Research Paper

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On Cordial Graphs

Mathematics

KEYWORDS : Product cordial labeling, Product cordial graph. 2000 Mathematics Subject Classification 05C78.

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ABSTRACT

A cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label |f(u)–f(v)| the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.A graph which cordial admits cordial labeling is the cordial graph.

A Resideo cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label (f(u)+f(v))(mod 2) the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.A graph which admits Resideo cordial labeling is the Resideo cordial graph.

In this paper, it is proved that every shadow graph is a cordial graph and that every shadow graph is a Resideo cordial graph.

1.Introduction :

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {uv} of vertices in E is called an edge or a line of G. In this paper , we proved that every shadow graph is a cordial graph and that every shadow graph is a Resideo cordial graph.

2.Preliminaries :

A cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label |f(u)–f(v)| the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.A graph which cordial admits cordial labeling is the cordial graph.

A Resideo cordial labeling of a graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label (f(u)+f(v)) (mod 2) the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.A graph which admits Resideo cordial labeling is the Resideo cordial graph.

In this paper, it is proved that every shadow graph is a cordial graph and that every shadow graph is a Resideo cordial graph.

Definition : 2.1 (Shadow Graph)

Let G be a connected Graph. A Graph, constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G_1 to the neighbours of the corresponding vertex v in G_2 ,that is for every vertex u in G_1 there exists v in G_2 such that N (u) = N (v). The resulting Graph is known as shadow Graph and it is denoted by $D_2(G)$.

3. Main Results

Theorem: 3.1

Every shadow graph is a cordial graph. (OR) For any graph G, D_2 (G) is cordial.

Proof:

Let G = (V, E) be a connected graph.
Let |V (G)|= p and |E (G)|= q.
Let G_1 and G_2 be two copies of G.
Let $V(G_1) = \{ [u_i : 1 \le i \le p] \}$ and
$V(G_2) = \{ [v_i : 1 \le i \le p] \}$.
$E(G_1) = \{ [u_iu_j : 1 \le i,j \le p] \}$ and
$E(G_2) = \{ [v_iv_j : 1 \le i,j \le p] \}$.
Let D_2 (G) be the shadow graph of G.

Then $V[D_2(G)] = \{ [u_i ,v_i : 1 \le i \le p] \}$ and
$E[D_2(G)] = \{ [(u_iv_j) ,(v_iu_j) : 1 \le i,j \le p] \}$.
Define $f: V[D_2(G)] \to \{0,1\}$ by
$f(u_i) = 0 ; 1 \le i \le p$.
$f(v_i) = 1; \ 1 \le i \le p$.

Then by definition, every edge, e = u_iu_j (i≠j) in G_1 the induced edge labeling is ,
$f^*(u_iu_j) = 0 ; 1 \le i,j \le p$ and for every edge e=v_iv_j (i≠j) in G_2 the induced edge labeling is
$f^*(v_iv_j) = 0 ; 1 \le i,j \le p$
Further in G_1 ,$v_f(0) = p$, $v_f(1) = 0$, $e_f(0) = q$, $e_f(1) = 0$.
In G_2 , $v_f(0) = 0$, $v_f(1) = p$ $e_f(0) = q$, $e_f(1) = 0$.
In D_2 (G),
For every edge e=u_iv_j ,the induced edge labeling $f^*(u_iv_j) = 1 ; 1 \le i,j \le p$ and $e_f(0) = 0$,$e_f(1) = 2q$.
Therefore, in D_2(G),
$v_f(0) = p$, $v_f(1) = p$,$e_f(0) = 2q$, $e_f(1) = 2q$.
And $| v_f(0) - v_f(1) | = 0$.
$|e_f(0) - e_f(1) | = 0$.

Hence D_2 (G) is cordial.

For example the cordial labeling of D_2 (P_3) as shown in figure 3.2 .

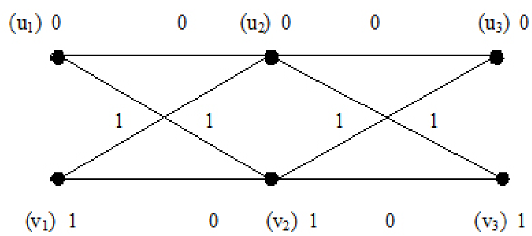

Figure 3.2 : $D_2(P_3)$

Theorem: 3.3

Every shadow graph is a Resideo cordial graph. (OR) For any graph G, D_2(G) is Resideo cordial.

Proof:

Let G = (V, E) be a connected graph.
Let |V (G)|= p and |E (G)|= q.
Let G_1 and G_2 be two copies of G.
Let $V(G_1) = \{ u_i : 1 \le i \le p\}$ and
$V(G_2) = \{ v_i : 1 \le i \le p\}$.
$E(G_1) = \{ [u_iu_j : 1 \le i,j \le p] \}$ and
$E(G_2) = \{ [v_iv_j : 1 \le i,j \le p] \}$.
Let D_2 (G) be the shadow graph of G.

Then $V[D_2(G)] = \{ u_i ,v_i : 1 \le i \le p\}$.
$E[D_2(G)] = \{ [(u_iv_j) ,(v_iu_j) : 1 \le i,j \le p] \}$.
Define $f: V[D_2(G)] \to \{0,1\}$ by
$f(u_i) = 0 ; 1 \le i \le p$.
$f(v_i) = 1; \ 1 \le i \le p$.

Then by definition, every edge, $e = u_iu_j$ ($i \neq j$) in G_1 the induced edge labeling is ,
$f^*(u_iu_j) = 0$; $1 \leq i,j \leq p$.
and for every edge $e=v_iv_j$ ($i \neq j$) in G_2 the induced edge labeling is
$f^*(v_iv_j) = 0$; $1 \leq i,j \leq p$.
Further in G_1 ,$v_f(0) = p$, $v_f(1) = 0$,
$e_f(0) = q$, $e_f(1) = 0$.
In G_2 , $v_f(0) = 0$, $v_f(1) = p$
$e_f(0) = q$, $e_f(1) = 0$.
In D_2 (G),
For every edge $e=u_iv_j$,the induced edge labeling $f^*(u_iv_j) =1$; $1\leq i,j \leq p$
and $e_f(0) = 0$
$e_f(1) = 2q$.
Therefore, in D_2(G),
$v_f(0) = p$, $v_f(1) = p$, $e_f(0) = 2q$, $e_f(1) = 2q$.
And $| v_f(0) - v_f(1) | = 0$.
$|e_f(0) - e_f(1) | = 0$.
Hence D_2 (G) is Resideo cordial.

For example the Resideo cordial labeling of $D_2(C_3)$ is shown in figure 3.4.

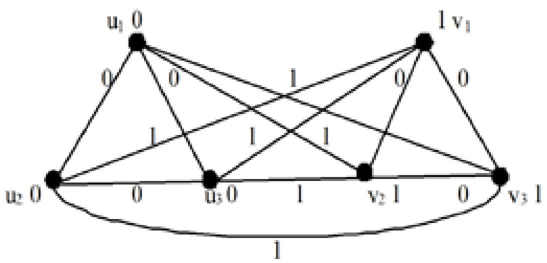

Figure : 3.4 : D_2 (C_3)

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