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On Uniform Continuity of Polynomials

Mathematics

KEYWORDS : Continuity , Uniform continuity

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ABSTRACT

In this paper we shall find conditions on polynomial function with real coefficients so that it becomes uniformly continuous on R.

1. INTRODUCTION

In this paper we shall find conditions on polynomials with real coefficients so that it becomes uniformly continuous on $\mathbb{R}$. There are several applications of continuity and of uniform continuity in every branch of sciences as well as in our daily life uses.

2 PRELIMINARIES

Definition 1.1 Let f be a real valued function defined on a set $S \subseteq \mathbb{R}$. Then f is said to be uniformly continuous on S if

For every $\epsilon > 0$ there exists $\delta > 0$ such that $x, y \in S$ and $|x - y| < \delta$ imply $|f(x) - fy<\varepsilon$

Theorem 1.1[1] A real valued function f on (a,b) is uniformly continuous on (a,b) if and only if it can be extended to a continuous function $\tilde{f}$ on [a,b]

Theorem 1.2[1] Let f be a continuous function on an interval I [I may be bounded or unbounded]. Let I° be the interval obtained by removing the end points of I . If f is differentiable on I° and if $f^{'}$(x) is bounded on I°,then f is uniformly continuous on I.

Result 1.1[3] Suppose that f : $[1,\infty) \to \mathbb{R}$ is uniformly continuous. Then there is a positive M such that $\frac{|f(x)|}{x} \leq M$ for $x \geq 1$.

Proof: By unform continuity of f on $[1,\infty)$ there exists $\delta > 0$ such that $|f(x) - f(\tilde{x})| < 1$ if $|x - \widetilde{x| \leq \delta}$. Any $x \geq 1$ can be written in the form $x = 1 + n\delta + r$, where $n \in \mathbb{N} \cup \{0\}$ and $0 \leq r < \delta$. Hence

$$|f(x)| \leq |f(1)| + |f(x) - f(1)|$$

$$\leq |f(1)| + (n + 1)$$

Dividing by x gives

$$\frac{|f(x)|}{x} \leq \frac{|f(1)|+n+1}{1+n\delta+r} \leq \frac{|f(1)|+2}{\delta} = M$$

Result 1.2 If a function f: $(0,\infty) \to \mathbb{R}$ be continuous and satisfies the condition that if

$\lim_{x\to 0} f(x)$ and $\lim_{x\to\infty} f(x)$ are finite reals then the function f is bounded on the interval $(0,\infty)$.

Proof : Since $\lim_{x\to\infty} f(x)$ is finite say l then by $[k,\infty)$. Again since $\lim_{x\to 0} f(x)$ is a finite real number so we can extent the function continuously on the interval $[0,k]$ and hence the function is bounded on the the interval$[0,k]$. In either way we can say that the function f is bounder on the interval$(0,\infty)$.

Result 1.3[3] If f : $[a,\infty) \to \mathbb{R}$ is continuous and $\lim_{x\to\infty} f(x)$ is finite, then f is uniformly continuous on $[a,\infty)$.

3.MAIN RESULTS.

First of all if our polynomials is of degree one that is of the form $f(x) = ax + b$ then clearly $f^{'}$(x) is bounded and hence by theorem 1.2 becomes uniformly continuous on $\mathbb{R}$.

Now if our polynomials is of degree more than one that is of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_0$ where n≥ 2. Then $\lim_{x\to\infty} \frac{|f(x)|}{x} = \infty$ because for arbitrary large values of x sign of $\frac{|f(x)|}{x}$ is same as that of x^{n-1} which is ∞ that is $\frac{|f(x)|}{x}$ is not bounded on $[1,\infty)$ and hence $f(x)$ is not uniformly continuous on $\mathbb{R}$ as there is no M that satisfy the conditions of the result 1.1.

Again for $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_0$ with any n becomes uniformly continuous on any bounded interval as its derivative is bounded on bounded interval.

4.CONCLUSION.

From the above discussion we conclude that any polynomial function of degree greater than one is not uniformly continuous on $\mathbb{R}$ but become uniformly

REFERENCE

[1]Ross K.A. (2010), "Elementary Analysis: The theory of calculus" Springer(2000). | [2]Rudin W.,"Principles of Mathematical analysis." McRRAW-HIL INTERNATIONAL EDITION(1976). | [3]Kaczor W.J. and Nowak M.T.203, (2003), "Problems in Mathematical Analysis 2 Continuity and Differentation." AMS STUDENT MATHEMATICAL LIBRARY VOLUME 12(2000). | [4] Kaczor W.J. and Nowak M.T.203, (2003), "Problems in Mathematical Analysis 1 real numbers , sequences and series." AMS STUDENT MATHEMATICAL LIBRARY VOLUME 12(2000). |