline{R}_{gg}[(\underline{F}^H\underline{X}^H\underline{X}\underline{F})^{-1}\sigma_N^2 + \underline{R}_{gg}]\underline{F}^{-1}\hat{H}_{LS}$$

...(4.7)

$$= \underline{R}_{HH}[\underline{R}_{HH} + \sigma_N^2(\underline{X}\underline{X}^H)^{-1}]^{-1}\hat{H}_{LS}$$

...(4.8)

The MMSE estimator yields much better performance than LS estimators, especially under complexity. Extensive computer simulations have been conducted to demonstrate the performance and complexity of the proposed channel estimation. The comparisons are investigated by computer simulations using MATLAB 7.8.

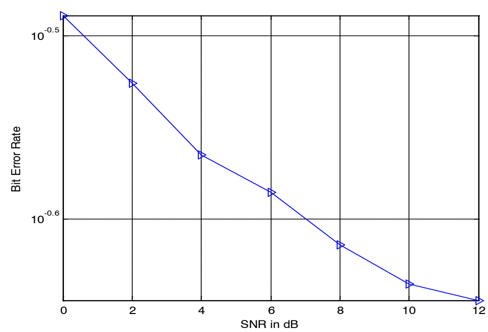
SIMULATION RESULTS

LS ESTIMATION

The fundamental requirements and the parameters of the OFDM systems investigated for simulation are shown in Table1.

SIMULATION PARAMETER

PARAMETER	SPECIFICATION
FFT size	N=64,128,256,512
Cyclic Prefix	N/4
Pilot interval	8
Modulation	2
Modulation types	M-QAM, M-PSK
Channel length	16
Number of iteration	500
System bandwidth	10 MHz



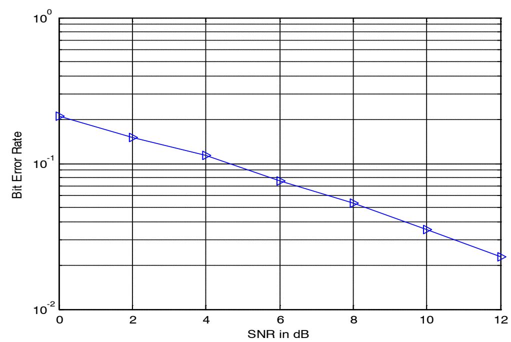


Figure (a)BER v/s SNR for different value of SNR for total number of sub channel 64 and 128

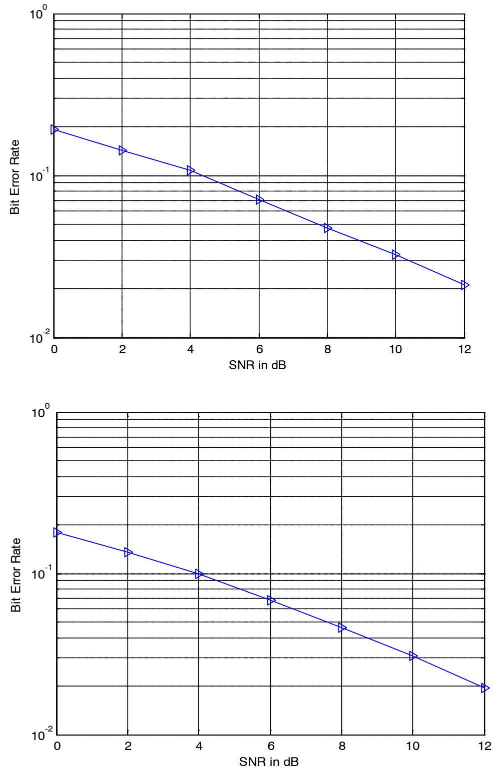


Figure (b)BER v/s SNR for different value of SNR for total number of sub channel 256 and 512

The figures showing the analysis of BER vs. SNR for the performance of LSE in OFDM channel estimation by using different sub channels. Simulation results show the performance of different estimators in terms of BER For the sake of simplicity and without loss of generality, we assume Rayleigh flat fading Figure 5.1(a) and 5.1(b) shows the BER of LS estimator As shown in above figure, increasing the number of sub-channel leads to increase the performance of LS estimator.

PERFORMANCES OF THE LS AND THE MMSE CHANNEL ESTIMATION

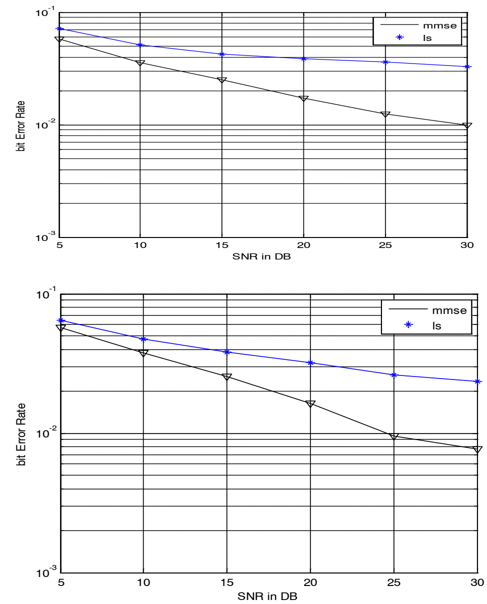


Figure (a)BER v/s SNR for different value of SNR for total number of sub channel 64 and 128

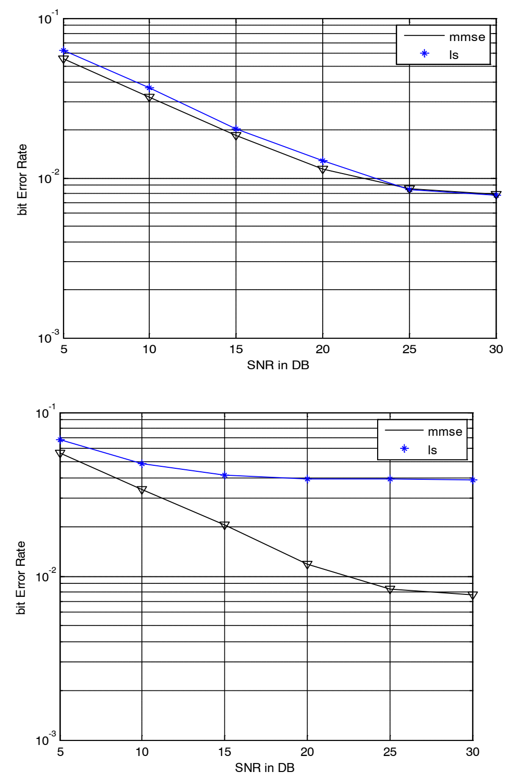


Figure (b) BER v/s SNR for different value of SNR for total number of sub channel 256 and 512

Figure (a) and (b) shows the BER performance comparison between LS and MMSE estimators. As can be observed from the figure, the MMSE performs better than the LS estimation. From the all above

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figure, it can be observed that the BER performance of MMSE estimation, but much better than LS estimation.

Compared with LS-based techniques, MMSE-based techniques yield better performance because they additionally exploit and require prior knowledge of the channel correlation and SNR. However, the channel correlation is sometimes not a priori known, which makes MMSE-based techniques infeasible.

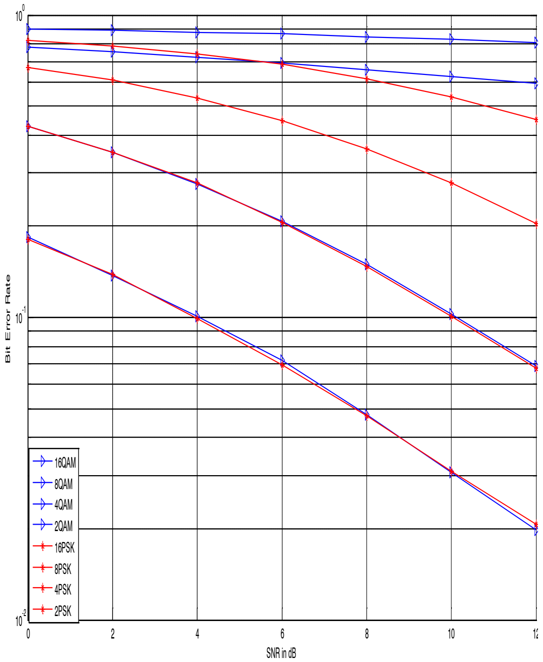

Figure (c) BER v/s SNR for different value of SNR for total number of sub channel 256 and 512 varying different value of 16 QAM, 8 QAM, 4QAM, 2QAM,16PSK,8PSK, 4PSK,2PSK.

CONCLUSION

From all the analysis it can be concluded that BER performance of MMSE is better as compare to LS. The investigation shows that for LS estimation the difference in performance is mainly due to the loss of power efficiency caused by the insertion of pilots and for this estimator the increased maximum delay of the compound channel limits the maximum pilot spacing. Also, the investigation shows that the performance of the MMSE estimator degrades considerably and it is inversely proportional to pilot density.

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