Research Paper
Volume : 3 | Issue : 2 | February 2014 • ISSN No 2277 - 8179

Fixed Point Theorem in Fuzzy Metric Space Satisfying Integral Inequality

Mathematics

KEYWORDS : Fuzzy metric space, non compatible maps, weakly compatible maps, common fixed point, EA property

Geeta Modi	Head & Professor, Department of Mathematics, Govt. M.V.M. Bhopal, MP, India.
Arvind Gupta	Professor, Department of Mathematics, Govt. M.V.M. Bhopal, MP, India.
Varun Singh	Research Scholar Govt. M.V.M. Bhopal, MP, India.

ABSTRACT

We prove common fixed point theorem for weakly compatible maps in fuzzy metric space by using the concept of E A property.

Introduction

In the study of common fixed points of compatible mappings we often require assumption on completeness of the space or continuity of mappings involved besides some contractive condition but the study of fixed points of non comapatible mappings can be extend to the class of non expansive or Lipschitz type mapping pairs even without assuming the continuity of the mappings involved or completeness of the space. Aamri and El Moutawakil [6] generalized the concepts of non comapatibility by defining the notion of (E.A) property and proved common fixed point theorems under strict contractive condition.

We prove common fixed point theorems for weakly compatible maps in fuzzy metric space by using the concept of (E.A) property, however, without assuming either the completeness of the space or continuity of the mappings involved.

Preliminaries

Definition: - A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if $([0,1], *)$, is an abelian topological monoid with the unit 1 such that $a * b \le c * d$ and whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$

Definition:-The triplet $(X, M, *)$ is a Fuzzy Metric Space if X is an arbitrary set, * is a continuous t-norm, and M is a Fuzzy set in $X^2 \times [0,1]$ satisfying the following conditions:

(i) $M(x, y, 0) = 0,$
(ii) $M(x, y, t) = 1$ for all $t > 0$ iff $x = y,$
(iii) $M(x, y, t) = M(y, x, t),$
(iv) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s),$
(v) $M(x, y, \cdot): [0, \infty) \to [0, 1]$ is continuous for all $x, y, z \in X$ and $s, t > 0.$

Example: - Let (X, d) be a metric space. Define $a * b = min\{a, b\}$ and

$$M(x, y, t) = \frac{t}{t + d(x, y)}$$

For all $x, y \in X$

and all $t > 0$ .Then $(X, M, *)$ is a fuzzy metric space. It is called the Fuzzy metric space induced by d.

Definition: - Let U and V be two self maps of a fuzzy metric space $(X, M, *)$ U and V are said to be compatible if $M(UVx_n, VUx_n) \to 1$ as $n \to \infty$ whenever $\{x_n\}$ is a sequence in X such that $UVx_n, VUx_n \to z$ as $n \to \infty$, for some $z \epsilon X$

Definition :- Two self maps U and V of Fuzzy metric space $(X, M, *)$ are said to be weakly compatible if they commute at their coincidence point, i.e. $UVx = VUx$ whenever $Ux = Vx \quad x \epsilon X$.

Clearly each pair of compatible self maps is weakly compatible but the converse is not true always.

Definition:-Let A and B be two self-maps of a Fuzzy metric space $(X, M, *)$**.** We say that A and B satisfy the property (E.A) if there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z \ for\ some\ z \epsilon X$$

Note that weakly compatible and property (E.A) are independent to each other.

Definition: - Let f and g be two self maps of a metric space (X, d) and f and g to be weakly commuting if
$d(fgx, gfx) \leq d(gx, fx)$ for all $x \epsilon X$.

It can be seen that commuting maps $(fgx = gfx \ \forall x \epsilon X)$ *are weakly compatible, but converse is false.*

In 2002, A.Branciari[1] analyzed the existence of fixed point for mapping T defined on a complete metric space $(X; d)$ satisfying a general contractive condition of integral type in the following theorem.

Theorem[1] :- Let (X; d) be a complete metric space, $c \epsilon (0,1)$ and let $T : X \to X$ be a mapping such that for each $x, y \epsilon X$,

$$\int_0^{d(Tx,Ty)} \varphi dt \leq c \int_0^{d(Tx,Ty)} \varphi dt \qquad (i)$$

where $\varphi[0, +\infty) \to [0; +\infty)$ is a Lesbesgue-integrable mapping which is summable (i.e. with finite integral) on each compact subset of [0;+∞), non-negative, and such that for each

$$\varepsilon > 0, \ \int_0^{\varepsilon} \varphi \, dt \geq 0$$

then T has a unique fixed point $a \epsilon X$ such that for each $x \epsilon X$,

$$\lim_{n\to\infty} T^n x = a$$

After the paper of Branciari[1], a lot of research works have been carried out on generalizing contractive conditions of integral type for different contractive mappings satisfying various known properties. A fine work has been done by Rhoades[2] extending the result of Branciari[1] by replacing the condition (i) by the following

$$\int_0^{d(Tx,Ty)} \varphi \, dt \leq \int_0^{\max\left\{\begin{matrix} d(x,y), \\ d(x,Tx), \\ d(y,Ty), \\ \frac{1}{2}[d(x,Ty), d(y,Tx)] \end{matrix}\right\}} \varphi \, dt$$

Main Results run as

Theorem:- Let f and g be two weak – compatible self maps of fuzzy metric space $(X, M, *)$ satisfying the property E.A. and

(i) $fX \subset gX$

(ii) $\int_0^{M(fx,fy,kt)} \emptyset(t)dt \geq \int_0^{M(gx,gy,t)} \emptyset(t)dt$, $k \geq 0$

(iii) $\int_0^{M(fx,ffx,t)} \emptyset(t)dt > \int_0^{\min\left\{\begin{matrix} M(ffx,gx,t), \\ M(ggx,fx,t), \\ M(ffx,ggx,t) \end{matrix}\right\}} \emptyset(t)dt$

Whenever $fx \neq f^2x$
If the range of f or g is complete subspace of X, then f and g have a common fixed point

Proof:-
Since f and g are satisfy the property E.A., so there exists a sequence $\{x_n\}$ in X such that

$$fx_n, gx_n \to z \ \text{as}\ n \to \infty \ \text{for}\ z \epsilon X$$

Since $z \epsilon fX$ and $fX \subset gX$, there exists some point s in X such that

$$z = gs \text{ where } gx_n \to z \ as\ n \to \infty$$

$$\int_0^{M(fx_n,fs,kt)} \emptyset(t)dt \geq \int_0^{M(gx_n,gs,t)} \emptyset(t)dt$$

Taking limit $n \to \infty$ we get

$$\int_0^{M(gs,fs,kt)} \emptyset(t)dt \geq \int_0^{M(gs,gs,t)} \emptyset(t)dt$$
$$= \int_0^1 \emptyset(t)dt$$

Not possible
Hence $fs = gs$

Since f and g are weakly compatible so $fgs = gfs$, therefore

$$fgs = ffs = gfs = ggs$$

If $ffs \neq fs$ then

$$\int_0^{M(fs,ffs,t)} \emptyset(t)dt > \int_0^{\min\left\{\begin{matrix} M(ffs,gs,t), \\ M(ggs,fs,t), \\ M(ffs,ggs,t) \end{matrix}\right\}} \emptyset(t)dt$$
$$= \int_0^{\min\left\{\begin{matrix} M(ffs,gs,t), \\ M(ggs,fs,t), \\ M(ffs,ffs,t) \end{matrix}\right\}} \emptyset(t)dt$$
$$= \int_0^{\min\left\{\begin{matrix} M(ffs,fs,t), \\ M(ffs,fs,t),1 \end{matrix}\right\}} \emptyset(t)dt$$
$$= \int_0^{M(ffs,fs,t)} \emptyset(t)dt$$

This is contradiction and so $ffs = fs$

$$\Rightarrow fs = ffs = fgs = gfs = ggs$$

Hence fs is a common fixed point of $f$and g .
The case when fX is a complete subspace X is similar to the above since$fX \subset gX$.

Conclusion

we prove fixed point theorem for weakly compatible maps in fuzzy metric space satisfying integral type inequality by using E.A. property.

REFERENCE

[1]. A. Branciari (2002), "A fixed point theorem for mappings satisfying a general contractive condition Of integral type" IJMMS 29:9 , 531-536. | | | | [2] B.E Rhoades (2003)., "Two fixed point theorem for mapping satisfying a general contractive condition of integral type" . Int. J. Math. Sci., 3: 4007-4013. | | [3] B.E. Rhoades and G. Jungck (1998), "Fixed points for set valued function without continuity" , Indian J. Pure Appl. Math. 29(3), 227-238. | | [4] G. Jungck (1986), "Compatible mappings and common fixed points", Int. J.Math. & Math. Sci. 9, 771-779. | | | [5] Grabiec, M. (1988), Fixed Point in Fuzzy Metric Spaces , Fuzzy Sets and Syestems 27 385-389. | | [6] M. Aamri and D. El Moutawakil (2002), "Some new common fixed point theorem under strict contractive conditions" J. Math. Anal. Appl.,270, 181-188. | | [7] Mantu saha(2012),"Fixed point theorems for A-contraction mappings of integral type" J. Non linear science and application, 5, 84-92. | | [8] S.N. Mishra (1994), N. Sharma and S.L. Singh, "Common fixed points of maps on fuzzy metric space", Int. J. Math. & Math. Sci. 17, 253-258. | | [9] Zadeh, (1965), L. A., "Fuzzy sets, Inform. Control" 8 ,338-353. |