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Prime Right Nearrings

Mathematics

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ABSTRACT

The natural to look for comparable results on right nearrings ,and this has been done [1],[2],[3] and [4]. The strong commutativity preserving (SCP)-derivations are motivated by recent studies of mappings F in rings having the property that [F(x),F(z)]=0 Whenever [x,z]=0. In [4], Bell and son established commutativity of right nearrings admitting derivations which are SCP derivations on its subsets. The aim of this section is to study the commutativity of right nearrings with the following constraints: First, with suitably – restricted right cancellation property of N,we prove main theorem1,secondly we deal with a type of derivation .Which is more general than SCP-derivations defined in [5].Finally, we establish that a right nearring N turn out to be a commutative ring if N satisfies [F(x),D(z)]=[x,z] for all x and z in some well-behaved ideal of N.

In this section, we prove that (N,+) is abelian if N has right cancellation property and N is commutative ring if N has no zero divisors with nonzero derivation D and a mapping F such that [F(x),D(z)]=[x,z] for all x,z□N.

Introduction:

A right nearring N is distributively generated (d-g) if it contains a multiplicative sub-semigroup of distributive elements which generates the additive group (N,+).

An element z of N is said to be distributive if z(x+y)=zx+zy for all x,yϵ N; N is said to be distributive if all the elements of N are distributive.

An ideal of a nearring N is defined to be a normal subgroup I of (N,+) such that

(i) NI I
(ii) x(a+z)-xz ϵ I for all x,zϵ N and aϵ I
In a (d-g) nearring (ii) may be replaced by (ii)1
(iii) IN I

A right nearring N is called zero symmetric if z0=0, for all zϵN. A right nearring N is said to be prime if aRb={0},implies that a=0 or b=0

Throughout this section, N will denote a zero Symmetric right nearing with the multiplicative center Z.

Main Section:

Theorem 1: Let N be a right nearring which has right cancellation property. If N admits a mapping F and a nonzero derivation D such that [F(x),D(z)]=[x,z] for all x,z ϵN, then (N,+) is abelian.

Proof: By our hypothesis we have,

[D(z D(z),F(z)] =[zD(z),z] for all zϵ N

This gives that [zD^2(z)+D(z)2,F(z)]=z[D2(z),F(z)]

In view of result(1), this yields

zD2(z)F(z)+D(z)2F(z)-(F(z) z D2(z)+F(z) D(z)2)=D2(z)z F(z)-F(z)zD2(z)

This implies that zD2(z)F(z)+D(z)2F(z)-F(z)D(z)2=D2(z)zF(z) for all zϵN

Clearly by our hypothesis [D(z),F(z)]=0,

The last equation implies that zD2(z)F(z)=zF(z)D2(z) for all zϵ N

Now two cases arise: (i) If D2(z)=0, then D(z) is a constant and hence by lemma1, D(z) is central, in particular ,[z,D(z)]=0 for all zϵ N.

If D(z)≠0 ,then D2(z) can be cancelled and we find [z,F(z)]=0 for all zϵN, that is, F is commuting on N, which yields ,by lemma (2), D is commuting on N.

Combining of result (3) and the obtained result, we get the required result.¨

Theorem 2: Let N be a right nearring having no zero –divisors if admits a mapping F and a nonzero commuting derivation D such that [F(x),D(y)]=[x,z] for all x,zϵN, then N is a commuting ring with no idemptotent except 0 or 1.

Proof: For all zϵN, [z,D(z)]=0 ,in view of lemma 2 yields that [z,D(z)]=0 for all zϵN. For any x,zϵN. We have,

[x,z]z=[xz,z]=[D(xz),F(z)]=[xD(z)+D(x)z,F(z)] by an application of result1, it gives [x,z]z=zD(x]F(z)+D(z)xF(z)-(F(z) zD(x)+xF(z) D(z)

Further, in view of result 3, (N,+) is abelian and since [D(z),F(z)]=0, the last equation reduces to [x,z]z=[D(x),F(z)] z=[D(x),F(z)]z+[F(z),x]D(z)

This implies that [x,F(z)]D(z)=0, for all x,y ϵ N
(1) Hence

[x,F(z)]=0 (2)

Replacing x by D(x) in (2), we have

0=[D(x),F(z)]=[x,z] for all x,zϵ N.

Which yields N is a commutative ring.

Taking e≠0, an idemptotent element in N. Then we have,

D(e)=D(e2)=eD(e)+D(e)e=2eD(e)

This gives eD(e)=2eD(e) ,i.e, eD(e)=0.

Thus D(e)=0, e is a constant, which is central by lemma(1),since (ez-z)e=0 for all zϵN, e is a right identity element which is central, it follows that e=1. ¨

Theorem 3: Let N be a nonzero right nearring such that NZ = N for all nonzero zϵN. If N admits a mapping F and a derivation D such that [F(x),D(z)]=[x,z] for all x,zϵ N, then N is a division ring.

Proof: Taking any nonzero element nϵN.

Then there exists an idempotent element e in N such that en=n, e2n=en and

(e2-e)n=0.

This shows that N has zero divisors, the last equation implies that e is a nonzero idempotent which must be a left identity.

Clearly, D(e)=D(e2)=eD(e)+D(e)e and hence D(e)=D(e)+D(e)e,

i.e. D(e)e=0.

Thus D(e)N=D(e)eN=0.

This gives D(e)=0,that is e is a constant ,by lemma(1),e϶Z.

Thus N has 1.Therefore, NZ=N for all 0≠z϶N, by an application of lemma(3), shows that N is distributive .

In addition, using lemma(1), (N,+) is abelian and hence ,by

Lemma (3), N is a ring which must be a division ring.¨

Lemma 1: Let N be a right nearring which admits a mapping F and derivation D such that [F(x), D(z)] = [x,z] for all x,z϶N, then constants in N are multiplicatively central. In addition, if N has identity 1, then (N,+) is abelian.

Proof: Let C be a constant in N. Replacing x by C in the hypotheses , we get

[C,z] = [D(c),F(z)] = [0,F(z)]=0 for all z϶ N . This implies that C϶Z.

Next, if N has unity 1, then 1+1϶Z and hence

[x+z , 1+1] = 0 for all x,z϶N.

This implies that x+z+x+z = x+x+z+z and hence z+x = x+z gives (N,+) is abelian. ¨

Lemma 2: Let N be a right nearring which admits a mapping F and a derivation D such that [D(x),F(z)]=[x,z] for all x,z϶ N. Then F is commuting on N if and only if D is commuting on N.

Proof: If F is commuting on N, then

0=[D(x),F(D(x)]=[x,D(x)] for all x϶N , that is , D is commuting on N, then 0=[D(F(z)],F(z)]=[F(z),z] for all z϶N.¨

Lemma 3: Let N be a right nearring with identity 1 which admits a mapping F and a derivation D such that [D(x),F(z)] = [x,z] for all x,z϶ N. then (yx + y) z = yxz + yz for all x,y and z϶N.

[D(x),F(z)] = [x,z] we have (x+1)z = xz + z for all x϶N.

Right multiplying by y yields the required result.¨

We also prove the following results which show that nearring N turn out to be a commutative ting if N satisfies the property [D(x),F(z)] = [x,z] for all x,z϶I, where I is an ideal of N. The following theorem1 is a generalization of [6, theorem 3] or [4, theorem 3] and theorem 2 is an extension of [4,theore m 6].

Theorem 4: Let N be a right nearring and U be a nonzero ideal of N which contains non zero divisors of N. If N admits a mapping F with the property that F(u)U, and a nonzero derivation D such that D is commuting on U and [D(x), F(z)] = [x,z] for all x,z϶U, then N is a commutative ring.

Proof: Without loss of generality, we first claim that:

If D is a nonzero derivation of N, then D is also a nonzero derivation of u.

Taking D(u)=0 for all u϶U.

Then D(un)=0 for all n϶N and u϶U, and hence uD(n)=0, this gives that D(n)=0 for all n϶N.

Secondly, we establish that: If u is a nonzero element of U, then (N,+) is abelian.

By application of result (2), it follows that additive commutator (u,z) is constant for all z϶N and u϶U. this implies that (u,z) n=(un,zn) is also constant for any n϶N.

Thus (u,z)D(n)=0 .But (u,z)϶U and hence cannot be a nonzero divisors of zero.Thus (u,z)=0 and (U,+) is abelian.

Further ,if u is a nonzero element of U and x,z϶N then (xn,zn)=(x,z)n=0, yields that (x,z)=0 for all x,z϶N.

So we get (N,+) is abelian.

Thirdly, we prove that N is a commutative ring:Note that arguments used in the proof of theorem 2 of relation1 are still valid in the situation.

Hence [x,F(z)]D(z)=0 for all x,z϶U.

Clearly,[x,F(z)]϶U and hence the last equation implies that if D(z)=0 then 0=[F(x),D(z)]=[x,z].

In particular,[x,F(z)]=0 all for x,z϶U.

But since D is a nonzero on U and hence [x,F(z)]=0 for all x,z϶U.

Replacing x by xD(x) in the last obtained result, we have

0=[xD(x),F(z)]=x[D(x),F(z))=x[x,z] for all x,z϶U.

We conclude that[x,z]=0 for all x,z϶U.

Now ,if u is a nonzero element of U and n,m ϶N then

[n,m]u2 = nmu2-u2nm

= n(mu)u-u(un)m

= munu-unum=0

Thus, [n,m]=0 for all n,m϶N.

Hence N is a commutative ring.¨

Theorem 5: Let N be prime right nearring and U a nonzero ideal of N which is distributively generated (d-g) right nearring with identity.If N admits a mapping F and a derivation D such that [D(x),F(z)}=[x,z] for all x,z϶U,then N is a commutative ring.

Proof: Let e be an identity element of U.

Then ue=u for all u϶U and hence we have D(u)=eD(u)+D(e)u.

This gives uD(e)e=0 for all u϶U.

So D(e)e=0.thus for each uÎU.

D(e)u=D(e)ue=0 that is D(e)u={0}.

This implies that D(e)=0 and hence D(e+e)=0.

Since lemma (1), we obtain that both e and e+e commute with elements of U, and (U,+) is abelian.

Trivially, one can see that (n,m)u={0} for all n,m϶N.

Thus (n,m)=0 for all n,m϶N, yields

(N,+) is abelian.

Since U is a (d-g) right nearring with identity and (U,+) is abelian, application of result (4) gives that U is distributive.

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Let u,vϵU and m,n ϵN.

Then {(m+n)v-(mv+nv)}u=(mu+nu)v-(mvu+nvu)=0.

This implies that (m+n)v=mv+nv.

Putting of v by yv for any yϵ N gives that (m+n)yv=myv+nyv.

We obtain {(m+n)y-(my+ny)}U={0} and hence (m+n)y=my+ny for all n,myϵN

i.e, N is distributive.

This indicates that N is a ring which is commutative by Lemma (4).¨

Corollary: Let N be a ring admitting a derivation D and U a nonzero ideal of N with identity, then U=N.¨

Example 1: Let N_1 be a noncommutative prime right nearring and N2 a noncommutative right nearing admitting a nonzero commuting derivation δ. Then N=N1N2 is a noncommutative semiprime right nearing. Define D:N→N by D(x,z)=(δ(x1,0)). Then D is a nonzero commuting derivation on N. Now, we define F:N→N by D(x,z)=(0,z1). Then [F(x),D(z)]=[x,z] for all x,zϵN.

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