

Phenomenological Study of Superdeformed Bands in ^{80}Sr



Physics

KEYWORDS : Superdeformed bands, Transition energies, Kinematic moment of inertia, RTEOS.

Ajay Kumar Sharma

Department of Physics & Electronics, University of Jammu, Jammu (180006)- India

Dr. S.K. Khosa

Department of Physics & Electronics, University of Jammu, Jammu (180006)- India

ABSTRACT

Two phenomenological methods, namely ab formula and power expression, have been used to study the systematics of superdeformed (SD) bands in A~80 mass region by taking the case of ^{80}Sr . Calculations have been performed on four SD bands of ^{80}Sr for transition energies and results are compared with their experimental counterparts. Kinematic moment of inertia have also been extracted for these SD bands and are plotted against rotational frequencies. Besides this, ratio of transition energy over spin (RTEOS) has also been plotted against spin and compared with the available experimental data in order to check the validity and reliability of the phenomenological methods applied. The calculated results show an overall good agreement with the corresponding experimental data for all the SD bands.

1. Introduction

The discovery of superdeformation is one of the significant advances in nuclear structure physics. Superdeformed (SD) bands were first observed in fission isomers in the actinide region [1]. With rapid advancements in the experimental facilities, particularly the detectors and data acquisition systems, SD shapes at higher spins were discovered in the last decade of 20th century. The first discrete line SD shapes were found in ^{152}Dy nucleus [2]. Since then vast experimental and theoretical studies have been undertaken. At present numerous SD bands have been observed in various mass region A= 30, 60, 80, 130, 150 and 190 [3,4].

The detailed experimental investigations of SD bands have revealed their interesting features which differ from normal deformed (ND) bands. A superdeformed nucleus is highly deformed with axes ratios around 2:1:1 giving it ellipsoidal shape. Because of the high degree of deformation their quadrupole moments are very large and the most important degree of freedom is the collective rotation. A SD nucleus is thus a good quantum rotor and its rotational energy spectrum is characterized by sequence of highly collective electric quadrupole (E2) transitions with regular energy spacing. One of the outstanding features exhibited by SD states is identical band phenomenon. The transition energies and dynamic moment of inertia of different SD cascades are nearly equal in such cases [5].

Although a general understanding of SD bands has been achieved, there are still some striking features which remain partially understood. The excitation energy and firm assignment of spin-parity are not known for most of the SD bands because of the near absence of information of the linking transitions between ND and SD bands except in a few cases. Several phenomenological formulae have been proposed to fit the transition energies and assign spin angular momentum to the observed levels of SD bands. The transition energies, spins and identical band phenomenon for SD bands in the mass 150 and 190 regions have been predicted by various two and three parameter expressions viz. VMI, ab formula, Harris expansion, $\text{SU}_q(2)$ model, [6-11].

The present work aims to describe the nuclear properties SD bands of ^{80}Sr nucleus. We have employed the two-parameter ab formula and the power expression to calculate the transition energies and kinematic moment of inertia for the four SD bands of ^{80}Sr and compared it with the experimental data. The applicability of the two models to the SD bands has been verified by the plots of transition energy over spin versus spin. The present paper is organised as follows. Section 2 of the paper presents a brief theory of the empirical methods used in the present work. In section 3, the discussion of the results obtained through present calculations is presented along with their comparison with the available experimental data. Finally, the paper is summarised in Section 4.

2. Phenomenological Methods

In phenomenological analysis of the rotational band spectra of even-even nuclei, Bohr and Mottelson [12] pointed out a series expansion of the form

$$E(I, K) = A\xi + B\xi^2 + C\xi^3 + \dots \quad (1)$$

where $\xi = I(I+1) - K^2$ with I the nuclear spin and K the projection of nuclear spin on symmetry axis, and A, B, C, ... are the parameters of expansion. For higher spins, the convergence of this series expansion is poor. Thus, other empirical expressions with two or three parameters were formulated and employed to characterize SD bands. The simple and effective phenomenological formula used to fit transition energies of SD bands is the two-parameter ab formula developed by Holmberg and Lipas [13].

$$E = a(\sqrt{I+1} - 1) \quad (2)$$

This formula was also derived theoretically by C.S. Wu et al. [14] for rotational spectrum of well-deformed nucleus with axial symmetry. The ab formula considers the variable moment of inertia and thus includes in itself the stretching effect. Using this formula, the transition energies of SD bands have been reproduced in many nuclei [15-16].

Transition energy from spin I to spin I-2 can be obtained from ab formula as

$$E_{\gamma}(I) = E(I) - E(I-2) = a(\sqrt{I+1} - \sqrt{I-1}) \quad (3)$$

The observed transition energies of SD band can be used to fit the equation (3) and the value of the parameters are determined using which the transition energies of the SD band are calculated.

Gupta et al. [17] suggested a single-term expression for ground state band of a soft rotor. They replaced the concept of the arithmetic mean of the two terms used in the Bohr-Mottelson expression by the geometric mean and introduced a two-parameter formula called the power expression

$$E = aI^b \quad (4)$$

where a and b are two parameters. The index b is the measure of the deformation of the nuclear core, being 1.0 for a spherical nucleus and about 2.0 for a deformed rotor. The coefficient a plays the role of inverse of moment of inertia. The parameters a, b can be determined by fitting the E2 transitions of SD band.

$$E_{\gamma}(I) = E(I) - E(I-2) = a(I^b - (I-2)^b) \quad (5)$$

The values of the parameters are then used to obtain the transition energies of the SD band.

The kinematic moment of inertia $\mathfrak{J}^{(1)}$ for SD bands can be ex-

tracted from the nuclear energy spectra by using the equation.

$$\mathfrak{S}^{(I)}(I) = \frac{(2I+1)\hbar^2}{E_\gamma(I \rightarrow I-2)} \quad (6)$$

Its value is nearly constant with nuclear spin for SD bands.

To check the validity of the phenomenological expressions to SD bands we have also calculated the transition energy over spins (RTEOS)

$$\frac{E_\gamma(I)}{I - \frac{1}{2}}$$

3. Results and Discussion

Four SD bands of ⁸⁰Sr, viz., SD1, SD2, SD3 and SD4, have been studied in this work. For SD1 and SD2 bands, the values of the parameters have been determined by fitting the E_γ(20→18) and E_γ(22→20) transitions for both ab formula and power expression. The experimental data for transition energies and the spin assignment of levels has been adopted from ref [3]. We have calculated the transition energies for both these SD bands up to spin 38ħ. Theoretical energies so obtained for the SD1 and SD2 bands have been compared with the corresponding experimental values in Table 1.

Table 1. Comparison of theoretical and experimental results on transition energies of SD1 and SD2 bands.

Spin (I)	E _γ (I) (MeV)					
	SD1			SD2		
	Expt.	power	ab	Expt.	power	ab
20	1.443	1.443	1.443	1.688	1.688	1.688
22	1.613	1.613	1.613	1.821	1.821	1.821
24	1.776	1.785	1.791	1.950	1.951	1.945
26	1.948	1.958	1.977	2.092	2.078	2.061
28	2.117	2.133	2.174	2.255	2.203	2.169
30	2.282	2.310	2.385	2.364	2.325	2.269
32	2.441	2.488	2.610	2.575	2.446	2.362
34	2.594	2.667	2.853	-	2.565	2.448
36	2.744	2.848	3.119	-	2.682	2.527
38	2.860	3.029	3.413	-	2.797	2.601

It is clear from the Table 1 that the theoretical transition energies obtained by using the power expression agree well with the experimental data for SD1 band. The experimental transition energies are nicely reproduced by the ab formula for the SD1 band upto spin 30ħ. Beyond that discrepancy between the theoretical and experimental transition energies increases as the nuclear spin increases. For SD2 band, experimental data for transition energies is available only upto spin 32ħ. We have calculated the theoretical values for this band upto spin 38ħ. It may further be noted from Table 1 that the transition energies obtained theoretically by using power expression for SD2 band show satisfactory agreement with the corresponding experimental ones.

For SD3 and SD4 bands the experimental transition energies and the spins of energy levels of the bands has been adopted from ref [3]. We have computed the values of parameters for both ab formula and power expression by fitting the experimental transition energies for E_γ(22→20) and E_γ(24→22) transitions. Transition energies for both of these SD bands have been calculated up to spin 38ħ. A comparison of the theoretical energies so obtained for SD3 and SD4 bands by using the two phenomenological formulae with the corresponding experimental values has been presented in Table 2. It should be noted that the experimental data for transition energies for SD3 and SD4

bands is available only upto spins 34ħ and 30ħ respectively.

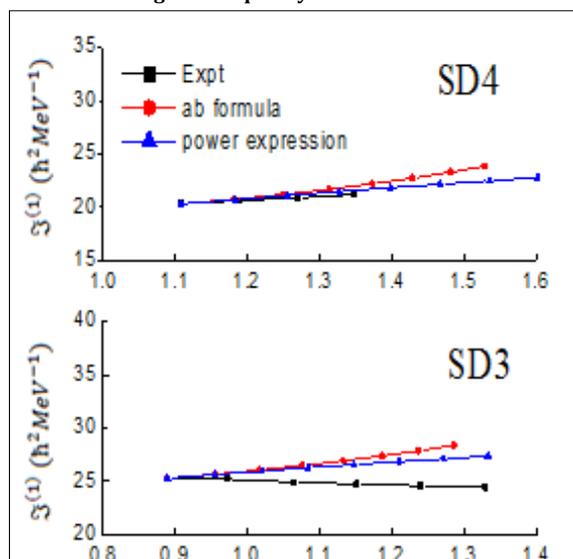
Table 2. Comparison of theoretical and experimental results on transition energies of SD3 and SD4 bands.

Spin (I)	E _γ (I) (MeV)					
	SD3			SD4		
	Expt.	power	ab	Expt.	Power	ab
22	1.712	1.712	1.712	2.140	2.140	2.140
24	1.845	1.845	1.845	2.291	2.291	2.291
26	2.039	1.976	1.972	2.459	2.439	2.433
28	2.216	2.105	2.094	2.621	2.584	2.565
30	2.390	2.232	2.210	2.764	2.726	2.689
32	2.571	2.358	2.320	-	2.865	2.805
34	2.747	2.483	2.425	-	3.003	2.912
36	-	2.606	2.524	-	3.138	3.012
38	-	2.728	2.618	-	3.272	3.105

Table 2 shows that the theoretical transition energies obtained for SD3 band by using both the theoretical models show good agreement with the experimental values upto spin 28ħ. Theoretical calculations beyond this spin reasonably agree with the experimental data for higher spins but the deviations from experimental results increase as spin increases beyond 28ħ. However, experimental transition energies for SD4 band are nicely reproduced by both the phenomenological expressions.

We have also studied the variation of kinematic moment of inertia with angular frequency for all the four SD bands of the ⁸⁰Sr nucleus by using the ab formula as well as power expression. The theoretical results so obtained have been compared with the results computed from experimental data. Figure 1 shows that the experimental kinematic moment of inertia is almost constant with spin. Similar trend is displayed by the theoretical results except at higher angular frequency where the theoretical values exceed the experimental ones. However, the overall agreement is excellent for power expression and quite reasonable for ab formula. It is pertinent to mention here that the parameters for both the empirical formulae were derived for the lowest spins, therefore, discrepancy results at higher angular frequencies.

Figure 1. Plots showing variation of Kinematic moment of Inertia with angular frequency.



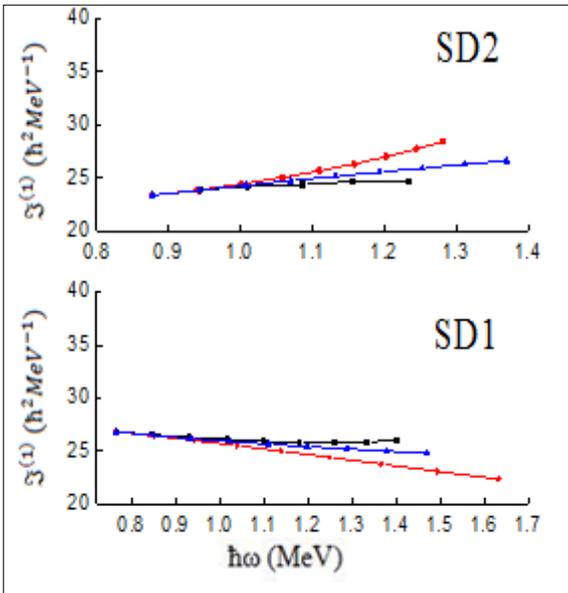
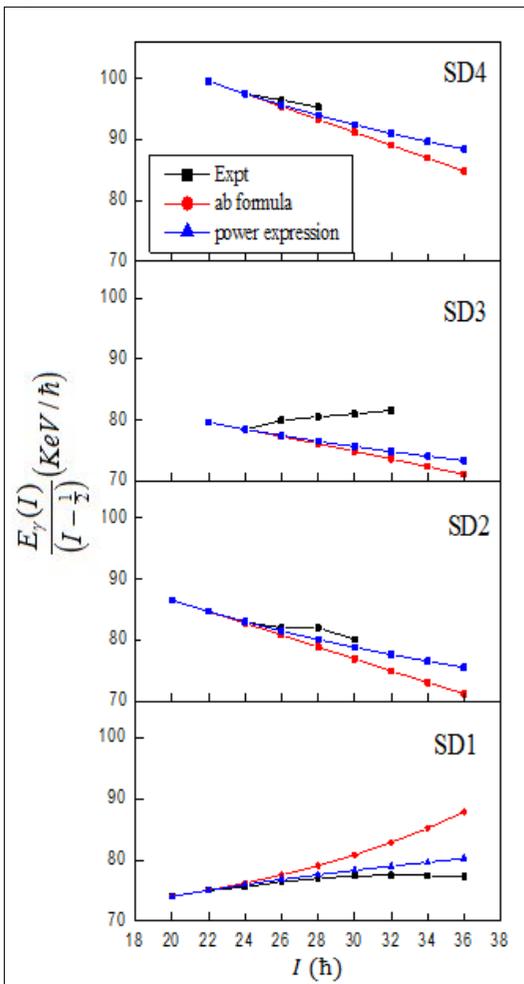


Figure 2 shows that the experimental values of RTEOS are nearly constant with spin for all the SD bands. For SD1, SD2 and SD4 bands the results obtained by power expression are in reasonably good agreement with experimental results except for higher spins. Similar results have been obtained by the ab formula for these bands except that the deviation from experimental plot is more as compared to power expression for the higher spins. In case of SD3 band, the experimental value of transition energy over spin remains constant as spin is increased. But the theoretical values predicted by the two expressions decrease as spin increases. The deviation from experimental results is more for the ab formula than power expression in higher spin domain in all the SD bands, however, the overall agreement is quite satisfactory. The discrepancy between experimental and theoretical results at higher spins arises owing to the fact that the values of

In order to verify the validity of the two theoretical formulae, we have plotted experimental transition energies over spin (RTEOS) versus spin for four SD bands of ⁸⁰Sr as shown in Figure 2. Plots of RTEOS against spin obtained from theoretical models have also been presented in the same figure for the sake of comparison.

Figure 2. Plots of RTEOS versus spin.



REFERENCE

- [1] M. Brack, J. Daamgaard, A.S. Jensen, H.C. Pauli, V.M. Strutinsky, and C.Y. Wong, *Rev. Mod. Phys.* 44, 320 (1972). | [2] P.J. Twin et al., *Phys. Rev. Lett.* 57, 811 (1986). | [3] B. Singh, R. Zywna and R.B. Firestone, *Nucl. Data Sheets* 97, 241 (2002). | [4] R. Janssens and T. Khoo, *Ann. Rev. Nucl. Part. Sci.* 41, 321 (1991). | [5] F.S. Stephens et al., *Phys. Rev. Lett.* 64, 2623 (1990). | [6] J.A. Becker et al., *Phys. Rev. C* 46, 889 (1992). | [7] J. Meng, C.S. Wu and J.Y. Zeng, *Phys. Rev. C* 44, 2545 (1991). | [8] C.S. Wu, J.Y. Zeng, Z. Xing, X. Q. Chen, and J. Meng, *Phys. Rev. C* 45, 261 (1991). | [9] S. X. Liu and J. Y. Zeng, *Phys. Rev. C* 58, 3266 (1998). | [10] D. Bonastos, S.B. Drenska, P.P. Raychev, R.P. Roussev and Yu F Smirnov, *J. Phys. G* 17, L67 (1991). | [11] Y. Liu, J. Song, Hong-zhou, Jia-jun Wang and Enguang Zhao, *J. Phys. G* 24, 117 (1998). | [12] A. Bohr and B. Mottelson, *Nuclear Structure Vol. II* (Benjamin, New York). | [13] P. Holmberg and P.O. Lipas, *Nucl. Phys. A* 117, 552 (1968). | [14] C.S. Wu and J. Zeng, *Commun. in Theor. Phys.* 8, 51(1987). | [15] X.Q. Chen and Z. Xing, *J. Phys. G* 19, 1869 (1993). | [16] Z. Xing, Z.X. Wang, and X.Q. Chen, *Chin. Phys. Lett.* 15, 170 (1998). | [17] J.B. Gupta, A.K. Kavathekar and R. Sharma, *Phys. Scr.* 51, 316 (1995). |