Research Paper

Cyclic Fuzzy Neutrosopic Soft Group

Mathematics

KEYWORDS : Soft set, Neutrosophic set, Fuzzy neutrosophic set, Fuzzy Neutrosophic soft group, Cyclic group, Characteristic soft group, identity fuzzy neutrosophic soft group.

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ABSTRACT

Neutrosophic soft set theory proposed by S.Broumi and F.Samarandach[14] has been regarded as an effective mathematical tool to deal with uncertainties. In this paper, we apply the concept of fuzzy neutrosophic soft set to group theory. The notion of fuzzy neutrosophic soft groups[FNSG] is introduced and their basic properties are presented. Union, intersection and difference operations of fuzzy neutrosophic soft groups are defined. Further we have defined cyclic fuzzy neutrosophic soft group[CFNSG] and studied some related properties with supporting proofs.

1.INTRODUCTION: The fuzzy set was introduced by Zadeh[19] in 1965 where each element had a degree of membership. The intuitionistic fuzzy set on a universe was introduced by K.Atanasov[1] in 1983, as a generalization of fuzzy set, where besides the degree of membership and the degree of non membership of each element. The concept of Neutrosophic set was introduced by F.Samarandach[14] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The theory of Neutrosophic set which is the generalization of the classical sets, conventional fuzzy set[19], intuitionistic fuzzy set[1] and interval valued fuzzy set[16] was introduced by F.Samarandach[14] .This concept has recently motivated new approach in several directions such as databases,medical diagnoses problems,decision making problem, topology, control theory and on so on. The concept of neutrosophic set handles indeterminate data where as fuzzy theory and intuitionistic fuzzy set theory failed when the relations are indeterminate. The concept of cyclic fuzzy set discussed in [12].Our objective is to introduce the concept of cyclic fuzzy neutrosophic soft groups[CFNSG] and its properties. The remaining part of this paper is organized as follows. Section 2 contains basic definitions and notations that are used in the remaining parts of the paper. We investigated main results of fuzzy neutrosophic soft groups and some operations in Section 3. In section 4, we present fuzzy characteristic neutrosophic soft groups and its

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properties. Cyclic fuzzy neutrosophic soft groups and its characterization are studied in Section-5.Finally, conclusion is made in section 6.

2 Preliminaries

2.1 Definition:[19] Let U be a non-empty set. Then by a fuzzy set on U is meant a function A : U → [0,1].A is called the membership function, A(x) is called the membership grade of x in A. We also write A= {(x, A(x)): x∈U}

Example: Consider U = { a, b, c ,d } and A : U → [0,1] defined by A(a)=0, A(b)=0.7, A(c)=0.4, A(d)=1.

2.2 Definition:[8] Let U be the initial universe set and E be the set of parameters. Let P(U) denote the power set of U. Consider a non empty set A, A ⊂ E.A pair (F,A) is called a soft set over U, where F : A→P(U).

Example: Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_5}.

Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_8}, where e_1, e_2, . . ., e_8 stand for the attributes “expensive”, “beautiful”, “wooden”, “cheap”, “modern”, and “in bad” “repair”,respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on.

For example, the soft set (F, A) that describes the “attractiveness of the houses” in the opinion of

a buyer, say Thomas, may be defined like this: A={e_1, e_2, e_3, e_4, e_5};

F(e_1) = {h_2, h_3, h_5}, F(e_2) = {h_2, h_4}, F(e_3) = {h_1}, F(e_4) = U, F(e_5) = {h_3, h_5}.

2.3 Definition :[14] A Neutrosophic set ‘A’ on the universe of discourse X is defined as A= $< x, T_A(x), I_A(x),\ F_A(x) >, x \in X$ where $T_A(x): X \to [0^-, 1^+]$, $I_A(x): X \to [0^-, 1^+]$,

$F_A(x): X \to [0^-, 1^+]$ and $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$, $T_A(x), I_A(x)$ and $F_A(x)$ are respectively truth membership, indeterminacy membership and falsity membership.

From philosophical point of view, the neutrosophic set takes from real standard and non-standard values of $[0^-, 1^+]$. So instead of $[0^-, 1^+]$ we need to take the interval [0,1] for technical applications, because $[0^-, 1^+]$ will be difficult to apply in all the real applications such as scientific and engineering problems.

A Neutrosophic set 'A' is contained in another neutrosophic set B, i.e. $A \subseteq B$ then

$T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x),\ F_A(x) \geq F_B(x)$ for all $x \in X$.

2.4 Definition :[2] A fuzzy neutrosophic set 'A' on the universe of discourse X is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$ where T: $X \to [0,1]$, I: $X \to [0,1]$, F: $X \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + F(x) \leq 2$.

Example: Assume that the universe of discourse $U=\{x_1,x_2,x_3\}$,where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is fuzzy neutrosophic set (FNS) of U, such that, $A = \{< x_1, 0.3, 0.5I, 0.4 >,< x_2, 0.4I, 0.2, 0.6 >,< x_3, 0.7, 0.3, 0.5I >\}$, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5I and degree of falsity of capability is 0.4 etc.

2.5 Definition: [2] Let U be the initial universal set and E be a set of parameters. Let P(U) denote the set of all fuzzy neutrosophic set of U. Consider a non-empty set A, $A \subset E$. The collection (F,A) is termed to be the fuzzy neutrosophic soft set (FNSS) over U, where F: $A \to P(U)$.

Example: Let U be the set of blouses under consideration and E is the set of parameters (or qualities). Each parameter is a fuzzy neutrosophic word or sentence involving fuzzy neutrosophic words. Consider E = { Bright, Cheap, Costly, very costly, Colorful, Cotton, Polystyrene, long sleeve , expensive }. In this case, to define a fuzzy neutrosophic soft set means to point out in the universe U given by, $U = \{b_1, b_2, b_3, b_4, b_5\}$ and the set of parameters $A = \{e_1, e_2, e_3, e_4\}$,where each e_i is a specific criterion for blouses:

e_1 stands for 'Bright',
e_2 stands for 'Cheap',
e_3 stands for 'costly',
e_4 stands for 'Colorful',

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Suppose that,

F(Bright)={<b_1, 0.5, 0.6I, 0.3>,<b_2,0.4, 0.7, 0.2I>,<b_3,0.6I, 0.2, 0.3>,<b_4, 0.7I, 0.3, 0.2> ,<b_5,0.8, 0.2, 0.3I>}.

F(Cheap)={<b_1, 0.6I, 0.3, 0.5>,<b_2, 0.7, 0.4I, 0.3>,<b_3, 0.8I, 0.1, 0.2>,<b_4, 0.7, 0.1, 0.3I> ,<b_5, 0.8I, 0.3, 0.4}.

F(Costly)={<b_1, 0.7I, 0.4, 0.3>,<b_2, 0.6, 0.1I, 0.2>,<b_3, 0.7, 0.2, 0.5I>,< b_4, 0.5I, 0.2, 0.6 > ,< b_5, 0.7, 0.3I, 0.2 >}.

F(Colorful)={<b_1, 0.8, 0.1I, 0.4>,<b_2, 0.4, 0.2I, 0.6>,<b_3, 0.3I , 0.6, 0.4>,<b_4, 0.4,0.8,0.5I> ,< b_5, 0.3, 0.5I, 0.7 >}.

The fuzzy neutrosophic soft set (FNSS) (F, E) is a parameterized family of all fuzzy neutrosophic sets of U and describes a collection of approximation ofan object. The mapping F here is 'blouses (.)', where dot(.) is to be filled up by a parameter ei ∈E. Therefore, F(e_1) means 'blouses (Bright)' whose functional-value is the fuzzy neutrosophic set {< b_1, 0.5, 0.6I, 0.3 >,< b_2, 0.4, 0.7, 0.2I >, < b_3, 0.6I, 0.2, 0.3 >,< b_4, 0.7I, 0.3, 0.2>,< b_5, 0.8, 0.2, 0.3I >}.

Thus we can view the fuzzy neutrosophic soft set (FNSS) (F,A) as a collection of approximation as below:
(F, A) = { Bright blouses= {< b_1, 0.5, 0.6I, 0.3 >,< b_2, 0.4, 0.7, 0.2I >, < b_3, 0.6I, 0.2, 0.3 >, <b_4,0.7I,0.3,0.2 >,< b_5,0.8,0.2,0.3I >}, Cheap blouses= {< b_1,0.6I,0.3,0.5 >,< b_2,0.7,0.4I,0.3 >, <b_3,0.8I,0.1,0.2 >, < b_4, 0.7,0.1,0.3I >,< b_5, 0.8I,0.3,0.4 >}, costly blouses= {< b_1,0.7I,0.4,0.3 > ,< b_2,0.6, 0.1I, 0.2 >,< b_3, 0.7, 0.2, 0.5I >,< b_4, 0.5I,0.2,0.6 >,< b_5, 0.7, 0.3I, 0.2 >}, Colorful blouses= {<b_1, 0.8, 0.1I,0.4 >,< b_2, 0.4, 0.2I,0.6 >,< b_3, 0.3I, 0.6,0.4 >, < b_4, 0.4, 0.8, 0.5I>, < b_5, 0.3, 0.5I, 0.7 >}}. In order to store a fuzzy neutrosophic soft set in a computer, we could represent it in the form of a table as shown below (corresponding to the fuzzy neutrosophic soft set in the above example). In this table, the entries are c_{ij} corresponding to the blouse b_i and the parameter e_j, where c_{ij} = (true-membership value of b_i ,indeterminacy-membership value of b_i, falsity membership value of b_i) in F(e_j). The fuzzy neutrosophic soft set (F, A) described as above

$$
\begin{matrix}
b1 \\ b2 \\ b3 \\ b4 \\ b5
\end{matrix}
\begin{pmatrix}
(0.5, 0.6I, 0.3) & (0.6I, 0.3, 0.5) & (0.7I, 0.4, 0.3) & (0.8, 0.1I, 0.4) \\
(0.4, 0.7, 0.2I) & (0.7, 0.4I, 0.3) & (0.6, 0.1I, 0.2) & (0.4, 0.2I, 0.6) \\
(0.6I, 0.2, 0.3) & (0.8I, 0.1, 0.2) & (0.7, 0.2, 0.5I) & (0.3I, 0.6, 0.4) \\
(0.7I, 0.3, 0.2) & (0.7, 0.1, 0.3I) & (0.5I, 0.2, 0.6) & (0.4, 0.8, 0.5I) \\
(0.8, 0.2, 0.3I) & (0.8I, 0.3, 0.4) & (0.7, 0.3I, 0.2) & (0.3, 0.5I, 0.7)
\end{pmatrix}
$$

2.6 Definition:[2] Let X be a non-empty set and A = < $x, T_A(x), I_A(x), F_A(x)$ >,

B = < $x, T_B(x), I_B(x), F_B(x)$ > are fuzzy neutrosophic soft sets. Then union, intersection and difference sets defined as

$T_{A\cup B}(x)$ = max $\{T_A(x), T_B(x)\}$, $I_{A\cup B}(x)$ = max $\{I_A(x), I_B(x)\}$, $F_{A\cup B}(x)$ = min $\{F_A(x), F_B(x)\}$,

$T_{A\cap B}(x)$ = min $\{T_A(x), T_B(x)\}$, $I_{A\cap B}(x)$ = min $\{I_A(x), I_B(x)\}$, $F_{A\cap B}(x)$ = max $\{F_A(x), F_B(x)\}$ for all $x \in X$. *and* $A/B = T_{A/B}(x)$ = min $\{T_A(x)$, = $I_{A/B}(x)$ = min $\{I_A(x), 1 - I_B(x)\}$ = $F_{A/B}(x)$ = max $\{F_A(x), T_B(x)\}$

2.7 Definition: A pair (F,A) is called Fuzzy neutrosophic soft group if the following conditions are satisfied:

(FNSG1): $T_A(xy) \geq$ min $\{T_A(x), T_A(y)\}$, $F_A(xy) \leq$ max $\{F_A(x), F_A(y)\}$, $I_A(xy) \leq$ max $\{I_A(x), I_A(y)\}$ for all $x, y \in$ X.

(FNSG2): $T_A(x^{-1}) \geq T_A(x)$, $F_A(x^{-1}) \leq F_A(x)$, $I_A(x^{-1}) \leq I_A(x)$ for all $x \in$ X.

Example:

(1) Assume that the universe of discourse X = {*x, y, z*}. A = {< *x*, 0.1, 0.3I, 0.5 >,

< *y*,0.2,0.5,0.6I >,< *z*,0.3I,0.4,0.5>} where the degrees of goodness of capability is 0.1, degree of Indeterminacy of capability is 0.3I and degree of falsity capability is 0.5.

(2) Consider a universe X = {DOG, CAT, RAT}. A FNS 'A' of X could be

A={ < DOG,(0.3I,0.2,0.1)>, < CAT,(0.3,0.4I,0.6)>,<RAT,(0.1,0.3,0.4I)>}

2.8 Definition :A fuzzy neutrosophic soft set is said to be zero FNS if $T_A(x)$= 0, $I_A(x)$= 0, $F_A(x)$= 1 for all $x \in$ X. It is denoted by 0_N. A fuzzy neutrosophic soft set is said to be unit FNS if $T_A(x)$= 1, $I_A(x)$= 1, $F_A(x)$= 0 for all $x \in$ X. It is denoted by 1_N.

2.1Proposition: Zero FNS and unit FNS of a group X are trivial FNSG of X.

It is obvious.

2.9 Definition: The α-cut of the FNS A is a crisp subset A_α of the set X is given by

A_α= { x, $x \in$ X / $T_A(x) \geq \alpha$ }.

2.2Proposition: Let A be FNSG of a group X.Then for $\alpha \in [0,1]$, α-cut A_α is a crisp subgroup of X.

Proof: For all $x, y \in A_\alpha$. We have $T_A(x) \geq \alpha, T_A(y) \geq \alpha$.

Now $T_A(xy^{-1}) \geq \min \{T_A(x), T_A(y)\} = \alpha$. Therefore $xy^{-1} \in A_\alpha$. Hence proved.

3.Main results

3.1Theorem: Let A be a FNSG and S be a FN soft subset of A. Then S is a FN subgroup of A (written as S<A) iff $T_S(xy^{-1}) \geq \min \{T_S(x), T_S(y)\}$, $I_S(xy^{-1}) \leq \max \{I_S(x), I_S(y)\}$,

$F_S(xy^{-1}) \leq \max \{F_S(x), F_S(y)\}$ for all $x, y \in$ S.

Proof: Let S be a fuzzy neutrosophic soft subgroup of A.

Form S is a FNSG, (FNSG1) and (FNSG2) are satisfied. Hence we obtain that

$$T_S(xy^{-1}) \geq \min \{T_S(x), T_S(y^{-1})\}$$
$$= \min \{T_S(x), T_S(y)\}.$$

Conversely, let the inequality $T_S(xy^{-1}) \geq \min \{T_S(x), T_S(y)\}$ be satisfied.

Choosing $y= x$, we get that

$$T_S(xx^{-1}) = T_S(e) \geq \min \{T_S(x), T_S(x^{-1})\} = T_S(x).$$

Hence for $x=e$

$$T_S(ey^{-1}) = T_S(y^{-1}) \geq \min \{T_S(e), T_S(y)\} = T_S(y).$$

Consequently,

$$T_S(x(y^{-1})^{-1}) \geq \min \{T_S(x), T_S(y^{-1})\}$$
$$= \min \{T_S(x), T_S(y)\}.$$

Also $$I_S(xy^{-1}) \leq \max \{I_S(x), I_S(y^{-1})\}$$

$= \max\{I_S(x), I_S(y)\}$.

Conversely, let $I_S(xy^{-1}) \le \max\{I_S(x), I_S(y^{-1})\}$ inequality be satisfied.

Choosing $y = x$, we get that

$$I_S(xx^{-1}) = I_S(e) \le \max\{I_S(x), I_S(x^{-1})\} = I_S(x).$$

Hence for $x=e$

$$I_S(ey^{-1}) = I_S(y^{-1}) \le \max\{I_S(e), I_S(y)\} = I_S(y).$$

Consequently,

$$I_S(xy^{-1}) \le \max\{I_S(x), I_S(y^{-1})\} = \max\{I_S(x), I_S(y)\}.$$

Similarly, we can prove $F_S(xy^{-1}) \le \max\{F_S(x), F_S(y)\}$, for all $x, y \in$ S.

3.2Theorem: Let A and B be fuzzy neutrosophic soft group in X, then so is $A \cup B$.

Proof: Since A and B be fuzzy neutrosophic soft groups in X. Then clearly FNSG1 and FNSG2 are satisfied.

Now, Let x,y ε X

(FNGS1): $T_{A\cup B}(xy) = \max\{T_A(xy), T_B(xy)\}$

$$\ge \max\{\min\{T_A(x), T_A(y)\}, \min\{T_B(x), T_B(y)\}\}$$

$$\ge \min\{\max\{T_A(x), T_A(y)\}, \max\{T_B(x), T_B(y)\}\}$$

$$\ge \min\{\max\{T_A(x), T_B(x)\}, \max\{T_A(y), T_B(y)\}\}$$

$$\ge \min\{T_{A\cup B}(x), T_{A\cup B}(y)\}$$

$I_{A\cup B}(xy) = \max\{I_A(xy), I_B(xy)\}$

$$\le \max\{\max\{I_A(x), I_A(y)\}, \max\{I_B(x), I_B(y)\}\}$$

$$\le \max\{\max\{I_A(x), I_B(x)\}, \max\{I_A(y), I_B(y)\}\}$$

$$\le \max\{I_{A\cup B}(x), I_{A\cup B}(y)\}$$

$F_{A\cup B}(xy) = \min\{F_A(xy), F_B(xy)\}$

$$\le \min\{\max\{F_A(x), F_A(y)\}, \max\{F_B(x), F_B(y)\}\}$$

$\leq \max\{\min\{F_A(x), F_A(y)\}, \min\{F_B(x), F_B(y)\}\}$

$\leq \max\{\min\{F_A(x), F_B(x)\}, \min\{F_A(y), F_B(y)\}\}$

$\leq \max\{F_{A\cup B}(x), F_{A\cup B}(y)\}$

(FNSG2): $T_{A\cup B}(x^{-1}) = \max\{T_A(x^{-1}), T_B(x^{-1})\} \geq \max\{T_A(x), T_B(x)\} \geq T_{A\cup B}(x)$.

$I_{A\cup B}(x^{-1}) = \max\{I_A(x^{-1}), I_B(x^{-1})\} \leq \max\{I_A(x), I_B(x)\} \leq I_{A\cup B}(x)$

$F_{A\cup B}(x^{-1}) = \min\{F_A(x^{-1}), F_B(x^{-1})\} \leq \min\{F_A(x), F_B(x)\} \leq F_{A\cup B}(x)$

3.3Theorem: If A and B be fuzzy neutrosophic soft group in X, then so is $A \cap B$.

Proof: The proof is similar to that of Theorem3.2.

3.4 Theorem: If A and B be fuzzy neutrosophic soft group in X, then A/B also fuzzy neutrosophic soft group in X.

Proof: Let x,y ε X

Now $T_{A/B}(xy) = \min\{T_A(xy), F_B(xy)\}$

$\geq \min\{\min\{T_A(x), T_A(y)\}, 1\text{-}F_B^c(xy)\}$

$= \min\{\min\{T_A(x), T_A(y)\}, 1\text{-}\max\{F_B^c(x), F_B^c(y)\}\}$

$= \min\{\min\{T_A(x), T_A(y)\}, \min\{F_B(x), F_B(y)\}$

$= \min\{\min\{T_A(x), F_B(x)\}, \min\{T_A(y), F_B(y)\}$

$= \min\{T_{A/B}(x), T_{A/B}(y)\}$

Also $I_{A/B}(xy) = \min\{I_A(xy), 1 - I_B(xy)\}$

$\leq \min\{\max\{I_A(x), I_A(y)\}, \max\{1 - I_B(y), 1 - I_B(x)\}$

$\leq \max\{\min\{I_A(x), 1 - I_B(x)\}, \min\{I_A(y), 1 - I_B(y)\}$

$\leq \max\{I_{A/B}(x), I_{A/B}(y)\}$

Also $F_{A/B}(xy) = \max\{F_A(xy), T_B(xy)\}$

$\leq \max\{\max\{F_A(x), F_A(y)\}, 1\text{-}\ T_B^c(xy)\}$

$\leq \max\{\max\{F_A(x), F_A(y)\}, \max\{T_B(x), T_B(y)\}\}$

$\leq \max\{\max\{F_A(x), T_B(x)\}, \max\{F_A(y), T_B(y)\}\}$

$\leq \max\{F_{A/B}(x), F_{A/B}(y)\}$

$\therefore A/B$ is also fuzzy neutrosophic soft group in X.

4. Fuzzy Characteristic neutrosophic soft group

Our work in this section is to define fuzzy characteristic neutrosophic soft group (FCNSG) and to study their properties. For this, first of all we define the notationsT_A^θ, I_A^θ, F_A^θ which will be useful in our next discussion.

4.1Definition: Let A be a fuzzy neutrosophics soft set of a group G. Let θ:G→G be a map. Define the maps T_A^θ:G→[0,1], I_A^θ:G→[0,1],F_A^θ:G→[0,1] given by, respectively $T_A^\theta(x)= T_A(\theta(x))$,

$I_A^\theta(x)= I_A(\theta(x))$, $F_A^\theta(x)= F_A(\theta(x))$ for all $x \in$X.

4.2 Definition: A FNSG 'A' of a group G is called fuzzy characteristic neutrosophic soft group (FCNSG) of G if $T_A^\theta= T_A$, $I_A^\theta= I_A$ and $F_A^\theta= F_A$ for every automorphism θ of G.

we now prove the following properties

4.1Proposition: If A is FNSG of a group G and θ is a homomorphism of G, then the fuzzy neutrosophics soft set A^θ of G given by $A^\theta= \{< x, T_A^\theta, I_A^\theta, F_A^\theta > / x \in G\}$ also FNSG of G.

Proof: Let $x, y \in$G. Then

(FNSG1): $T_A^\theta(xy) = T_A(\theta(xy)) = T_A(\theta(x)\theta(y))$

$\geq \min\{T_A(\theta(x)), T_A(\theta(y))\}$

$= \min\{T_A^\theta(x), T_A^\theta(y)\}$.

$I_A^\theta(xy) = I_A(\theta(xy)) = I_A(\theta(x)\theta(y))$

$\leq \max\{I_A(\theta(x)), I_A(\theta(y))\}$

$= \max\{I_A^\theta(x), I_A^\theta(y)\}$.

$F_A^\theta(xy) = F_A(\theta(xy)) = F_A(\theta(x)\theta(y))$

$\leq \max\{F_A(\theta(x)), F_A(\theta(y))\}$

$= \max\{F_A^\theta(x), F_A^\theta(y)\}$.

And (FNSG2): $T_A^\theta(x^{-1}) = T_A(\theta(x^{-1})) \geq T_A(\theta(x)) = T_A(\theta(x))$.

Similarly, we can prove $I_A^\theta(x^{-1}) = I_A(\theta(x))$ and $F_A^\theta(x^{-1}) = F_A(\theta(x))$.

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Therefore Aθ is FNSG of G.

5. Cyclic fuzzy neutrosophic soft groups

The main underlying idea of this work on cyclic fuzzy neutrosophic soft group is based on cyclic group and fuzzy neutrosophic soft group. Before defining the cyclic fuzzy neutrosophic soft group, we will need the following well known definition.

If 'a' is an element of a group A, then the set S=$\{a^n / n \in Z\}$ is a subgroup of A.

The group S is cyclic iff there exists an element a in S such that S=$\{a^n / n \in Z\}$, and it will be denoted by $< a >$, the element a is called the generator of the group S.

Now we shall define a new class of fuzzy neutrosophic soft groups. Let A=$< a >$ be a cyclic group. If $\widetilde{A} = \{< a^n, (T_A(a^n)), (I_A(a^n)), (F_A(a^n)) > / n \in Z\}$ is fuzzy neutrosophic soft group, then $\widetilde{A}$ is called a cyclic fuzzy neutrosophic soft group[CFNSG] generated by $(a, T_A(a), I_A(a), F_A(a))$ and will be denoted by $< a, T_A(a), I_A(a), F_A(a) >$.

5.1Theorem: If 'A' is cyclic fuzzy neutrosophic soft group, then

$A^m = \{ (a^n, (T_A(a^n))^m, (I_A(a^n)), (F_A(a^n)) / n \in Z, m \in N\}$ is also a cyclic fuzzy neutrosophic soft group.

5.1Definition: Let 'e' be the identity element of the group A. We shall define identity fuzzy neutrosophic soft group E by

E = $\{ e, T_A(e), I_A(e), F_A(e) / T_A(e) = I_A(e) = F_A(e) = 1$ and e∈A$\}$.

5.2 Theorem: The fuzzy neutrosophic soft group A^n is a fuzzy neutrosophic soft subgroup of A^m if m≤n.

Proof: Immediate from the above theorem5.1.

5.3Theorem: If A^i and A^j are cyclic fuzzy neutrosophic soft groups, then $A^i \cup A^j$ is also cyclic fuzzy neutrosophic soft group, $i, j \in N$.

Proof: It is enough to consider only membership function. Let m≤n.

In this case (FNSG1), since $A^i \supset A^j$,

$$T_{A^i \cup A^j}(a^n a^m) = \max\{T_{A^i}(a^n a^m), T_{A^j}(a^n a^m)\}$$

$$\geq \max\{(T_A(a^n a^m))^i, (T_A(a^n a^m))^j\}$$

$$\geq \max\{ \min\{(T_A(a^n))^i, (T_A(a^m))^i\}, \min\{(T_A(a^n))^j, (T_A(a^m))^j\}\}$$

$\geq \min\{\max\{T_{A^i}(a^n), T_{A^i}(a^m)\}, \max\{T_{A^j}(a^n), T_{A^j}(a^m)\}\}$

$\geq \min\{\max\{T_{A^i}(a^n), T_{A^j}(a^n), \}, \max\{T_{A^i}(a^m), T_{A^j}(a^m)\}\}$

$\geq \min\{T_{A^i \cup A^j}(a^n), T_{A^i \cup A^j}(a^m)\}$

Similarly we can prove $F_{A^i \cup A^j}$ and $I_{A^i \cup A^j}$.

(FNSG2) $T_{A^i \cup A^j}(x^{-1}) = \max\{T_{A^i}(x^{-1}), T_{A^j}(x^{-1})\}$

$\geq \max\{(T_A(x^{-1}))^i, (T_A(x^{-1}))^j\}$

$\geq \max\{(T_A(x))^i, (T_A(x))^j\}$

$\geq \max\{T_{A^i}(x), T_{A^j}(x)\}$

$\geq T_{A^i \cup A^j}(x)$.

Similarly $F_{A^i \cup A^j}(x^{-1})$ and $I_{A^i \cup A^j}(x^{-1})$ is proved.

5.4Theorem: If A^i and A^j are cyclic fuzzy neutrosophic soft groups, then $A^i \cap A^j$ is also cyclic fuzzy neutrosophic soft group, $i, j \in N$.

Proof: This theorem may be proved similarly as the Theorem5.3.

5.2Definition: Let A be a cyclic fuzzy neutrosophic soft group. Then the following set of the cyclic neutrosophic soft group { A, A^2, A^3,…, A^m,….,E} is called cyclic fuzzy neutrosophic soft group family generated by A. It will be denoted by < A >.

5.5 Theorem: Let <A> = { A, A^2, A^3,…, A^m,….,E}. Then $\bigcup_{n=1}^{\infty} A^n$ = A and $\bigcap_{n=1}^{\infty} E$.

Proof: The proof is clear.

6.Conclusion and Future work:

In this paper, The notion of Fuzzy neutrosophic soft groups[FNSG] is introduced and their basic properties are presented. Union, intersection and difference operations of Fuzzy neutrosophic soft groups are defined. Further we have defined cyclic fuzzy neutrosophic soft group[CFNSG] and studied some related properties with supporting proofs. One can obtain the similar results by using these ideas into ring and ideal theory.

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