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A Study on Anti S-Fuzzy Subfield of A Field

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of anti S-fuzzy subfield of a field. 2000 AMS Subject classification: 03F55, 06D72, 08A72.

INTRODUCTION: After the introduction of fuzzy sets by L.A.Zadeh[13], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R [4, 5]. In this paper, we introduce the some theorems in anti S-fuzzy subfield of a field.

1. PRELIMINARIES:

1.1 Definition: A S-norm is a binary operation S : [0, 1]x [0, 1] → [0, 1] satisfying the following requirements;

(i) 0 S x = x, 1 S x = 1 (boundary condition)

(ii) x S y = y S x (commutativity)

(iii) x S (y S z) = (x S y) S z (associativity)

(iv) if x ≤ y and w ≤ z, then x S w ≤ y S z (monotonicity).

1.2 Definition: Let X be a non-empty set. A fuzzy subset A of X is a function A : X → [0, 1].

1.3 Definition: The **union** of two fuzzy subsets A and B of a set X is defined by (A∪B)(x) = max { A(x), B(x) }, for all x in X.

1.4 Definition: Let (F, +, ·) be a field. A fuzzy subset A of F is said to be an **anti S-fuzzy subfield** (anti fuzzy subfield with respect to S-norm) of F if the following conditions are satisfied:

(i) A(x+y) ≤ S (A(x), A(y)), for all x and y in F,

(ii) A(- x) ≤ A(x), for all x in F,

(iii) A(xy) ≤ S (A(x), A(y)), for all x and y in F,

(iv) A(x^{-1}) ≤ A(x), for all x in F−{0}, where 0 is the additive identity element of F.

2 – PROPERTIES OF ANTI S-FUZZY SUBFIELDS:

2.1 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then A(-x) = A(x), for all x in F and A(x^{-1}) = A(x), for all x in F−{0} and A(x) ≤ A(0), for all x in F and A(x) ≤ A(1), for all x in F, where 0 and 1 are identity elements in F.

Proof: For x in F and 0, 1 are identity elements in F. Now, A(x) = A(-(-x)) ≤ A(-x) ≤ A(x). Therefore, A(- x) = A(x), for all x in F. And, A(x) = A(($x^{-1})^{-1}$) ≤ A(x^{-1}) ≤ A(x). Therefore, A(x^{-1}) = A(x), for all x in F−{0}. And, A(0) = A(x-x) ≤ S (A(x), A(-x)) = A(x). Therefore, A(0) ≤ A(x), for all x in F. And, A(1) = A(xx^{-1}) ≤ S(A(x), A(x^{-1})) = A(x). Therefore, A(1) ≤ A(x), for all x in F.

2.2 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then (i) A(x-y) = A(0) gives A(x) = A(y), for all x and y in F,

(ii) A(xy^{-1}) = A(1) gives A(x) = A(y), for all x and y ≠ 0 in F, where 0 and 1 are identity elements in F.

Proof: Let x and y in F and 0, 1 are identity elements in F. (i) Now, A(x) = A(x-y+y) ≤ S(A(x-y), A(y)) = S(A(0), A(y)) = A(y) = A(x-(x-y)) ≤ S(A(x-y), A(x)) = S (A(0), A(x)) = A(x). Therefore, A(x) = A(y), for all x and y in F. (ii) Now, A(x) = A($xy^{-1}y$) ≤ S(A(xy^{-1}), A(y)) = S(A(1), A(y)) = A(y) = A(($xy^{-1})^{-1}x$) ≤ S(A(xy^{-1}), A(x)) = S(A(1), A(x)) = A(x). Therefore, A(x) = A(y), for all x and y≠ 0 in F.

2.3 Theorem: A is an anti S-fuzzy subfield of a field (F, +, ·) if and only if A(x-y) ≤ S (A(x), A(y)), for all x and y in F and A(xy^{-1}) ≤ S (A(x), A(y)), for all x and y ≠ 0 in F.

Proof: It is trivial.

2.4 Theorem: Let A be a fuzzy subset of a field (F, +, ·). If A(e) = A(e^1) = 0, A(x-y) ≤S(A(x), A(y)), for all x and y in F and A(xy^{-1}) ≤ S(A(x), A(y)), for all x and y ≠ e in F, then A is an anti S-fuzzy subfield of F, where e and e^1 are identity elements of F.

Proof: Let e and e^1 be identity elements of F and x, y in F. Now A(-x) = A(e-x) ≤ S (A(e), A(x)) = S (0, A(x)) = A(x). Therefore, A(-x) ≤ A(x), for all x in F. And, A(x^{-1}) = A(e^1x^{-1}) ≤ S (A(e^1), A(x)) = S (0, A(x)) = A(x). Therefore, A(x^{-1}) ≤ A(x), for all x in F−{e}. And, A(x+y) = A(x-(-y)) ≤ S(A(x), A(-y)) ≤ S(A(x), A(y)). Therefore, A(x+y) ≤ S(A(x), A(y)), for all x and y in F. And, A(xy) = A(x($y^{-1})^{-1}$) ≤ S(A(x), A(y^{-1})) ≤ S (A(x), A(y)). Therefore, A(xy) ≤ S(A(x), A(y)), for all x and y in F. Hence A is an anti S-fuzzy subfield of F.

2.5 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then H = { x / x∈F: A(x) = 0 } is either empty or is a subfield of F.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then A(x-y) ≤ S(A(x), A(-y)) = S(A(x), A(y)) = S(0, 0) = 0. Therefore, A(x-y) = 0, for all x and y in F. We get x-y in H. And, A(xy^{-1}) ≤ S(A(x), A(y^{-1})) = S(A(x), A(y)) = S(0, 0) = 0. Therefore, A(xy^{-1}) = 0, for all x and y ≠ e in F. We get xy^{-1} in H. Therefore, H is a subfield of F. Hence H is either empty or is a subfield of F.

2.6 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·), then H ={ x∈F: A(x) = A(e) = A(e^1) } is either empty or is a subfield of F, where e and e^1 are identity elements of F.

Proof: If no element satisfies this condition, then H is empty. If x and y satisfies this condition, then A(-x) = A(x) = A(e), for all x in F and A(x^{-1}) = A(x) = A(e^1), for all x in F−{e}. Therefore, A(-x) = A(e), for all x in F and A(x^{-1}) = A(e^1), for all x in F−{e}. Hence -x, x^{-1} in H. Now, A(x-y) ≤ S(A(x), A(-y)) ≤ S(A(x), A(y)) = S(A(e), A(e)) = A(e). Therefore, A(x-y) ≤ A(e) -----------(1). And, A(e) = A((x-y)-(x-y)) ≤ S(A(x-y), A(-(x-y))) ≤ S(A(x-y), A(x-y)) = A(x-y). Therefore, A(e) ≤ A(x-y) ----------(2). From (1) and (2), we get A(e) = A(x-y), for all x and y in F. Now, A(xy^{-1}) ≤ S(A(x), A(y^{-1})) ≤ S(A(x), A(y)) = S(A(e^1), A(e^1)) = A(e^1). Therefore, A(xy^{-1}) ≤ A(e^1) ------------- (3). And, A(e^1) = A((xy^{-1})($xy^{-1})^{-1}$) ≤ S(A(xy^{-1}), A(($xy^{-1})^{-1}$)) ≤ S(A(xy^{-1}), A(xy^{-1})) = A(xy^{-1}). Therefore, A(e^1) ≤ A(xy^{-1}) ------------- (4). From (3) and (4), we get A(e^1) = A(xy^{-1}), for all x and y ≠ e in F. Hence A(e) = A(x-y), A(e^1) = A(xy^{-1}). We get x-y, xy^{-1} in H. Hence H is either empty or is a subfield of F.

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2.7 Theorem: Let A be an anti S-fuzzy subfield of a field (F, +, ·). Then (i) if A(x-y) = 0, then A(x) = A(y), for x and y in F. (ii) if $A(xy^{-1}) = 0$, then A(x) = A(y), for all x and y≠ e in F, where e and e^1 are identity elements of F.

Proof: Let x and y in F. Now, A(x) = A(x-y+y) ≤ S(A(x-y), A(y)) = S(0, A(y)) = A(y) = A(-y) = A(-x+x-y) ≤ S(A(-x), A(x-y)) = S(A(-x), 0) = A(-x) = A(x). Therefore, A(x) = A(y), for all x and y in F. And, $A(x) = A(xy^{-1}y) \leq S(A(xy^{-1}), A(y)) = S(0, A(y)) = A(y) = A(y^{-1}) = A(x^{-1}xy^{-1}) \leq S(A(x^{-1}), A(xy^{-1})) = S(A(x^{-1}), 0) = A(x^{-1}) = A(x)$. Therefore, A(x) = A(y), for all x and y≠ e in F.

2.8 Theorem: If A is an anti S-fuzzy subfield of a field (F, +, ·) , then (i) if A(x-y) =1, then either A(x) = 1or A(y) = 1, for x and y in F,

(ii) if $A(xy^{-1}) =1$, then either A(x) = 1or A(y) = 1, for all x and y≠ e in F.

Proof: Let x and y in F. By the definition A(x-y) ≤ S(A(x), A(y)), which implies that 1≤ S(A(x), A(y)). Therefore, either A(x) = 1or A(y) = 1, for all x and y in F. And by the definition $A(xy^{-1}) \leq S(A(x), A(y))$, which implies that 1≤ S(A(x), A(y)). Therefore, either A(x) = 1or A(y) = 1, for all x and y ≠ e in F.

2.9 Theorem: Let (F, +, ·) be a field. If A is an anti S-fuzzy subfield of F, then A(x+y) = S(A(x), A(y)), for all x and y in F and A(xy) = S(A(x), A(y)), for all x and y in F with A(x) ≠A(y).

Proof: Let x and y belongs to F. Assume that A(x) < A(y). Now, A(y) = A(-x+x+y) ≤ S(A(-x), A(x+y)) ≤ S(A(x), A(x+y)) = A(x+y) ≤ S(A(x), A(y)) = A(y). Therefore, A(x+y) = A(y) = S(A(x), A(y)), for all x and y in F. And, $A(y) = A(x^{-1}xy) \leq S(A(x^{-1}), A(xy)) \leq S(A(x), A(xy)) = A(xy) \leq S(A(x), A(y)) = A(y)$. Therefore, A(xy) = A(y) = S(A(x), A(y)), for all x and y in F.

2.10 Theorem: If A and B are any two anti S-fuzzy subfields of a field (F, +, ·), then their union A∪B is an anti S-fuzzy subfield of F.

Proof: Let x and y belongs to F and A = {⟨x, A(x)⟩ / x∈F} and B = {⟨ x, B(x)⟩ / x∈F }. Let C = A∪B and C = {⟨x, C(x) ⟩ / x∈F}, C(x) = max {A(x), B(x) }. (i) C(x-y) = max { A(x-y), B(x-y) } ≤ max { S(A(x), A(y)), S(B(x), B(y)) } ≤ S (max { A(x), B(x) }, max { A(y), B(y)}) = S(C(x), C(y)). Therefore, C(x-y) ≤ S(C(x), C(y)), for all x and y in F. (ii) $C(xy^{-1}) = \max \{A(xy^{-1}), B(xy^{-1})\} \leq \max \{ S(A(x), A(y)), S(B(x), B(y)) \} \leq S(\max \{ A(x), B(x)\}, \max \{A(y), B(y) \}) = S(C(x), C(y))$. Therefore, $C(xy^{-1}) \leq S(C(x), C(y))$, for all x and y ≠ 0 in F. Hence A∪B is an anti S-fuzzy subfield of a field F.

2.11 Theorem: The union of a family of anti S-fuzzy subfields of a field (F, +, ·) is an anti S-fuzzy subfield of F.

Proof: Let $\{A_i\}_{i\in I}$ be a family of anti S-fuzzy subfields of a field F and $A = \bigcup_{i\in I} A_i$. Then for x and y belongs to F, we have (i) $A(x-y) = \sup_{i\in I} A_i(x-y) \leq \sup_{i\in I} S(A_i(x), A_i(y)) \leq S(\sup_{i\in I}(A_i(x)), \sup_{i\in I}(A_i(y)))$ = S(A(x), A(y)). Therefore, A(x–y) ≤ S(A(x), A(y)), for all x and y in F. (ii) $A(xy^{-1}) = \sup_{i\in I} A_i(xy^{-1}) \leq \sup_{i\in I} S(A_i(x), A_i(y)) \leq S(\sup_{i\in I}(A_i(x)), \sup_{i\in I}(A_i(y))) = S(A(x), A(y))$. Therefore, $A(xy^{-1}) \leq S(A(x), A(y))$, for all x and y ≠ 0 in F. Hence the union of a family of anti S-fuzzy subfields of a field F is an anti S-fuzzy subfield of F.

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