A Simple Analytical Expression of Non-Linear Boundary Value Problem for Steady State Thermal Criticality in Viscous Reactive Flows Through Channels With A Sliding Wall

Mathematics

KEYWORDS : Lubricant hydrodynamics; Sliding wall; Reaction diffusion equation; Non-linear boundary value problem; Homotopy perturbation method.

V. Ananthaswamy	Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India.
S. Mythili	Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India.

ABSTRACT

In this article, we present the steady state solutions for viscous reactive flows through channels with a sliding wall. The reaction is assumed to be strongly exothermic under arrhenius kinetics, neglecting the consumption of the material. The simple analytical expression of the dimensionless temperature is derived for non-linear boundary value problem using the Homotopy perturbation method. We also compared our analytical results with perturbation method (previous work) and a satisfactory agreement is noted. The present approaches are very less computational, simple and are applicable for solving other strongly non-linear boundary value problems.

INTRODUCTION

The study of the thermal effect of a sliding plate on a viscous reacting fluid is extremely important in understanding lubricants hydrodynamics in engineering systems as well as plasma [1] and fluid physics [2,3,6,9,13]. Lubricant is a thin viscous film used to prevent solid-to-solid contact during sliding motion.Generally speaking, most lubricants used in both engineering and industrial processes are reactive e.g. hydrocarbon oils, polyglycols, synthetic esters, polyphenylethers, etc., and their efficiency depends largely on the temperature variation fromtime to time. Hence, it is important to determine the thermal criticality conditions for viscous reactive fluid effectiveness as lubricants. In this particularproblem, we have assumed that the pressuregradient is zeroand the flow is driven solely by uniform velocity at the upper plate, i.e. the well-known plane Couette flow [3]. The resulting velocity profile is linear with zero value atthe lower fixed plate and maximum value at the upper moving plate.

In this analysis consists nonlinear boundary value problem by using of the Homotopy perturbation method. The steady state flow consist a single a class of parallel flows in fluid mechanics. Mostly this of flow views between two sliding walls due to pressure gradient as well as temperature. Besides viscous reacting fluid is the importance of understanding the concepts of lubricants, which means a viscous film and it is used to prevent solid-to-solid contact during motion.

Meanwhile, analytical solutions of the highly non-linear ordinary differential equations governing transient heating in a slab of combustible material due to exothermic impossible or extremely difficult to obtain. In this particular problem we have postulated that the pressure gradient=zero, the resulting velocity profile is linear with zero(0) value at the lower fixed plate and maximum value at the upper moving plate.

Mathematical formulation of the problem

The mathematical formulation for the momentum and heat balance in one dimension together with the boundary conditions can be written as [3,9,10,13]

$$\frac{d^2u}{d\bar{y}^2}=0 \tag{1}$$

$$\frac{d^2T}{d\bar{y}^2}+\frac{QC_0A}{k}e^{\frac{E}{RT}}+\frac{\mu}{k}\left(\frac{du}{d\bar{y}}\right)^2=0, \tag{2}$$

$$u(a)=U,\ T(a)=T_0,\ u(0)=0,\ T(0)=T_0 \tag{3}$$

where T is the absolute temperature, U the upper wall characteristic velocity, T_0 the geometry wall temperature, k the thermal conductivity of the material, the heat of reaction, A the rate constant, E the activation energy, R the universal gas constant, C_0 the initial concentration of the reactant species, a the channel width, $\bar{y}$ the distance measured in the normal direction and μ the fluid dynamic viscosity coefficient [1–4,5,9]. We introduce the following dimensionless variables into the eqns. (1)-(3) are as follows:

$$\theta=\frac{E(T-T_0)}{RT_0^2},\quad y=\frac{\bar{y}}{a},\quad \lambda=\frac{QEAa^2C_0e^{-\frac{E}{RT_0}}}{T_0^2Rk} \tag{4}$$

$$W=\frac{u}{U},\ \beta=\frac{\mu U^2e^{\frac{E}{RT_0}}}{QAa^2C_0},\ \text{and}\ \varepsilon=\frac{RT_0}{E} \tag{5}$$

Using these dimensionless variables we can obtain the governing equation and the boundary conditions as

$$\frac{d^2\theta}{dy^2}+\lambda\left(e^{\left(\frac{\theta}{1+\varepsilon\theta}\right)}+\beta\right)=0 \qquad (6)$$

and

$$\theta(0)=0, \text{ and } \theta(1)=0 \qquad (7)$$

Solution of the boundary value problem using HPM

Linear and non-linear phenomena are of fundamental importance in various fields of science and engineering. Most models of real – life problems are still very difficult to solve. Therefore, approximate analytical solutions such as Homotopy perturbation method (HPM) [14-25] were introduced. This method is the mosteffective and convenient ones for both linear and non-linear equations.Perturbation method is based on assuming a small parameter. The majority of non-linear problems, especially those having strongnon-linearity, havenosmall parameters at all and the approximate solutions obtained by the perturbation methods, in most cases, are valid only for small values of the small parameter. Generally, the perturbation solutions are uniformly valid as long as a scientific system parameter is small. However, we cannot rely fully on the approximations, because there is no criterion on which the small parameter should exists. Thus, it is essential to check the validity of the approximations numerically and/or experimentally. To overcome these difficulties, HPM have been proposed recently.

Recently, many authors have applied the Homotopy perturbation method (HPM) to solve the non-linear boundary value problem in physics and engineering sciences [14-17]. Recently this method is also used to solve some of the non-linear problem in physical sciences [18-20]. This method is a combination of Homotopy in topology and classic perturbation techniques. Ji-Huan He used to solve the Lighthill equation [18], the Diffusion equation [19] and the Blasius equation [20-21]. The HPM is unique in its applicability, accuracy and efficiency. The HPM uses the imbedding parameter p as a small parameter, and only a few iterations are needed to searchfor an asymptotic solution. The approximate analytical solution of eqn.(3) using Homotopy perturbation method [22-25] is given by

$$\begin{aligned}\theta(y)&=\lambda\left(\frac{1+\beta}{2}\right)\left(y-y^2\right)\\&-\lambda^2\left(\frac{1+\beta}{2}\right)\left(\frac{y^3}{6}-\frac{y^4}{12}\right)\\&+\lambda^3\varepsilon\left(\frac{(1+\beta)^2}{4}\right)\left(\frac{y^4}{12}+\frac{y^6}{30}-\frac{y^5}{10}\right)\\&+\left(\lambda^2\left(\frac{1+\beta}{24}\right)-\lambda^3\varepsilon\left(\frac{(1+\beta)^2}{240}\right)\right)y\end{aligned} \qquad (8)$$

Previous work

The solution for the dimensionless temperature using Hermite-Pad'e approximation [1] is as follows:

$$\begin{aligned}\theta(y)&=-\frac{\lambda}{2}(\beta+1)\left(y^2-y\right)+\\&\frac{\lambda^2}{24}(\beta+1)\left(y^2-y\right)\left(y^2-y-1\right)\\&+\frac{\lambda^3}{1440}(\beta+1)\left(y^2-y\right)\\&\left(\begin{array}{l}-6y^4\beta+12y^4\beta\varepsilon+12y^3\beta-24y^3\beta\varepsilon\\+6y^2\beta\varepsilon-3y^2\beta+6y\beta\varepsilon-3y\beta\\+6\beta\varepsilon-3\beta+12y^4\varepsilon-8y^4-24y^3\varepsilon\\+16y^3+6y^2\varepsilon+y^2-9y+6y\varepsilon\\+6\varepsilon-9\end{array}\right)+O\left(\lambda^4\right)\end{aligned} \qquad (9)$$

Results and Discussions

In this section, we validate the above perturbation method, and we conclude the solution for the nonlinear boundary value problem for thermal criticality in viscous reactive flows through channels with a sliding wall. First we have to set the channels as well as sliding walls, because of the occurrence of the thermal power on viscous reactive flow method. The non-existence of a steady state solution for non-linear reaction diffusion problems for a certain parameter value is extremely important for the application perception.

The concept of criticality or non-existence of a steady state solution for nonlinear reaction diffusion problems for a certain parameter value is extremely important from the application point of view. Figure (1) is the schematic diagram for the reactive viscous material. Figures 2 (a) -(d) shows the dimensionless distance y versus the dimensionless temperature $\theta(y)$. From these figures it is evident that, when the variable viscosity heating parameter β increases, the corresponding dimensionless temperature $\theta(y)$ also increases in some fixed values of the Frank Kamenetskii parameter λ and the activation energy parameter ε. Figure 3 (a)-(d) is the dimensionless distance y versus the dimensionlesstemperature $\theta(y)$. From these figures, it is inferred that when the Frank Kamenetskii parameter λ increases, the dimensionless temperature $\theta(y)$ also increases in some fixed values of β and ε .Figure 4 also shows the dimensionless distance y versus the dimensionlesstemperature $\theta(y)$. From Fig. 4, we conclude that when ε increases, the temperature $\theta(y)$ also increases very small in the fixed values of λ and β.

Furthermore, it is an interesting note that an increase in the magnitude of the viscous shears heating parameter (β) due to the sliding motion of the upper plate will lower magnitude of thermal critically, hence enhancing the development of lubricant using the Homotopy perturbation method.

Conclusions

In this analysis we have discussed about simple analytical solutions for the nonlinear boundary value problem. Besides, it will be used in all types of non-linear types of problem. It will be extended to advanced techniques of lubricant, viscosity flow principles for other technical field; it is more effective than existing, when we applied in the sliding wall by using Homotopy perturbation method. It is also related to applications of fluid mechanism in the engineering field. It evolves more advantageous methods to solve the criticality of thermal in viscous fluids. we used the following parameters, λ, β and ε. We also compared our analytical results to the perturbation technique. In future it will be used for more applications related to non-linear as well as thermal and viscous flows in multiple channels. Future implementation of this work includes effective solution for this problem in various fields by using of our efficient perturbation method.

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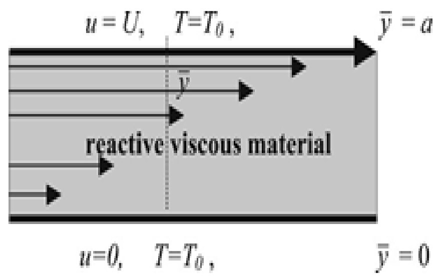
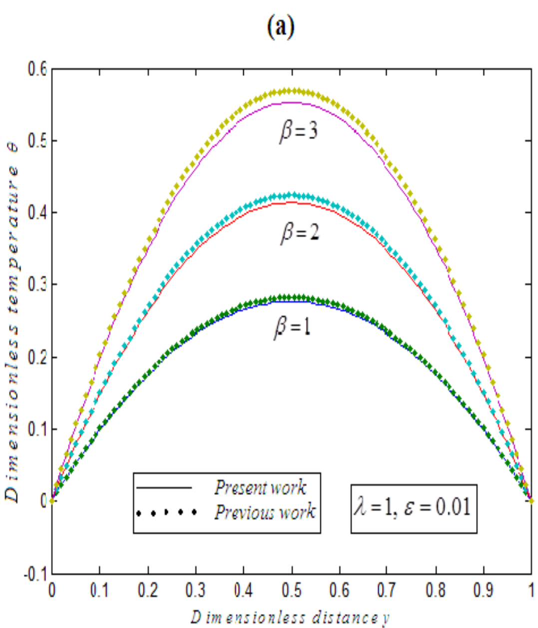
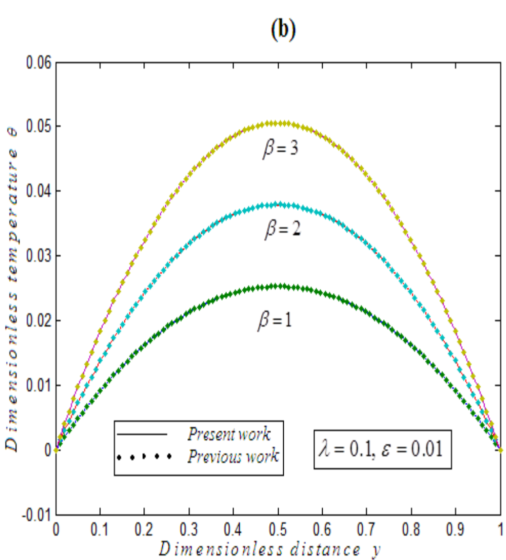
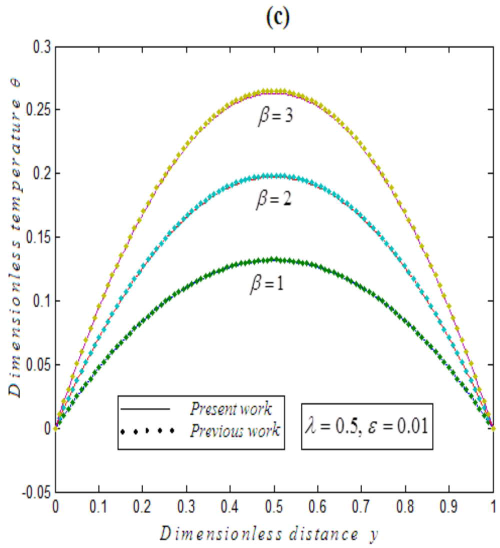


Fig. 1. Geomety of the problem







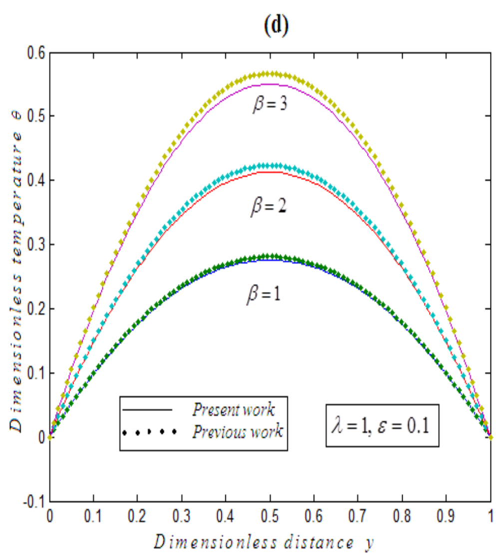
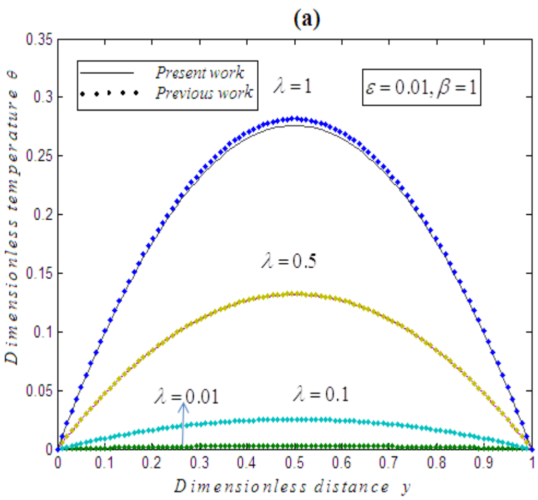
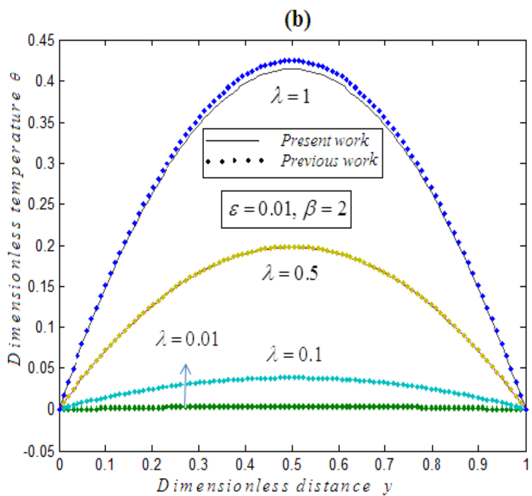
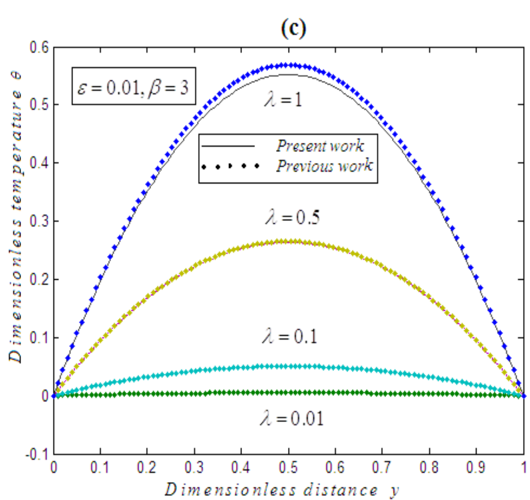


Figure:2 Dimensionless temperature θ versus the dimensionless vertical distance y .The temperature $\theta(y)$ were computed using eqn. (6) for various values of the dimensionless parameter β when (a) $\lambda = 1, \varepsilon = 0.01$,(b) $\lambda = 0.1, \varepsilon = 0.01$,(c) $\lambda = 0.5, \varepsilon = 0.01$ and (d) $\lambda = 1, \varepsilon = 0.1$.







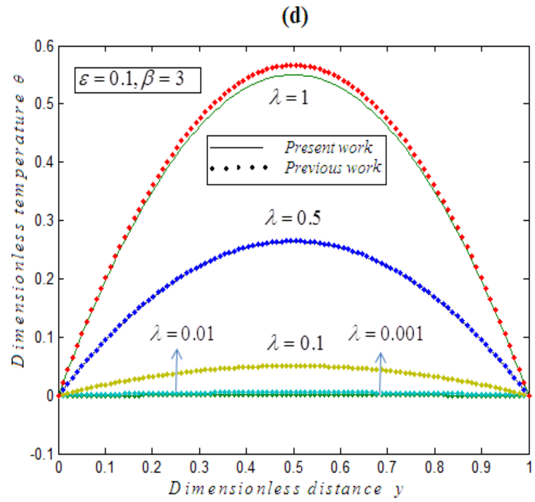


Figure:3 Dimensionless temperature θ versus the dimensionless vertical distance y. The temperature $\theta(y)$ were computed using eqn.(6) for various values of the dimensionless parameter λ when (a) $\beta = 1, \varepsilon = 0.01$, (b) $\beta = 2, \varepsilon = 0.01$, (c) $\beta = 3, \varepsilon = 0.01$ and (d) $\beta = 3, \varepsilon = 0.1$.

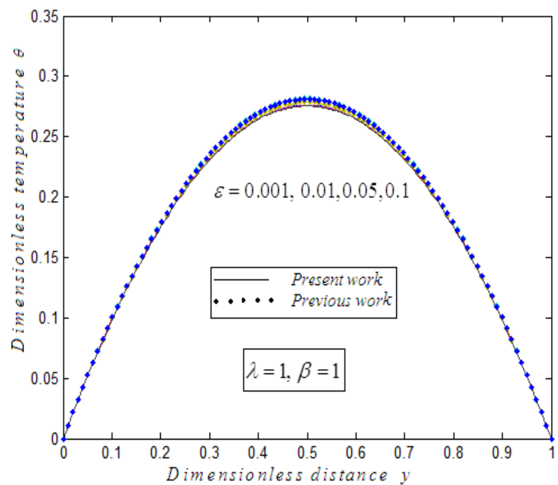


Figure: 4 Dimensionless temperatures θ versus the dimensionless vertical distance y. The temperature were computed using eqn. (6) for various values of the dimensionless parameters ε and some fixed value λ and β.

Appendix A.

Basic concepts of the Homotopy perturbation method [14-25]

To explain this method, let us consider the following function:

$$D_o(u) - f(r) = 0, \quad r \in \Omega \tag{A.1}$$

with the boundary conditions of

$$B_o(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \Gamma \tag{A.2}$$

where D_o is a general differential operator, B_o is a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω. In general, the operator D_o can be divided into a linear part L and a non-linear part N. Equation (A.1) can therefore be written as

$$L(u) + N(u) - f(r) = 0 \tag{A.3}$$

By the Homotopy technique, we construct a Homotopy $v(r, p) : \Omega \times [0,1] \to \Re$ that satisfies

$$H(v, p) = (1-p)[L(v) - L(u_0)] + p[D_o(v) - f(r)] = 0 \tag{A.4}$$

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \tag{A.5}$$

where $p \in [0, 1]$ is an embedding parameter, and u_0 is an initial approximation of eqn. (A.1) that satisfies the boundary conditions. From the eqn. (A.4) and the eqn. (A.5), we have

$$H(v,0) = L(v) - L(u_0) = 0 \tag{A.6}$$

$$H(v,1) = D_o(v) - f(r) = 0 \tag{A.7}$$

When p=0, then the eqn. (A. 4) and the eqn. (A.5) become linear equations. When p =1, they become non-linear equations. The process of changing p from zero to unity is that of $L(v) - L(u_0) = 0$ to $D_o(v) - f(r) = 0$. We first use the embedding parameter p as a "small parameter" and assume that the solutions of the eqns. (A.4) and (A.5) can be written as a power series in p :

$$v = v_0 + pv_1 + p^2 v_2 + ... \tag{A.8}$$

Setting $p = 1$ results in the approximate solution of the eqn. (A.1) is

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + ... \tag{A.9}$$

This is the basic idea of the HPM.

Appendix: B

Solution of non-linear equation (6) and (7) using HPM

In this Appendix, we indicate how the eqn. (8) in this paper is derived. To find the solution of the eqn.(6) . When $\varepsilon\theta$ small, the eqn.(6) reduces to

$$\frac{d^2\theta}{dy^2} + \lambda\left(1 + \theta - \varepsilon\theta^2 + \beta\right) = 0 \tag{B.1}$$

We construct the Homotopy as follows:

$$(1-p)\left[\frac{d^2\theta}{dy^2} + \lambda + \lambda\beta\right] + p\left[\frac{d^2\theta}{dy^2} + \lambda\left(1 + \theta - \varepsilon\theta^2 + \beta\right)\right] = 0 \tag{B.2}$$

The analytical solution of an eqn.(B.1) is

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + ... \tag{B.3}$$

Substituting the eqn.(B.3) into an eqn.(B.2) we get

$$(1-p)\left[\frac{d^2\left(\theta_0 + p\theta_1 + p^2\theta_2 + ...\right)}{dy^2} + \lambda + \lambda\beta\right] + p\left[\frac{d^2\left(\theta_0 + p\theta_1 + p^2\theta_2 + ...\right)}{dy^2} + \lambda\left(\begin{array}{l} 1 + \left(\theta_0 + p\theta_1 + p^2\theta_2 + ...\right) \\ - \varepsilon\left(\theta_0 + p\theta_1 + p^2\theta_2 + ...\right)^2 + \beta \end{array}\right)\right] = 0 \tag{B.4}$$

Comparing the coefficients of like powers of p in the eqn.(B.4) we get

$$p^0 : \frac{d^2\theta_0}{dy^2} + \lambda + \lambda\beta = 0 \tag{B.5}$$

$$p^1 : \frac{d^2\theta_1}{dy^2} + \lambda\theta_0 - \lambda\varepsilon\theta_0^2 = 0 \tag{B.6}$$

The initial approximations is as follows

$$\theta(0) = 0, \qquad \theta(1) = 0, \tag{B.7}$$

Solving the eqns.(B.5) and (B.6) and using the boundary condition eqn.(B.7) we obtain the following results:

$$\theta_0 = \frac{\lambda(1+\beta)}{2}\left(y - y^2\right) \tag{B.8}$$

$$\theta_1 = -\frac{\lambda^2(1+\beta)}{2}\left(\frac{y^3}{6}-\frac{y^4}{12}\right) + \frac{\lambda^3\varepsilon(1+\beta)^2}{4}\left(\frac{y^4}{12}+\frac{y^6}{30}-\frac{y^5}{10}\right) + \left[\frac{\lambda^2(1+\beta)}{24}-\frac{\lambda^3\varepsilon(1+\beta)^2}{240}\right]y \quad (B.9)$$

According to the HPM, we can conclude that

$$\theta = \lim_{p\to 1}\theta(y) = \theta_0 + \theta_1 \quad (B.10)$$

After putting the eqns. (B.8) and (B.9) into an eqn.(B.10), we obtain the solutionin the text eqn.(8).

Appendix C:

Nomenclature

T	Absolute temperature
U	Upper wall characteristic velocity
T_0	Geometry wall temperature
k	Thermal conductivity of the material
Q	Heat of reaction
A	Rate constant
E	Activation energy
R	Universal gas constant
C_0	Initial concentration of the reactant species
a	Channel width
$\overline{y}$	Distance measured in the normal direction
μ	Fluid dynamic viscosity coefficient
λ	Frank Kamenetskii parameter
ε	Activation energy parameter
β	Viscous heating parameter
θ	Dimensionless temperature
y	Dimensionless distance

REFERENCE

[1] Makinde , O.D. (2008), " Thermal criticality in viscous reactive flows through channels with a sliding wall: An exploitation of the Hermite–Pad´eapproximation method", Mathematical and Computer Modeling , 47.312-317. | [2] Balakrishnan.E., Swift A., Wake G.C., (1996), "Critical values for some non-class A geometries in thermal ignition theory", Math. Comput.Modelling 24 (8).1-10. | [3] Batchelor G.K. (1967), An Introduction to Fluid Dynamics, Cambridge University Press. | [4] Bebernes J., Eberly D. (1989), Mathematical Problems from Combustion Theory, Springer-Verlag, New York. | [5] Bowes, P.C. (1984), Self-Heating: Evaluating and Controlling the Hazard, Elsevier, Amsterdam. | [6] Frank, D.A. (1969), Kamenetskii, Diffusion and Heat Transfer in Chemical Kinetics, Plenum Press, New York. | [7] Guttamann, A.J. Asymptotic analysis of power-series expansions, in: Domb, C.Lebowitz, J.K. (Eds.) 1989, Phase Transitions and CriticalPhenomena, Academic Press, New York.. | [8] Hunter, D. L., Baker. G. A. (1979),"Methods of series analysisIII: integral approximant methods". Phys. Rev. B 19, 3808-3821. | [9]Johnsand. L.E., Narayanan, R.(1997),"FrictionalHeating in plane-Couette flow".Proc.Royal Soc.London A, 453:1653-1670. | [10] Makinde, O.D. (2005) ,"Strong exothermic in a cylindrical pipe: A case study of series summation | Technique",Mech. Res. 32, 191-195. | [11] Sergeyev, A.V. Goodson D.Z. (1998),"Summation of asymptotic expansions of multiple-valuedFunctions using algebraic approximants: Application toAn-harmonicoscillators", J. Phys. A 31, 4301-4317. | [12] Tourigny, Y.Drazin, P.G. (2000),"The asymptotic behaviour of algebraic approximants", Proc. R. Soc. Lond., A 456.1117-1137. | [13] Subrahmaniam, N. Johns, L.E. Narayanan R.(2002), "Stability of frictional heating in plane Couette flow at fixed power input",Proc. Roy. Soc.A 458, 2561-2569. | [14] Ghori, Q.K. Ahmed,M.andSiddiquiA. M.(2007), "Application of Homotopy perturbation method to squeezing flow of a Newtonian fluid", Int. J. Nonlinear Sci. Numer. Simulat, 8, 179-184. | [15] Ozis,T.andYildirim,A.(2007),"A Comparative study of He's Homotopy perturbation method for determining frequency-amplitude relation of a nonlinear oscillator with discon-tinuities", Int. J. Nonlinear Sci. Numer. Simulat, 8.243-248. | [16] Li,S. J. and Liu,Y. X.(2006), An Improved approach to nonlinear dynamical system identification using PID neural networks, Int. J. Nonlinear Sci. Numer.Simulat, 7. 177-182. | [17] Mousa,M.M.,Ragab,S.F. and Nturforsch.Z.(2008),"Application of the Homotopyperturbation method to linear and nonlinear Schrödinger equation",Zeitschrift für Naturforschung, 63 .140-144. | [18] He, J.H.(1999),Homotopy perturbation technique, Comp Meth.Appl. Mech. Eng, 178,257-262. | [19] He, J. H. (2003), "Homotopy perturbation method: a new nonlinear analytical technique", Appl. Math.Comput, 135,73-79. | [20] He, J. H. (2003),"A simple perturbation approach to Blasius equation", Appl. Math. Comput, 140. 217-222. | [21] Ariel, P.D. (2010),"Alternative approaches to construction of Homotopyperturbation algorithms", Nonlinear. Sci. Letts. A., 1. 43-52. | [22] Ananthaswamy, V. and Rajendran, L. (2012), "Analytical solution of two-point non-linear boundary value problems in a porous catalyst particles", International Journal of Mathematical Archive vol. 3 (3), pp. 810-821. | [23] Ananthaswamy,V. and Rajendran,L.(2012), " Analytical solutions of some two-point non-linear elliptic boundary value problems", Applied Mathematics, Vol.3, 1044-1058. | [24] Ananthaswamy,V. and Rajendran,L.(2012)," Analytical solution of non-isothermal diffusion- reaction processes and effectiveness factors", Article ID 487240, 2012,1-14. | [25] Ananthaswamy,V. Ganesan,SP. and Rajendran,L.(2013)," Approximate analytical solutionof non-linear boundary value problem of steady state flow of a liquid film: Homotopy perturbation method, International Journal of Applied Science and Engineering Research (IJASER), 2 (5), 569-577. |