Volume : 3 | Issue : 1 | January 2014 • ISSN No 2277 - 8179

Research Paper

# On Clique Matrix in the Formation of a Government

Mathematics

**KEYWORDS :** Clique of a graph, clique matrix, incidence matrix.

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## ABSTRACT

*The clique problem refers to any of the problems related to finding particular complete sub graphs ("cliques") in a graph, i.e., sets of elements where each pair of elements is connected. Let us consider a social network, where the graph's vertices represent people, and the graph's edges represent mutual acquaintance. Along with its applications in social networks, the clique problem also has many applications in bioinformatics and computational chemistry [3]. Although complete subgraphs have been studied for longer in mathematics [2], the term "clique" and the problem of algorithmically listing cliques both come from the social sciences, where complete subgraphs are used to model social cliques, groups of people who all know each other. The "clique" terminology comes from Luce & Perry (1949), and the first algorithm for solving the clique problem is that of Harary & Ross (1957), who were motivated by the sociological applications.*

## 1.INTRODUCTION

The clique problem refers to any of the problems related to finding particular complete sub graphs ("cliques") in a graph, i.e., sets of elements where each pair of elements is connected. Let us consider a social network, where the graph's vertices represent people, and the graph's edges represent mutual acquaintance. Along with its applications in social networks, the clique problem also has many applications in bioinformatics and computational chemistry [3]. Although complete subgraphs have been studied for longer in mathematics [2], the term "clique" and the problem of algorithmically listing cliques both come from the social sciences, where complete subgraphs are used to model social cliques, groups of people who all know each other. The "clique" terminology comes from Luce & Perry (1949), and the first algorithm for solving the clique problem is that of Harary & Ross (1957), who were motivated by the sociological applications *Monica Pătruț et. al.(2010) discussed* Political Discourse Analysis through Solving Problems of Graph Theory.

## 2. SOME DEFINITIONS

Following definitions are used in this paper:

### Adjacency Matrix

The adjacency matrix of a graph G with vertices and no parallel edges is an *n* by *n* matrix

**A = {$a_{ij}$}** whose elements are given by

**$a_{ij}$** = 1, if there is an edge between $i^{th}$ and $j^{th}$ vertices and

= 0, if there is no edge between them.

### Incidence Matrix

Consider a undirected graph $G = (V, E)$ which has $n$ vertices and $m$ edges all labeled. The incidence matrix **B = {$b_{ij}$}**, is then $n$ by $m$, whose elements are given by

**$b_{ij}$** = 1, when edge $e_j$ is incident with $v_i$

= 0, otherwise.

### Clique of a Graph

By a graph ***G = ( V, E )*** we mean a finite undirected graph without loops or multiple edges, where ***V*** is the set of vertices and ***E*** is the set of edges. A clique of a graph ***G*** is a maximal complete subgraph of ***G*** [6]. A clique is called dominant if it is maximal, i.e. if it is not contained (when considered as a set of vertex) in any larger clique.

Alternatively, a clique can also be defined as follows:

A subset S of vertices satisfying the following properties:

- S contains three or more vertices.
- Each pair of vertices in S has an edge connecting them.
- S is maximal (there is no larger set of vertices that satisfies the second property and contains S)

### Clique number

The clique number of a graph *G*, denoted by , is the maximum number of vertices in a complete subgraph of G.

### Maximal Clique

A maximal clique is a clique, which is not a subgraph of a clique. In other word, a clique is maximal if it is not contained in any larger clique.

## 3. CLIQUE MATRIX OF A GRAPH

We define a clique matrix ***C* = [$c_{ij}$]** of a graph in which the rows correspond to the vertices of the graph and the column to the number of cliques formed [5] :

**$c_{ij}$** = 1 ; if $i^{th}$ clique contains $j^{th}$ vertex

= 0 ; otherwise.

For example, let us consider a graph with five vertices and eight edges as shown in the

**Fig. 3.1 below:**

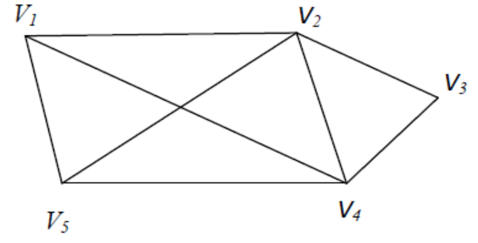


**Fig. 3.1**

Then cliques of this graph are:

$C_1 = \{v_1, v_2, v_5\}$

$C_2 = \{v_1, v_4, v_5\}$

$C_3 = \{v_1, v_2, v_4\}$

$C_4 = \{v_2, v_3, v_4\}$

$C_5 = \{v_2, v_4, v_5\}$

$C_6 = \{v_1, v_2, v_4, v_5\}$

Volume : 3 | Issue : 1 | January 2014 • ISSN No 2277 - 8179

The clique matrix formed by the definition is:

$$C=\begin{bmatrix}1 & 1 & 1 & 0 & 0 & 1\\ 1 & 0 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 1 & 1 & 1 & 1 & 1\\ 1 & 1 & 0 & 0 & 1 & 1\end{bmatrix}$$

**Observations :**
From the clique matrix the following observations can be made:

(i) The number of ones in each row represents the number of cliques formed by the vertex.
(ii) The number of ones in each column represents the size of the clique i.e. whether it is 2-vertex clique, 3-vertex clique etc.
(iii) A row with all the entries zero corresponds to a non clique vertex i.e. a weakly connected vertex and does not form clique with any other vertex.

Although a pictorial representation of a graph is very convenient, matrix representation is also a better and convenient way of representing a graph useful for computing process. Matrices are easy for manipulation and can be readily applied to study the structural properties of graph from an algebraic point of view. In many applications of graph theory, such as electrical network, control problems and operational research, matrix also turn out to be the natural way of expressing the problem. The clique matrix is the generalization of the incidence matrix as the incidence matrix is also a clique matrix where the cliques formed is the number of edges corresponding to the number of vertices. We introduce the terminology clique matrix for the incidence matrix since each column express which vertex form a clique. The incidence matrix is a special case, namely a 2-vertex clique matrix. The number of columns in the clique matrix is the number of cliques formed and each column describes the clique either it is 2- vertex clique, 3-vertex clique or more.

**4. FORMULATION OF THE PROBLEM**
A political party problem can be modeled by a graph where nodes represent the political party and the edges represent the relationships among the parties. Let us consider six political parties, *x, y, z, u, v, w*, together with eight edges where edges represent the relation between the political parties. Suppose *x* has a relation with *y* and *y* has a relation with *z* as well as *u*, and *z* has a relation with *u* and *u* has a relation with *v* as well as *w* and *v* has a relation with *w*. Then the graph representing these relations is as shown in the **Fig. 4.1** below :

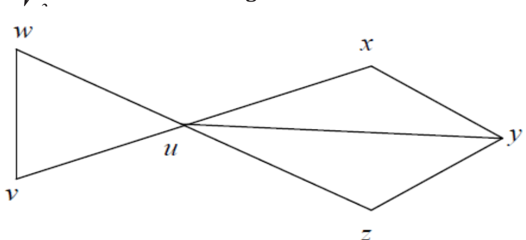

**Fig.4.1**

According to the definition of the clique and clique matrix, we have three cliques for the **Fig. 4.1** and the cliques are:

$C_1 = \{x, y, u\}$

$C_2 = \{y, z, u\}$

$C_3 = \{u, v, w\}$

The matrix representation of the graph is

$$C=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 0 & 1 & 0\\ 1 & 1 & 1\\ 0 & 0 & 1\\ 0 & 0 & 1\end{bmatrix}$$

As the number of columns represents the clique matrix formed by the graph, which also depicts the number of solutions that can be obtained from the graph. Here the numbers of cliques formed are three and as clique of the graph represents the maximum connected subgraph, it is therefore necessary to find the political party that can support maximum number of political parties simultaneously at a time.

**5. RESULT**
In the clique matrix a row with all entries one represents a strongly connected node and it form cliques with all other vertices of the graph, here in this example the vertex *u* is compatible with all other vertices. So, *u* is the most crucial party, because clique with more number of one's form government. Therefore, the party who have support from *u*, that will form a government. On the other hand, a row with all the entries zero represents a node that does not form clique is not compatible with other political parties. Thus from here we can conclude that, without political party *u*, the government cannot be formed.

**6. CONCLUSON**
In this paper, have use graph theoretic tool in the formation of government of the political party, in this problem we have consider six political parties but idea can be generalized for a large number of political parties in which the number of cliques formed will also be more. So clique of a graph is the useful tool for finding a solution of the political party problem from the clique matrix.

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