Research Paper

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Transmission Error Analysis of Involute Gears in Mesh by Fem

Engineering

KEYWORDS : Involute Gear, Transmission error Analysis, FEM.

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ABSTRACT

The characteristics of an involute gear system and transmission errors of gears in mesh. Gearing is one of the most critical components in mechanical power transmission systems. Transmission error is considered to be one of the main contributors to noise and vibration in a gear set. Transmission error measurement has become popular as an area of research on gears and is possible method for quality control. To estimate transmission error in a gear system, the characteristics of involute spur gears were analyzed by using the finite element method. Transmission error can be achieved by inserting a contact element placed in between the two areas where contact occurs. The results of the two dimensional FEM analyses from ANSYS are presented. These stresses were compared with the theoretical values. Both results agree very well. This indicates that the FEM model is accurate.

INTRODUCTION AND DEFINITION OF TRANSMISSION ERROR

Getting and predicting the static transmission error (TE) is a necessary condition for reduction of the noise radiated from the gearbox. In the previous literature to obtain TE the contact problem was seldom included because the nonlinear problem made the model too complicated. This chapter deals with estimation of static transmission error including the contact problem and the mesh stiffness variations of spur gears. For this purpose, an FEA numerical modeling system has been developed. For spur gears a two dimensional model can be used instead of a three dimensional model to reduce the total number of the elements and the total number of the nodes in order to save computer memory. This is based on a two dimensional finite element analysis of tooth deflections. Two models were adopted to obtain a more accurate static transmission error, for a set of successive positions of the driving gear and driven gear. Two different models of a generic gear pair have been built to analyze the effects of gear body deformation and the interactions between adjacent loaded teeth. Results are from each of the two models average values [1].

It is generally accepted that the noise generated by a pair of gears is mainly related to the gear transmission error. The main source of apparent excitation in gearboxes is created by the meshing process. Researchers usually assume that transmission error and the variation in gear mesh stiffnesses are responsible for the noise radiated by the gearbox. The static transmission error is defined by Smith [2].

The term transmission error is used to describe the difference between the theoretical and actual relative angular rotations between a pinion and a gear. Its characteristics depend on the instantaneous positions of the meshing tooth pairs. Under load at low speeds (static transmission error) these situations result from tooth deflections and manufacturing errors. In service, the transmission error is mainly caused by:

* Tooth geometry errors: including profile, spacing and runout errors from the manufacturing process;
* Elastic deformation: local contact deformation from each meshing tooth pair and the deflections of teeth because of bending and shearing due to the transmitted load;
* Imperfect mounting: geometric errors in alignment, which may be introduced by static and dynamic elastic deflections in the supporting bearings and shafts.

The first two types of transmission errors are commonly referred to in the literature [3, 4, 5]. The first case has manufacturing errors such as profile inaccuracies, spacing errors, and gear tooth runout. When the gears are unloaded, a pinion and gear have zero transmission error if there is no manufacturing error. The second case is loaded transmission error, which is similar in principle to the manufactured transmission error but takes into account tooth bending deflection, and shearing displacement and contact deformation due to load. In chapter 5 the second case is considered.

The static transmission error of gears in mesh at particular positions throughout the mesh cycle was generated in this study by rotating both solid gears one degree each time then creating a finite element model in that particular position. In order to develop representative results, a large number of finite element models at the different meshing positions were undertaken for this investigation. One of the most important criteria for each model was that the potential contact nodes of both surfaces would be created on the nodes near the intersection point between the pressure line and the involute curve for that particular tooth. The additional problem of determining the penalty parameter at each contact position could be user-defined or a default value in the finite element model. At each particular meshing position, after running ANSYS the results for angular rotation of the gear due to tooth bending, shearing and contact displacement were calculated. In the pinion reference frame: the local cylindrical system number 12 was created by definition in ANSYS. By constraining the all nodes on the pinion in radius and rotating θg with the gear having a torque input load the model was built. In this case, $\theta p = 0$ and θg is in the opposite direction to that resulting from forward motion of θp changing the TE result to positive as seen by equation (1)

$$TE = \theta g - (z)\, \theta p \qquad (1)$$

Where Z is the gear ratio and θg is the angular rotation of the input and output gears in radians respectively. In relation to the gear reference frame: the local cylindrical system number 11, the gear was restrained with degrees of freedom in radius and rotating θp with the pinion having the torque input load and the resulting angular rotation of the pinion was computed. In this second case θg =0 and the TE will be positive for forward motion of θg. After compensating for torque and angular rotation for the particular gear ratio, the results from these two models should be the same, and so the mean of these two angular rotations would give the best estimate of the true static transmission error of the involute profile gears under load.

THE COMBINED TORSIONAL MESH STIFFNESS

Because the number of the teeth in mesh varies with time, the combined torsional mesh stiffness varies periodically. When a gear with perfect involute profiles is loaded the combined torsional mesh stiffness of the gear causes variations in angular rotation of the gear body. The gear transmission error is related directly to the deviation of the angular rotation of the two gear bodies and the relative angular rotation of the two gears is inversely proportional to the combined torsional mesh stiffness, which can be seen from the results of ANSYS later in this document. The combined torsional mesh stiffness is different throughout the period of meshing position. It decreases and increases dramatically as the meshing of the teeth change from the double pair to single pair of teeth in contact [3].

In other words under operating conditions, the mesh stiffness variations are due to variations in the length of contact line and tooth deflections. The excitation located at the mesh point generates dynamic mesh forces, which are transmitted to the housing through shafts and bearings. Noise radiated by the gearbox is closely related to the vibratory level of the housing.

Sirichai [6] has developed a finite element analysis and given a definition for torsional mesh stiffness of gear teeth in mesh. The combined torsional mesh stiffness is defined as the ratio between the torsional load and the angular rotation of the gear body. The development of a torsional mesh stiffness model of gears in mesh can be used to determine the transmission error throughout the mesh cycle.

The combined torsional mesh stiffness of gears is time dependent during involute action due to the change in the number of contact tooth pairs. Considering the combined torsional mesh stiffness for a single tooth pair contact zone, the single tooth torsional mesh stiffness of a single tooth pair in contact is defined as the ratio between the torsional mesh load (T) and the elastic angular rotation (θ) of the gear body. In the single tooth pair contact zone, as the pinion rotates, the single tooth torsional mesh stiffness of the pinion, KP is decreasing while the single tooth torsional stiffness of the gear, Kg, is increasing. When the pinion rotates to the pitch point P, the single tooth torsional stiffness of both gears is equal because both of them were assumed to be identical spur gears with ratio 1:1 in order to make the analysis simple. The single tooth torsional mesh stiffness of the pinion and the gear are given by [7],

$$K_P^B = \frac{T_P^B}{\theta_P^B} \qquad (2)$$

$$K_g^B = \frac{T_P^B}{\theta_g^B} \qquad (3)$$

where $K_P{}^B$ and $K_g{}^B$ are the single tooth torsional mesh stiffness of the single tooth pairs at B of the pinion and gear respectively. The torsional mesh stiffness can be related to the contact stiffness by considering the normal contact force operating along the line of action tangential to the base circles of the gears in mesh. The torsional mesh stiffness can be seen to be the ratio between the torque and the angular deflection. By considering the total normal contact force F, acting along the line of action, the torque T will be given by the force multiplied by the perpendicular distance (base circle radius rb) T =Frb if there is one pair gear on contact. The elastic angle of rotation θ of the gear body can then be calculated from related to the arc length c, by the base circle radius as θ=c / rb. The torsional mesh stiffness can then be given by

$$K_m = \frac{T}{Q} = \frac{Fr_b}{c/r_b} = \frac{Fr_b^2}{c} \qquad (4)$$

The tooth contact stiffness Kmb, can be seen to be the ratio of the normal contact force F to the displacement along the line of action, which gives Kmb=F / a, where the length a is equal to the arc c length for a small angles θ. The relationship between the linear contact stiffness and torsional mesh stiffness then becomes,

$$K_{mb} = \frac{K_m}{r_b^2} \qquad (5)$$

The contact between the gears is a nonlinear problem. This cannot be put in the form of a linear differential equation if the problem is solved by the equations so here ANSYS was used to study this problem. In this chapter the program ANSYS 7.1 was used to help to solve this nonlinear problem. The gears were modeled using quadratic two dimensional elements and the contact effect was modeled using 2D surface-to-surface (line-to-line) general contact elements that can include elastic Coulomb frictional effects. The torsional mesh stiffness of gears in mesh at particular positions throughout the mesh cycle was generated by rotating both solid gears one degree each time, then creating a finite element model in that particular position. The torsional mesh stiffness Km in mesh was automatically considered, when the transmission error was obtained from the results of the FEA model. Figure 1 shows how to apply load and how to define the input torque by a set of beam elements (beam 3) connected from the nodes on the internal cycle of rim to the center point of the pinion, while restraining all nodes on the internal circle of the output gear hub. The center node of pinion was constrained in the X and Y directions and it was kept the degree of freedom for rotation around the center of the pinion. The moment was applied on the center of the pinion.

After running ANSYS for the each particular position of the FEA model there were volumous results from the postprocessor. For example, the Von Mises stresses, contact stresses and deformations in the X and Y directions can easily be gotten. The static transmission error and the torsional mesh stiffness were then automatically obtained from ANSYS in the postprocessor. The vectors of displacement in the global system at one particular meshing position were shown in Figure 2. In Figure 2 θ represents TE at one position. Twenty-six positions were chosen and for each position ANSYS would produced numerous results. These results indicated that variation in the mesh stiffness is clearly evident as the gears rotate throughout the meshing cycle. The results here are based on FEA modeling and also on the tooth stiffness change.

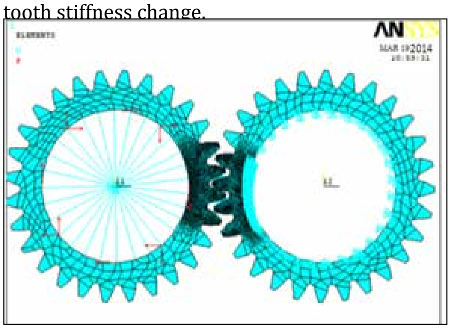

Figure 1 The beam elements were used in the FEA model

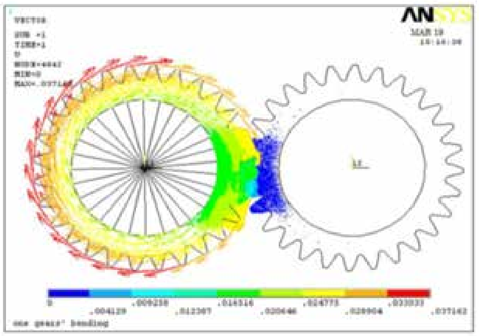

Figure 2 Vectors of displacement

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TRANSMISSION ERROR MODEL

Analysis of the Load Sharing Ratio

Under normal operating conditions, the main source of vibration excitation is from the periodic changes in tooth stiffness due to non-uniform load distributions from the double to single contact zone and then from the single to double contact zone in each meshing cycle of the mating teeth. This indicates that the variation in mesh stiffness can produce considerable vibration and dynamic loading of gears with teeth, in mesh. For the spur involute teeth gears, the load was transmitted between just one to two pairs of teeth gears alternately. The torsional stiffness of two spur gears in mesh varied within the meshing cycle as the number of teeth in mesh changed from two to one pair of teeth in contact. Usually the torsional stiffness increased as the meshing of the teeth changed from one pair to two pairs in contact. If the gears were absolutely rigid the tooth load in the zone of the double tooth contacts should be half load of the single tooth contact.However, in reality the teeth become deformed because of the influence of the teeth bending, shear, and contact stresses.

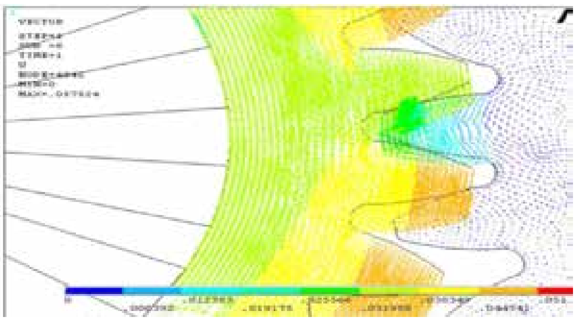

Figure 3 Vectors of displacement near the contact surfaces

These factors change the load distribution along the path of contact. In addition, every gear contains surface finishing and pitching errors. They alter the distribution of load. Because the teeth are comparatively stiff, even small errors may have a large influence. The elastic deformation of a tooth can result in shock loading, which may cause gear failure. In order to prevent shock loading as the gear teeth move into and out of mesh, the tips of the teeth are often modified so as the tooth passes through the mesh zone the load increases more smoothly. The static transmission error model of gears in mesh can be used to determine the load sharing ratio throughout the mesh cycle. Two identical spur gears in mesh are considered here. Table 1 shows the gear parameters

Gear Type	Standard Involute, Full-Depth Teeth
Modulus of Elasticity, E	200GPa
Module (M)	3.75 mm
Number of Teeth	27
Pressure Angle	20
Addendum, Dedendum	1.00*M, 1.25*M

Table 1: Gear Parameters Used in the Model

2D FEM TRANSMISSION ERROR MODEL

Usually calculation of the static transmission error requires estimation of the loaded teeth deflections. In order to evaluate these required quantities, Tavakoli [8] proposed to model gear teeth using a non-uniform cantilever beam. Tobe [9] used a cantilever plate, while numerous authors have developed finite element tooth modeling excluding the contact problem. Unfortunately, the hypotheses related to these models cannot be justified because characteristic dimensions of gear teeth are neither representative of a beam nor a plate for the calculation of the static transmission error and tooth deflection behavior changes because of non-linear contact. Most of the previously published FEA models for gears have involved only a partial tooth model. In this section to investigate the gear transmission error including contact elements, the whole bodies of gears have to be modeled because the penalty of parameter of the contact elements must account for the flexibility of the whole bodies of gears, not just the local stiffness. Here 2D plane 42 elements were used with 2 degrees of freedom per node. The whole model has 5163 nodes, 4751 elements. For the contact surface the contact element was Conta172 and for the target surface the target element was Targe169 shown in Figure 4 that matches the position in Figure 5. Figure 5 displays a meshing model of a spur gear. Fine meshing was used shown in Figure 6. The one or two sets of contact elements were enlarged for the single or the double pairs of gears in contact. This operation allows extracting the compliance due to bending and shear deformation, including the contact deformation. This procedure was successively applied to the pinion and the gear.

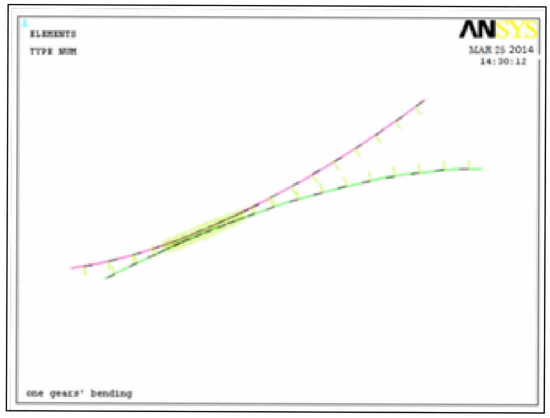

Fig.4 Contact elements between the two contact surfaces

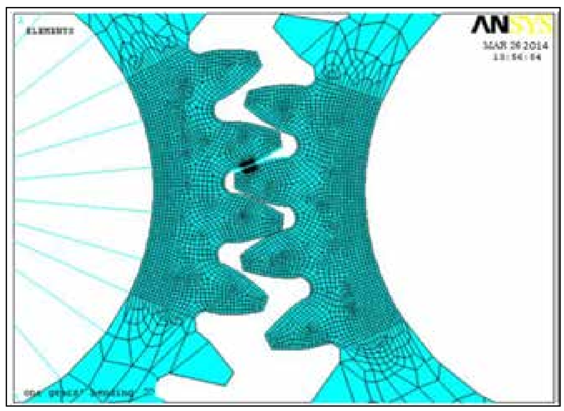

Fig.5 Meshing model for spur gears

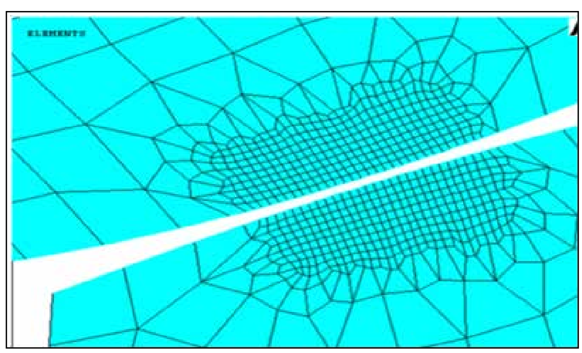

Figure 6: The fine mesh near the two contact surfaces

The 3D model first exhibited difficult convergence behavior. The output window always displayed: "The system process was out of virtual memory". Or "the value at the certain node is greater than current limit of 106 ." Several methods were used in order to overcome such difficulties. First, at the beginning a simple model was built. For example, the contact stress between the two square boxes or two circles was obtained using ANSYS. From this simple model, the author learned that it is necessary to make sure there is the enough computer memory for the 3D model so here the 2D model was chosen. It is also very important to allow the certain constraint conditions for the model to be modeled. If the constraints are inadequate, the displacement values at the nodes may exceed106. This generally indicates rigid body motion as a result of an unconstrained model so one must verify that the model is properly constrained.

THE RESULTS FROM ANSYS

Here Von Mises stresses and the contact stresses just for one position are shown below in Figure 7 and Figure 8. For the gears the contact stress was compared with the results from the Hertz equations, and the two results agree with each other well.

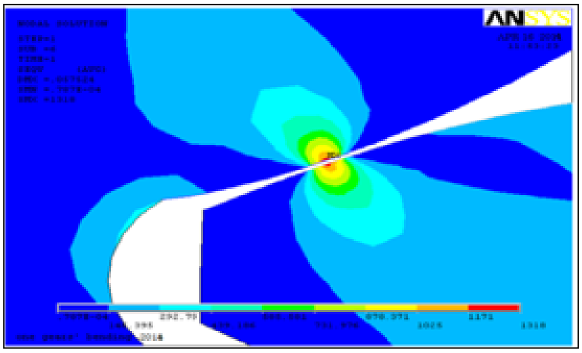

Figure 7 Von Mises stresses in spur gears

In this model, there are 4751 elements and 5163 nodes. For the contact surfaces there are more than eight nodes on each contact side. So the distribution of contact stresses is resonable. In this chapter the transmission error is emphasized and contact is a nonlinear problem so the solution will likely be done after a greater time compared with the time in linear analysis. It is much simpler to use "WIZARD BAR" and to create contact pair between the contact surfaces from "Preprocessor>Modeling>C reate>Contact Pair".

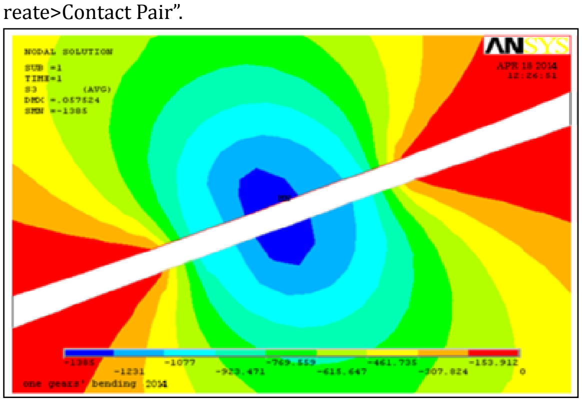

Fig.8: The distribution of contact stresses between two teeth

THE TRANSMISSION ERROR

The static transmission error is expressed as a linear displacement θ at the pitch point [10]. A kinematic analysis of the gear mesh allows determining the location of contact line for each loaded tooth pair. These contact lines were discretized. The total length of lines of contact grows with the applied load. For each position of the driving gear, the iterative procedures were used to solve the static equilibrium of the gear pair and to calculate the load distribution on the contact lines and the static transmission error. However the contact deformation was excluded in those models.

This section considers a FEA model, which was used to predict static transmission error of a pair of spur gears in mesh including the contact deformation of two pairs of teeth are meshing two sets of contact elements were established between the two contact bodies. When gears are unloaded, a pinion and gear with perfect involute profiles should theoretically run with zero transmission error. However, when gears with involute profiles are loaded, the individual torsional mesh stiffness of each gear changes throughout the mesh cycle, causing variations in angular rotation of the gear body and subsequent transmission error. The theoretical changes in the torsional mesh stiffness throughout the mesh cycle match the developed static transmission error using finite element analysis.

CONCLUSION

Mesh stiffness variation as the number of teeth in contact changes is a primary cause of excitation of gear vibration and noise. This excitation exists even when the gears are perfectly machined and assembled. Numerical methods using 2-D FEM modeling of toothed bodies including contact elements have been developed to analyze the main static transmission error for spur gear pairs. Numerous simulations allowed validating this method and showed that a correct prediction of transmission error needed an accurate modeling of the whole toothed bodies. The elasticity of those bodies modifies the contact between loaded tooth pairs and the transmission error variations. The developed numerical method allows one to optimize the static transmission error characteristics by introducing the suitable tooth modifications. These offer interesting possibilities as first steps of the development of a transmission system and can be also successfully used to improve to control the noise and vibration generated in the transmission system.

REFERENCE

1. Smith, J. D. 1983, "Gears and Their Vibration", Marcel Dekker, New York. | 2. Sachindra Kumar, Anjani Kumar Singh, Nitesh Kumar, Sushil Patel, Ajit Kumar, Contact Stress Analysis of Involute Gear through FEM, International Journal Of Scientific Research And Education,||Volume||2||Issue|| 6||Pages 948-962 |||2014|| ISSN (e): 2321-7545. | 3. Sachindra Kumar, Anjani Kumar Singh, Nitesh Kumar, Sushil Patel, Ajit Kumar, Bending Stress and Deformation Analysis of Spur Gear by FEM, International Global Journal for research and analysis, Volume: 3||Issue: 6||June 2014. |||2014|| ISSN No 2277-8160, PP 1-5. | 4. Wang, J., 2003, "Survey of Nonlinear Vibration of Gear Transmission Systems" Appl Mech Rev vol 56, No 3. | 5. Kohler, H., Regan, R., 1985, "The derivation of gear transmission error from pitch error records", Proc. Instn Mech. Engrs, Part C, Journal of Mechanical Engineering Science, 199(C3), 195-201. | 6. Sirichai, S. et al 1997, "Finite Element Analysis of gears in Mesh", Fifth International Congress on Sound and Vibration, Australia, pp 869-876. | 7. Chong, T. H., Bar, I., 2001, "Multiobjective optimal Design of Cylindrical Gear Pairs for the Reduction of Gear Size and Meshing Vibration", JSME International Journal, Vol.44, No. 1, pp 291-292 | 8. Tavakoli M., Houser D., 1986, "Optimum Profile Modifications for the minimization of Static Transmission Errors of Spur Gears", Journal of Mechanisms, Transmission and Automation in Design, Vol. 108, pp 86-95. | 9. Tobe T., Kato, M., Inoue K., 1978, "Bending of Stub Cantilever Plate and some Application to Strength of Gear Teeth", Journal of Mechanical Design, Vol. 100, pp 374-381. | 10. Rigaud, E. "Modelling and Analysis of Static Transmission Error – Effect of Wheel Body Deformation and Interactions Between Adjacent Loaded Teeth". |