Bipolar-Valued Fuzzy Normal Subgroups of a Group

Mathematics

KEYWORDS : Bipolar-valued fuzzy set, bipolar-valued fuzzy subgroup, bipolar-valued fuzzy normal subgroup.

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of bipolar-valued fuzzy normal subgroups and prove some results on these. 2000 AMS Subject classification: 03F55, 06D72, 08A72.

INTRODUCTION: In 1965, Zadeh [10] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [7]. Lee [6] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [–1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [–1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [6, 7]. We introduce the concept of bipolar-valued fuzzy normal subgroup and established some results.

1.PRELIMINARIES:

1.1 Definition: A bipolar-valued fuzzy set (BVFS) A in X is defined as an object of the form $A = \{ < x, A^+(x), A^-(x) >/ x\in X\}$, where $A^+ : X\to [0, 1]$ and $A^- : X\to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar-valued fuzzy set A. If $A^+(x) \neq 0$ and $A^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A^+(x) = 0$ and $A^-(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A^+(x) \neq 0$ and $A^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.1 Example: $A = \{ < x, 0.4, -0.5 >, < y, 0.3, -0.6 >, < z, 0.4, -0.6 >\}$ is a bipolar-valued fuzzy subset of $X= \{x, y, z\}$.

1.2 Definition: Let G be a group. A bipolar-valued fuzzy subset A of G is said to be a bipolar-valued fuzzy subgroup of G (BVFSG) if the following conditions are satisfied,

(i) $A^+(xy) \geq \min\{ A^+(x), A^+(y) \}$,
(ii) $A^+(x^{-1}) \geq A^+(x)$,
(iii) $A^-(xy) \leq \max\{ A^-(x), A^-(y) \}$,
(iv) $A^-(x^{-1}) \leq A^-(x)$, for all x and y in G.

1.2 Example: Let $G = \{1, -1, i, -i \}$ be a group with respect to the ordinary multiplication. Then $A = \{ < 1, 0.6, -0.6 >, < -1, 0.5, -0.4 >, < i, 0.3, -0.2 >, < -i, 0.3, -0.2 > \}$ is a bipolar-valued fuzzy subgroup of G.

1.3 Definition: Let (G, .) be a group. A bipolar-valued fuzzy subgroup A of G is said to be a bipolar-valued fuzzy normal subgroup (BVFNSG) of G if $A^+(xy) = A^+(yx)$ and $A^-(xy) = A^-(yx)$, for all x and y in G.

1.4 Definition: Let $A = \langle A^+, A^- \rangle$ and $B = \langle B^+, B^- \rangle$ be any two bipolar-valued fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as $A\times B = \{ \langle (x, y), (A\times B)^+(x, y), (A\times B)^-(x, y) \rangle$ / for all x in G and y in H }, where $(A\times B)^+(x, y) = \min \{ A^+(x), B^+(y) \}$ and $(A\times B)^-(x, y) = \max \{ A^-(x), B^-(y) \}$, for all x in G and y in H.

1.5 Definition: Let $A = \langle A^+, A^- \rangle$ be a bipolar-valued fuzzy subset in a set S, the strongest bipolar-valued fuzzy relation on S, that is a bipolar-valued fuzzy relation on A is V = { 〈 (x, y), $V^+(x, y)$, $V^-(x, y)$ 〉 / x and y in S} given by $V^+(x, y) = \min \{ A^+(x), A^+(y) \}$ and $V^-(x, y) = \max \{ A^-(x), A^-(y) \}$, for all x and y in S.

1.6 Definition: Let G and G^l be any two groups. Then the function f: G→ G^l is said to be an antihomomorphism if f(xy) = f(y)f(x), for all x and y in G.

1.7 Definition: Let X and X^l be any two sets. Let f : X→ X^l be any function and let A be a bipolar-valued fuzzy subset in X, V be a bipolar-valued fuzzy subset in f(X) = X^l, defined by $V^+(y) = \sup_{x\in f^{-1}(y)} A^+(x)$ and $V^-(y) = \inf_{x\in f^{-1}(y)} A^-(x)$, for all x in X and y in X^l. A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

2. SOME PROPERTIES:

2.1 Theorem: Intersection of any two bipolar-valued fuzzy subgroups of a group G is a bipolar-valued fuzzy subgroup of a group G.

2.2 Theorem: The intersection of a family of bipolar-valued fuzzy subgroups of group G is a bipolar-valued fuzzy subgroup of a group G.

2.3 Theorem: If A and B are any two bipolar-valued fuzzy subgroups of the groups G_1 and G_2 respectively, then A×B is a bipolar-valued fuzzy subgroup of $G_1 \times G_2$.

2.4 Theorem: Let A be a bipolar-valued fuzzy subset of a group G and V be the strongest bipolar-valued fuzzy relation of G. Then A is a bipolar-valued fuzzy subgroup of R if and only if V is a bipolar-valued fuzzy subgroup of G×G.

2.5 Theorem: Let A be a bipolar-valued fuzzy subgroup of a group H and f is an isomorphism from a group G onto H. Then A∘f is a bipolar-valued fuzzy subgroup of G.

2.6 Theorem: Let A be a bipolar-valued fuzzy subgroup of a group H and f is an anti-isomorphism from a group G onto H. Then A∘f is a bipolar-valued fuzzy subgroup of G.

2.7 Theorem: Let (G, .) be a group. If A and B are two bipolar-valued fuzzy normal subgroups of G, then A∩B is a bipolar-valued fuzzy normal subgroup of G.

Proof: Let x and y in G. Let A = { < x, $A^+(x)$, $A^-(x)$ > / x∈G }, B = { < x, $B^+(x)$, $B^-(x)$ > / x∈G } be bipolar-valued fuzzy normal subgroups of a group G. Let C = A∩B and C={ < x, $C^+(x)$, $C^-(x)$ > / x∈G }. Then, clearly C is a bipolar-valued fuzzy subgroup of a group G, since A and B are two bipolar-valued fuzzy subgroups of a group G. And, $C^+(xy) = \min \{ A^+(xy), B^+(xy) \} = \min \{ A^+(yx), B^+(yx)\} = C^+(yx)$, for all x and y in G. Therefore, $C^+(xy) = C^+(yx)$, for all x and y in G. And, $C^-(xy) = \max \{A^-(xy), B^-(xy) \} = \max \{ A^-(yx), B^-(yx) \} = C^-(yx)$, for all x and y in G. Therefore, $C^-(xy) = C^-(yx)$, for all x and y in G. Hence A∩B is a bipolar-valued fuzzy normal subgroup of the group G.

2.8 Theorem: Let (G, .) be a group. The intersection of a family of bipolar-valued fuzzy normal subgroups of G is a bipolar-valued fuzzy normal subgroup of the group R.

Proof: It is trivial.

2.9 Theorem: Let A and B be bipolar-valued fuzzy subgroup of the groups G and H, respectively. If A and B are bipolar-valued fuzzy normal subgroups, then A×B is a bipolar-valued fuzzy normal subgroup of G×H.

Proof: Let A and B be bipolar-valued fuzzy normal subgroups of the groups G and H respectively. Clearly A×B is a bipolar-valued fuzzy subgroup of G×H. Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in G×H. Now, $(A\times B)^+[(x_1, y_1)(x_2, y_2)] = (A\times B)^+(x_1x_2, y_1y_2) = \min\{A^+(x_1x_2), B^+(y_1y_2)\} = \min\{A^+(x_2x_1), B^+(y_2y_1)\} = (A\times B)^+(x_2x_1, y_2y_1) = (A\times B)^+[(x_2, y_2)(x_1, y_1)]$. Therefore, $(A\times B)^+[(x_1, y_1)(x_2, y_2)] = (A\times B)^+[(x_2, y_2)(x_1, y_1)]$. And, $(A\times B)^-[(x_1, y_1)(x_2, y_2)] = (A\times B)^-(x_1x_2, y_1y_2) = \max\{A^-(x_1x_2), B^-(y_1y_2)\} = \max\{A^-(x_2x_1), B^-(y_2y_1)\} = (A\times B)^-(x_2x_1, y_2y_1) = (A\times B)^-[(x_2, y_2)(x_1, y_1)]$. Therefore, $(A\times B)^-[(x_1, y_1)(x_2, y_2)] = (A\times B)^-[(x_2, y_2)(x_1, y_1)]$. Hence A×B is a bipolar-valued fuzzy normal subgroup of G×H.

2.10 Theorem: Let A be a bipolar-valued fuzzy subset in a group G and V be the strongest bipolar-valued fuzzy relation on G. Then A is a bipolar-valued fuzzy normal subgroup of G if and only if V is a bipolar-valued fuzzy normal subgroup of G×G.

Proof: Suppose that A is a bipolar-valued fuzzy normal subgroup of a group G. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in G×G. Clearly V is a bipolar-valued fuzzy subgroup of a group G. We have, $V^+(xy) = V^+[(x_1, x_2)(y_1, y_2)] = V^+(x_1y_1, x_2y_2) = \min\{A^+(x_1y_1), A^+(x_2y_2)\} = \min\{A^+(y_1x_1), A^+(y_2x_2)\} = V^+(y_1x_1, y_2x_2) = V^+[(y_1, y_2)(x_1, x_2)] = V^+(yx)$. Therefore, $V^+(xy) = V^+(yx)$, for all x and y in G×G. And, $V^-(xy) = V^-[(x_1, x_2)(y_1, y_2)] = V^-(x_1y_1, x_2y_2) = \max\{A^-(x_1y_1), A^-(x_2y_2)\} = \max\{A^-(y_1x_1), A^-(y_2x_2)\} = V^-(y_1x_1, y_2x_2) = V^-[(y_1, y_2)(x_1, x_2)] = V^-(yx)$. Therefore, $V^-(xy) = V^-(yx)$, for all x and y in G×G. This proves that V is a bipolar-valued fuzzy normal subgroup of G×G. Conversely assume that V is a bipolar-valued fuzzy normal subgroup of G×G, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in G×G, we know that A is a bipolar-valued fuzzy subgroup of G, and $\min\{A^+(x_1y_1), A^+(x_2y_2)\} = V^+(x_1y_1, x_2y_2) = V^+[(x_1, x_2)(y_1, y_2)] = V^+(xy) = V^+(yx) = V^+[(y_1, y_2)(x_1, x_2)] = V^+(y_1x_1, y_2x_2) = \min\{A^+(y_1x_1), A^+(y_2x_2)\}$. If $x_2 = 0$, $y_2 = 0$, we get, $A^+(x_1y_1) = A^+(y_1x_1)$, for all x_1 and y_1 in G. Also, $\max\{A^-(x_1y_1), A^-(x_2y_2)\} = V^-(x_1y_1, x_2y_2) = V^-[(x_1, x_2)(y_1, y_2)] = V^-(xy) = V^-(yx) = V^-[(y_1, y_2)(x_1, x_2)] = V^-(y_1x_1, y_2x_2) = \max\{A^-(y_1x_1), A^-(y_2x_2)\}$. If $x_2 = 0$, $y_2 = 0$, we get, $A^-(x_1y_1) = A^-(y_1x_1)$, for all x_1 and y_1 in G. Therefore A is a bipolar-valued fuzzy normal subgroup of G.

In the following Theorem ∘ is the composition operation of functions:

2.11 Theorem: Let A be a bipolar-valued fuzzy subgroup of a group H and f is an isomorphism from a group G onto H. If A is a bipolar-valued fuzzy normal subgroup of the group H, then A∘f is a bipolar-valued fuzzy normal subgroup of the group G.

Proof: Let x and y in G and A be a bipolar-valued fuzzy normal subgroup of a group H. Then we have, clearly A∘f is a bipolar-valued fuzzy subgroup of a group G. Now, $(A^+\circ f)(xy) = A^+(f(xy)) = A^+(f(x)f(y)) = A^+(f(y)f(x)) = A^+(f(yx)) = (A^+\circ f)(yx)$, which implies that $(A^+\circ f)(xy) = (A^+\circ f)(yx)$, for all x and y in G. And, $(A^-\circ f)(xy) = A^-(f(xy)) = A^-(f(x)f(y)) = A^-(f(y)f(x)) = A^-(f(yx)) = (A^-\circ f)(yx)$, which implies that $(A^-\circ f)(xy) = (A^-\circ f)(yx)$, for all x and y in G. Hence A∘f is a bipolar-valued fuzzy normal subgroup of a group G.

2.12 Theorem: Let A be a bipolar-valued fuzzy subgroup of a group H and f is an anti-isomorphism from a group G onto H. If A is a bipolar-valued fuzzy normal subgroup of the group H, then A∘f is a

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bipolar-valued fuzzy normal subgroup of the group G.

Proof: Let x and y in G and A be a bipolar-valued fuzzy normal subgroup of a group H. Then we have, clearly A∘f is a bipolar-valued fuzzy subgroup of a group G. Now, $(A^+\circ f)(xy) = A^+(f(xy)) = A^+(f(y)f(x)) = A^+(f(x)f(y)) = A^+(f(yx)) = (A^+\circ f)(yx)$, which implies that $(A^+\circ f)(xy) = (A^+\circ f)(yx)$, for all x and y in G. And, $(A^-\circ f)(xy) = A^-(f(xy)) = A^-(f(y)f(x)) = A^-(f(x)f(y)) = A^-(f(yx)) = (A^-\circ f)(yx)$, which implies that $(A^-\circ f)(xy) = (A^-\circ f)(yx)$, for all x and y in G. Hence A∘f is a bipolar-valued fuzzy normal subgroup of a group G.

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