

Decision-Making using the Analytic Hierarchy Process (AHP)



Engineering

KEYWORDS :

Virendra Rajput

ME Research Scholar, Department of Mechanical Engineering, Ujjain Engineering College, Ujjain (M.P.)

Dr.A.C.Shukla

Director, Mahakal Institute of Technology and Science, Ujjain (M.P.)

ABSTRACT

This paper aims at examining the applicability of Analytic Hierarchy Process (AHP) in Two Wheeler Industry. For this purpose, customer needs are determined through direct interviews, observation and data analyses.

The quantification and prioritization of customer needs are done on the hierarchy diagram providing accurate ratio-scale priorities. The relationship, which is between product and customer's requirements, is determined by Analytic Hierarchy Process (AHP). A well researched methodology has been adopted for the synthesis of priorities and the measurement of consistencies. A consistency ratio has also been calculated.

Introduction

The Analytic Hierarchy Process (AHP) method was developed by Professor Thomas Saaty (Saaty 1980). This method is used for decision making and it has wide application in many areas like economic, social and management science (Saaty 1993). The theory was developed to solve a specific problem in contingency planning (Saaty 1972) and a later major application was to design alternative futures for a developing country, the Sudan (Saaty 1977) The result was a set of priorities and an investment plan for projects to be undertaken there in the late 1980's. The ideas have gradually evolved through use in a number of other applications ranging from energy allocation (Saaty 1979) investment in technology under uncertainty, dealing with terrorism (Saaty 1977), buying a car to choosing a job. The method is based on "pair wise" comparisons of alternatives. AHP represents a theoretically founded approach to computing weights representing the relative importance of criteria. The AHP has attracted the interest of many researchers mainly due the nice mathematical properties of the method.

Literature Review

The Analytic Hierarchy Process (AHP) is a systematic decision-making approach that was first developed in 1971 by Thomas L Saaty (Saaty,1977,80,88,94,2000) . A very detailed literature review of the many applications of AHP is given by Vaidya & Kumar (Vaidya& Kumar 2006) who highlight just how broadly the process has been used. According to the study, AHP has been used in education, engineering, government, industry, management, manufacturing, finance sector and so forth. The reason why it has been so widely used is because of its simplicity, ease of use and flexibility Ho (Ho w 2008) The process does however have its critics with the earliest being Belton & Gear (Belton gear 1983), who state that they discovered many instances where the addition of an alternative causes a change in the relative importance of criteria and thus overall preferences order. They recommend that the pairwise comparison questions be more specific than those advocated in the original method. This view is supported by other studies such as one carried out by Aiqing & Jinli (2008) who propose a new method of rank preservation based on what they call the judgment matrix consistency. However, in spite of these perceived shortcomings, AHP in its original form still remains very powerful, especially when it is used in conjunction with other decision making techniques.

Methodology

AHP was introduced by Thomas Saaty (1980) it is an effective tool for dealing with complex decision making, and may aid the decision maker to set priorities and make the best decision. By reducing complex decisions to a series of pair wise comparisons, and then synthesizing the results, the AHP helps to capture both subjective and objective aspects of a decision. In addition, the AHP incorporates a useful technique for checking the consistency of the decision maker's evaluations, thus reducing the bias in the decision making process. Steps involved in AHP are.

Step 1 The complex problem is decomposed into smaller sub problems with goal hierarchy at top, followed by criteria sub criteria at lower levels and at the bottom decision variables.

Step 2 Saaty gave a nine point scale (table 1). Decision matrix is constructed and the priority score is determined. For an equal assessment numerical value 1is assigned and for moderately more important 3 is assigned, for strongly more 5 is assigned and 7 for very strongly, for extremely more important 9 is assigned. 2,4,6,8 are assigned for intermediate values of importance.

Table 1 Scale for pair wise comparisons

Relative Intensity	Definition	Explanation
1	Equal importance	Two elements are of equal value
3	Slightly more importance	Experience slightly favor one element over another
5	strong importance	Experience strongly favor on element over another
7	Very strong importance	An element is strongly favored and its dominance is demonstrated in practice
9	Extreme importance	The evidence favoring one over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values between two adjacent judgments	When compromise is needed
Rationales	Rationales Ratios arising from the scale	If consistency were to be forced by obtaining n numerical values to span the matrix

Step 3 The relative values are inserted in a matrix $n \times n$, where n is the number of the elements. By convention, the comparison is always done with the element in the column, on the left, against an element in the row, on top. For instance, in Table 3.1, the comparisons are done as the pairs: Req. 1 with Req. 2, Req.1 with Req. 3, Req. 1 with Req. 4, until the end of the first row. After that, the second row is evaluated and so on. This is a recommendation to the execution of the comparisons, to make it easier and ordered. The number of comparisons is defined by:

$$n * (n - 1) / 2 \text{ (n is the number of elements).}$$

For instance, if there are 7 requirements to be compared: $7 * (7 - 1) / 2 = 21$

In an example with 7 requirements, 21 comparisons are necessary. The matrix is filled with the values from the judgments that are explained in the next section.

Step 4 for checking the consistency of decision maker's judgment in consistency index or consistency ration is calculated using the equation $CI=(\lambda_{max}-n)/(n-1)$. As the CI nears zero, the consistency increases. The relevant index should be lower than 0.1 otherwise steps and 3 should be repeated.

Step 5 the comparison matrix is to be normalized by dividing each column by the sum of the entries of that column. The matrix so obtained will have sum of the entire elements in that column as 1.

Step 6 to obtain the relative weight of the criteria Eigen value of the n normalized matrix should be calculated. The equation $A.W=\lambda_{max}$. W should be verified, Where, A is pair wise comparison matrix, W is Eigen vector; λ_{max} max is highest Eigen value. The alternative with the highest coefficient value is chosen as the best alternative.

Step 7 The consistency can be compared with its value from random indices. The values in the random indices are obtained from randomly chosen judgments and corresponding reciprocals in the reverse positions in a matrix of the same size. Comparing the value of the consistency index with its random indices, how bad the consistency may be in a given problem can be estimated. The consistency indices of these random judgments are represented in Table 3.13. The first row represents the order of the matrix and the second one is respectively the consistency index.

Table 3.13 Random indices.

1	2	3	4	5	6	7	8	9	10
0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Step 8 The ratio of the Consistency Index to the average of the Random Indices for the same order matrix is called the Consistency Ratio (CR). The value of CR is defined dividing the CI by CR:

Main Result
Criteria's are

1. Fuel Economy, 2. Power, 3. Price, 4. Style ,5 Comfort

Table 2 Comparison matrix for Voice Of Customer.

VOICE OF CUSTOMER						
	FUEL ECONOMY	POWER	PRICE	STYLE	COMFORT	EIGEN VECTOR
FUEL ECONOMY	1	3	5	7	9	0.523
POWER	1/3	1	1	5	7	0.210
PRICE	1/5	1	1	4	5	0.171
STYLE	1/7	1/5	1/4	1	3	0.062
COMFORT	1/9	1/7	1/5	1/3	1	0.034
TOTAL						1

Consistency Index:

$$1*0.523+3*0.210+5*0.171+7*0.062+9*0.034= 2.748$$

$$1/3*0.523+1*0.210+1*0.171+5*0.062+7*0.034=1.171$$

$$1/5*0.523+1*0.210+1*0.171+4*0.062+5*0.034= 0.904$$

$$1/7*0.523+1/5*0.210+1/4*0.171+1*0.062+3*0.034= 0.323$$

$$1/9*0.523+1/7*0.210+1/5*0.171+1/3*0.062+1*0.034= 0.176$$

Dividing these values by the corresponding Eigenvectors we get.

$$2.749/0.523=5.25, 39 \text{ similarly we get } 5.57, 5.28, 5.20, \text{ and } 5.17$$

The mean of these values is 5.29 and that is our estimate for λ_{max} .

Now, Consistency Index= $(\lambda_{max}-n)/(n-1)$

Thus for n=5, we get **CI=0.07**

Consistency Ratio: $CR=0.07/1.11=0.063$

Consistency Ratio: =0.063

As a result of these calculations, if CR value is under 0.10, AHP analysis is consistent. If this number is more than 0.10, it shows that there might be a calculation mistake or decision maker's inconsistency. In this situation, whole process must be repeated.

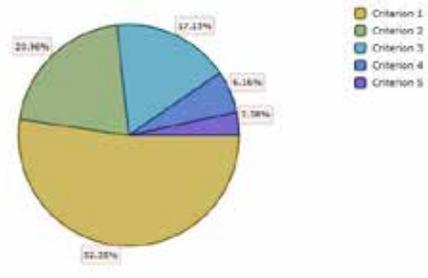


Figure .6.1 Pie Chart for Voice of Customers generated by AHP software

Figure shows the pie chart which shows the percentage importance of each criteria. Pie chart is generated using AHP software (Make It Rational).

Conclusions

Like all modeling methods, the AHP has strengths and weaknesses.

The main advantage of the AHP is its ability to rank choices in the order of their effectiveness in meeting conflicting objectives. If the judgments made about the relative importance of, in this example, the objectives of expense, operability, reliability and flexibility, and those about the competing machines' ability to satisfy those objectives, have been made in good faith, then the AHP calculations lead inexorably to the logical consequence of those judgments.

The limitations of the AHP are that it only works because the matrices are all of the same mathematical form – known as a positive reciprocal matrix. The reasons for this are explained in Saaty's book, which is not for the mathematically daunted, so we will simply state that point. To create such a matrix requires that, if we use the number 9 to represent 'A is absolutely more important than B', then we have to use 1/9 to define the relative importance of B with respect to A. Some people regard that as reasonable; others are less happy about it.

In short, the AHP is a useful technique for discriminating between competing options in the light of a range of objectives to be met. The calculations are not complex and, while the AHP relies on what might be seen as a mathematical trick, you don't need to understand the maths to use the technique. Do, though, be aware that it only shows relative value for money.

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