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An Application of Fuzzy quantifier in Fuzzy Transportation Problem

Mathematics

KEYWORDS : Fuzzytransportation problem, fuzzy quantifier, hexagonal fuzzy number, ranking of hexagonal fuzzy number, zero suffix method.

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ABSTRACT

This work presents a ranking procedure based on Hexagonal fuzzy numbers applied to a transportation problem with fuzzy quantifiers. By this ranking method any transportation problem can be converted into crisp valued problem to get an optimal solution. Then by using zero suffix method, we obtain the optimal solution for the fuzzy transportation problem. A numerical example is provided to illustrate the proposed algorithm.

Introduction

The transportation problem is one of the earliest applications of linear programming problems which deals with the distribution of single commodity from various sources of supply to various destinations of demand in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the modes must be uncertain due to several factors. These imprecise data may be represented by fuzzy numbers. The idea of fuzzy set was introduced by Zadeh [1] in 1965. Bellmann and Zadeh[1] proposed the concept of decision making in fuzzy environment.

A fuzzy transportation problem is a problem in which the transportation cost,

supply and demand quantities are fuzzy quantifiers. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total transportation cost while satisfying fuzzy supply and demand limits.

In this work, we investigate the more realistic problems, namely the transportation problem with fuzzy costs. Since the objective is to minimize the total cost or to maximize the total profit, subject to some fuzzy constraints, the objective function is also considered as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for numbers to find the best alternative. Rajarajeswari. P[3] have proposed a method for ranking hexagonal fuzzy number. The idea is to transform a problem with fuzzy parameters in the form of Linear Programming Problem and solve it by the zero suffix method.

Preliminaries

Fuzzy set

Let X be a non empty set. A fuzzy set A in X is characterized by its membership function A : X[0,1], where A(x) is interpreted as the degree of membership of element x in fuzzy A for each xX.

Fuzzy number

A fuzzy set of the real line R with membership function : R is called fuzzy number if

1) A must be normal and convex fuzzy set,

2) the support of , must be bounded

3) must be a closed interval for ever

Support

The support of a fuzzy set , S() is the crisp set of all xX such that

Fuzzy quantifier

Fuzzy quantifier are fuzzy number that take part in fuzzy propositions. Fuzzy quantifiers that characterize linguistic terms such as about 10, much more than 100, atleast about 5 of the first kind and almost all, about half, most and so on of the second kind.

Hexagonal Fuzzy number

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H(a_1, a_2, a_3, a_4, a_5, a_6)$ where $(a_1, a_2, a_3, a_4, a_5, a_6)$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below.

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & for\ a_2 \le x \le a_3 \\ 1 & for\ a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_6}\right) & for\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ 0 & for\ x > a_6 \end{cases}$$

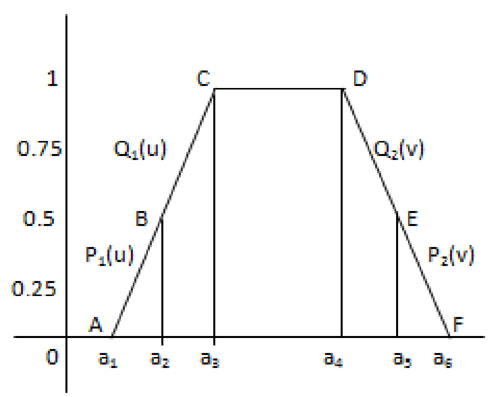

Figure 1

Graphical representation of a hexagonal fuzzy number

Arithmetic operations on Hexagonal fuzzy numbers

Let $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two hexagonal fuzzy numbers, then

(i) $\tilde{A}_H + \tilde{B}_H = (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3,\ a_4 + b_4,\ a_5 + b_5,\ a_6 + b_6)$

(ii) $\tilde{A}_H - \tilde{B}_H = (a_1 - b_1,\ a_2 - b_2,\ a_3 - b_3,\ a_4 - b_4,\ a_5 - b_5,\ a_6 - b_6)$

(iii) $\tilde{A}_H * \tilde{B}_H = (a_1 * b_1,\ a_2 * b_2,\ a_3 * b_3,\ a_4 * b_4,\ a_5 * b_5,\ a_6 * b_6)$

Ranking of Hexagonal fuzzy numbers

A number of approaches have been proposed for the ranking of fuzzy numbers. In this paper for a hexagonal fuzzy number = , $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$, a ranking method is devised based on the following formula.

$$R(\tilde{A}_H) = \left(\frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18}\right)\left(\frac{5}{18}\right)$$

Let $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two hexagonal fuzzy numbers, then

$$\tilde{A}_H \approx \tilde{B}_H \Leftrightarrow R(\tilde{A}_H) = R(\tilde{B}_H)$$

$$\tilde{A}_H \geq \tilde{B}_H \Leftrightarrow R(\tilde{A}_H) \geq R(\tilde{B}_H)$$

$$\tilde{A}_H \leq \tilde{B}_H \Leftrightarrow R(\tilde{A}_H) \leq R(\tilde{B}_H)$$

Mathematical formulation of a Fuzzy Transportation Problem-

Consider the following fuzzy transportation problem(FTP) having costs, sources and demands are fuzzy quantifiers.

(FTP) Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} \otimes \tilde{x}_{ij}$

Subject to $\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_v$ $for\ i = 1,2, \ldots \ldots m$

$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, \quad for\ j = 1,2, \ldots \ldots n$

$\tilde{x}_{ij} \geq \tilde{0}$

$for\ i = 1,2, \ldots \ldots m$ and $j = 1,2, \ldots \ldots n$

Where m = the number of supply points

n = the number of demand points

$\tilde{x}_{ij} \approx (x^1{}_{ij}, x^2{}_{ij}, x^3{}_{ij}, x^4{}_{ij}, x^5{}_{ij}, x^6{}_{ij})$ is the uncertain number of units shipped from supply point i to demand point j;

$\tilde{C}_{ij} \approx (c^1{}_{ij}, c^2{}_{ij}, c^3{}_{ij}, c^4{}_{ij}, c^5{}_{ij}, c^6{}_{ij})$ is the uncertain cost of shipping one unit from supply point i to demand point j;

$\tilde{a}_i \approx (a^1{}_i, a^2{}_i, a^3{}_i, a^4{}_i, a^5{}_i, a^6{}_i)$ is the uncertain supply at supply point i and

$\tilde{b}_j \approx (b^1{}_j, b^2{}_j, b^3{}_j, b^4{}_j, b^5{}_j, b^6{}_j)$ is the uncertain demand at demand point j. The necessary and sufficient condition for the linear programming problem given above

$$\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j$$

The above problem can be put in table namely fuzzy transportation problem given below:

Table 1

		Destination (j) 1 2 … j …. n	Supply
Source(i)	1	$\tilde{c}_{11}$ $\tilde{c}_{12}$ … $\tilde{c}_{1j}$ … $\tilde{c}_{1n}$	$\tilde{a}_1$
	2	$\tilde{c}_{21}$ $\tilde{c}_{22}$ … .$\tilde{c}_{2j}$ … $\tilde{c}_{2n}$	$\tilde{a}_2$
	.	. . … . …. .	.
	.	. . … . … . .	.
	.	. . … . ….. .	.
	i	$\tilde{c}_{i1}$ $\tilde{c}_{i2}$ … .$\tilde{c}_{ij}$ … $\tilde{c}_{in}$	$\tilde{a}_i$
	.	. . … . … .	.
	.	. . … . …. .	.
	.	$\tilde{c}_{m1}$ $\tilde{c}_{m2}$ …$\tilde{c}_{mj}$…. $\tilde{c}_{mn}$	.
	m		$\tilde{a}_m$
	Demand	b_1 b_2 ….b_j ……b_n	

Zero suffix method

The working rule of finding the optimal solution is as follows:

Step 1: Construct the transportation problem.

Step 2: Subtract each row entries of the assignment table from the row minimum element.

Step 3: Subtract each column entries of the assignment table from the column minimum element.

Step 4: In the reduced cost matrix there will be atleast one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S.

Therefore

$$S = \frac{\text{Add the cost of nearest adjacent sides of zeros}}{\text{No. of costs added}}$$

Step 5: Choose the maximum of S, if it has one maximum value then first supply to that demand corresponding to the cell. If it has more equal values then select $\{a_{\tilde{i}}, b_{\tilde{j}}\}$ and supply to that demand maximum possible.

Step 6: After the above step, the exhausted demands (column) or supplies (row) to be trimmed. The resultant matrix must possess at least one zero in each row and column, else repeat Step 2.

Step 7: Repeat Step 4 to Step 6 until the optimal solution is obtained.

Numerical Example

The fuzzy transportation cost for unit quantity of the product from i^{th} source to i^{th} destination is $\tilde{C}ij$ where

$$[\tilde{C}_{ij}]_{3\times 3} = \begin{pmatrix} \text{Extremely low} & \text{low} & \text{Very Very low} \\ \text{Fairly high} & \text{Extremely high} & \text{Medium} \\ \text{Very Very high} & \text{Fairly low} & \text{High} \end{pmatrix}$$

Fuzzy availability of the product at source are (Very low, Very Very High, High) and the fuzzy demand of the product at destinations are (Medium, Low, Very High) respectively. Here costs, sources and demands are fuzzy quantifiers which characterize the linguistic variables are replaced by fuzzy numbers. The problem is then solved by proposed method.

Solution:

The linguistic variables showing the qualitative data is converted into quantitative data using the following table.

Table 2

Extremely low	(-1,1,4,7,10,13)
Very Very low	(0,3,5,8,11,14)
Very low	(0,3,6,9,12,15)
Fairly low	(2,4,8,10,12,14)
Low	(1,4,7,10,13,16)
Medium	(2,5,8,11,14,17)
High	(-2,3,8,13,18,23)
Fairly High	(0,5,10,15,20,25)
Very High	(-1,5,11,17,23,29)
Very Very High	(4,8,12,16,20,24)
Extremely High	(2,7,13,18,23,28)

The fuzzy transportation problem are given in table 3.

Table 3

	D1	D2	D3	Supply
S1	(-1,1,4,7,10,13)	(1,4,7,10,13,16)	(0,3,5,8,11,14)	(0,3,6,9,12,15)
S2	(0,5,10,15,20,25)	(2,7,13,18,23,28)	(2,5,8,11,14,17)	(4,8,12,16,20,24)
S3	4,8,12,16,20,24)	(2,4,8,10,12,14)	(-2,3,8,13,18,23)	(-2,3,8,13,18,23)
Demand	(2,5,8,11,14,17)	(1,4,7,10,13,16)	(-1,5,11,17,23,29)	

Application of zero suffix method gives the following allocation in table 4.

Table 4

	D1	D2	D3	Supply
S1	2.08 1.56	1.56	1.88	2.08
S2	3.47	4.23	3.89 2.64	3.89
S3	0.56 3.89	2.36 2.35	2.92 4.34	2.92
Demand	2.64	2.36	3.89	8.89

The total cost associated with these allocations is 21.24. It can be seen that the value of the objective function obtained by ZSM-Method is same as the optimal value obtained by MODI Method. Thus the value obtained by ZSM-Method is also optimal.

Conclusion

In this paper, the transportation costs, sources and demands are considered as fuzzy quantifiers that characterize linguistic variables are represented by Hexagonal fuzzy numbers. The fuzzy transportation problem has been transformed into crisp transportation problem using ranking of fuzzy numbers. Numerical example shows that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. Thus it can be concluded that ZSM-Method provides an optimal solution directly, in fewer iterations, for the transportation problems. As this method consumes less time and is very easy to understand and apply, so it will be very helpful for decision makers who are dealing with logistic and supply chain problems.

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