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Bianchi Type Bulk Viscous String Cosmological Models in Modified Theory of Gravity

Cosmology

KEYWORDS : Bulk model, string model, new class of Bianchi type model, f(R,T).

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ABSTRACT

In this paper, we investigated the new class of Bianchi type string Cosmological model of the early universe with bulk viscosity in modified theory of gravity proposed by Harko et al.(Phys. Rev. D 84:024020,2011). To get determinate solution of the field equation, a special law of variation proposed by Berman is used. Some physical and geometrical properties of the model are also discussed. For the pressure and density, we also used the barotropic equation of state and bulk viscous pressure is assumed to be proportional to energy density.

Introduction:

Recently several modified theories of gravity have been developed and studied, in the view of the late time acceleration of the Universe and the existence of dark energy and dark matter. Noteworthy amongst them are the $f(R,T)$ theory of gravity proposed by Nojiri and Odintsov (2003a) and $f(R,T)$ theory of gravity formulated by Harko et al. (2011). Bertolami et al. proposed a generalization of $f(R,)$ theory of gravity by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. Nojiri and Odintsov developed a general solution for the modified $f(R,)$ gravity reconstruction from any realistic FRW Cosmology. They have showed that modified $f(R,)$ gravity indeed represents a realistic alternative to general relativity, being more consistent in dark epoch. Nojiri et al. developed a general program for the unification of matter-dominated era with acceleration epoch for scalar-tensor theory or dark fluid. Shamir proposed a physically viable $f(R,)$ gravity model, which showed the unification of early time inflation and late time acceleration.

Harko et al.(2011) developed $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor. It is to be noted that the dependence of T may be induced by exotic imperfect fluid or quantum effects. they have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion of test particles, which follows from the covariant divergence of the stress-energy tensor. They have derived some particular models corresponding to specific choices of function $f(R,T)$. They have also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of the function $f(R,T)$.

In $f(R,T)$ gravity, the field equations are obtained from a variation, Hilbert-Einstein type, principle.

The action principle for this modified theory $f(R,T)$ gravity is given by

$$S=\frac{1}{16\pi G}\int f(R,T)\sqrt{-g}d^4x+\int L_m\sqrt{-g}d^4x \quad . \qquad (1.1)$$

Where $f(R,T)$ is an arbitrary function of the Ricci scalar R, T is the trace of stress energy tensor of matter, T_{ij} and L_m is the matter Lagrangian density.

We define the stress energy tensor of matter as

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\partial(\sqrt{-g})}{\partial g^{ij}} L_m,$$

$$\Theta_{ij} = -2T_{ij} - pg_{ij} \qquad (1.2)$$

By varying the action principle (1.1) with respect to metric tensor, the corresponding field equations of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T) g_{ij}$$
$$+ (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j) f_R(R,T)$$
$$= 8\pi T_{ij} - f_T(R,T)T_{ij}$$
$$- f_T(R,T)\Theta_{ij} \qquad (1.3)$$

Where

$f_R = \frac{\delta f(R,T)}{\delta R}$, $\quad f_T = \frac{\delta f(R,T)}{\delta T}$ and $\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$

Here ∇_i is the covariant derivation and T_{ij} is standard matter energy-momentum tensor derived from the Lagrangian L_m.

It can be observed that when $f(R,T) = f(R)$, then (1.3) yield the field equations of $f(R)$ gravity.

It is mentioned here that these field equations depend on physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et. al.gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases}$$

Assuming,

$$f(R,T) = R + 2f(T) \qquad (1.4)$$

as a first choice, where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter

Then from (1.3) and (1.4), we get the gravitational field equation as

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2f'(T)T$$
$$- 2f'(T)\Theta_{ij} - f(T) g_{ij} \qquad (1.5)$$

where the overhead prime indicates differentiation with respect to the argument.

The Friedmann-Robertson-Walker models are the only globally acceptable perfect fluid space-times which are spatially homogenous and isotropic. The adequacy of isotropic cosmological models for describing the present state of the universe is no basis for expecting that they are equally suitable for describing the early stages of the evolution of the Universe. At the early stages of the evolution of Universe, it is, in general spatially homogenous and anisotropic. Bianchi spaces are useful tools for constructing spatially homogenous and anisotropic cosmological models in general relativity and scalar-tensor theories of gravitation. Reddy et. al. (2012a, 2012b) have obtained Kaluza-Klein cosmological model in the presence of perfect fluid source and Bianchi type III cosmological model in $f(R,T)$ gravity using the assumption of law of variation for the Hubble parameter proposed by Bermann (1983), Adhav (2012) has obtained LRS Bianchi type-I cosmological model in $f(R,T)$ gravity using the same assumption of law of variation for the Hubble parameter proposed by Bermann (1983). Shamir et al. obtained exact solution of Bianchi type-I and type-V cosmological model in $f(R,T)$ gravity. Chaubey and Shukla (2013) have obtained a new class of Bianchi cosmological models in $f(R,T)$ gravity. Reddy and Santi Kumar (2013) have presented some anisotropic cosmological models in this theory. Rao and Neelima (2013) have discussed perfect fluid Einstein-Rosen universe in $f(R,T)$ gravity. Recently Rao and Neelima (2013) have obtained perfect fluid Bianchi type VI_0 perfect fluid model in $f(R,T)$ gravity.

In order to study the evolution of universe, many authors constructed cosmological models containing viscous fluids. Bulk viscosity plays a significant role in the early evolution of the universe and contributes to the accelerated expansion phase of the universe popularly known as the inflationary phase. There are so many circumstances during the evolution of the universe in which bulk viscosity is could arises (Ellis 1971). At the time of particle creation in the early universe and during formation of galaxies (Hu 1983), when neutrinos decoupled from cosmic fluid (Misner 1968) viscosity arises. Many authors have studied bulk viscous cosmological models in general relativity. Johri and Sudarsan have investigated bulk viscous cosmological model in Brans-Dicke theory of gravitation. Bulk viscous cosmological model in Saez-Ballester theory of gravitation have been discussed be several authors. Recently Rao et al., Reddy et al., Naidu et al. have discussed the bulk viscous cosmological models in modified theoty of gravity proposed by Harko et. al (2011). Very recently Naidu et al. investigated Bianchi type V Bulk viscous string cosmological model in f(R,T) gravity.

Cosmic strings are theoretical fault lines in the universe, defective links between different regions of space created in the moments after the Big Bang. Strings arise as a random network of stable line like topological defects during the phase transition in the early universe. Massive closed loops of strings serve as seeds for the formation of large structures like galaxies and cluster of galaxies at the early stages of evolution of the universe. Many authors have investigated about different aspects of string cosmological models either in the frame work of Einstein's theory or in the modified theories of gravity.

In this paper, we intend to construct new class of Bianchi type string Cosmological model of the early universe with bulk viscosity in modified theory of $f(R,T)$ gravity. By using the barotropic equation of state and bulk viscous pressure is assumed to be proportional to energy density, solutions of field equations obtained. We have also discussed the physical and kinematical properties of the models.

2. The Metric and Field equations:-

We consider the new class of Bianchi type cosmological models, given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2x}dy^2 + C^2e^{-2mx}dz^2 \qquad (2.1)$$

where A, B, C are functions of t only. The metric (2.1) corresponds to a Bianchi type III model for $m=0$, Bianchi type V model for $m=1$ and Bianchi type VI_0 model for $m=-1$.

We consider the energy momentum tensor for a bulk viscous fluid containing one dimensional cosmic strings are given by

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p}g_{ij} - \lambda x_i x_j \qquad (2.2)$$

Where v_i and x_i satisfy condition

$$v^i v_i = -x^i x_i = -1, v^i x_i = 0 \qquad (2.3)$$

Where $\bar{p}$ is effective pressure, ρ is the proper energy density for cloud strings with particles attached to them, λ is the string tension density, v^i is the four-velocity of the particles and x^i is a unit space-time vector representing the direction of string. Here we also consider $\bar{p}$, ρ and λ are functions of t only.

The effective pressure $\bar{p}$ is given by

$$\bar{p} = p - 3\varsigma H \qquad (2.4)$$

Where $\varsigma(t)$ is the coefficient of bulk viscosity, $3\varsigma H$ is usually known as bulk viscous pressure, H is Hubble's parameter.

Using co-moving coordinates and Eqs. (2.2)-(2.4), the f(R,T) gravity field Eqs. (1.5) with the particular choice of the function

$f(T) = \mu T$, μ is constant. (2.5)

For the metric (2.1) take the form

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m}{A^2} = -\bar{p}(8\pi + 7\mu) + \lambda(8\pi + 3\mu) + \mu\rho \qquad (2.6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -\bar{p}(8\pi + 7\mu) + \lambda\mu + \mu\rho \qquad (2.7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = -\bar{p}(8\pi + 7\mu) + \lambda\mu + \mu\rho \qquad (2.8)$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{(1+m+m^2)}{A^2} = \rho(8\pi + 3\mu) - 5\mu\bar{p} + \lambda\mu \qquad (2.9)$$

$$(1+m)\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - m\frac{\dot{C}}{C} = 0 \qquad (2.10)$$

Where overhead dot denotes differentiation with respect to t.

Spatial volume and the scale factor for the metric (2.1) are given respectively, by

$$V^3 = ABC \qquad (2.11)$$

$$a = (ABC)^{\frac{1}{3}} \qquad (2.12)$$

The physical quantities of observational interest in cosmology are the expansion scalar θ, the mean anisotropy parameter A_h and shear scalar σ^2 which are given as

$$\theta = 3H = (\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) \qquad (2.13)$$

Where H is the mean Hubble parameter,

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \ \Delta H_i = H_i - H, i = 1,2,3 \qquad (2.14)$$

$$\sigma^2 = \frac{1}{2}\sigma^{ik}\sigma_{ik}\sum_{i=1}^{3} H_i^{\,2} - 3H^2 = \frac{3}{2}A_h H^2 . \qquad (2.15)$$

3. Solutions and the model

The field equations (2.6) –(2.10) reduce to the following independent equations

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{m-m^2}{A^2} = -\lambda(8\pi + 2\mu) \qquad (3.1)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{m^2-1}{A^2} = 0 \qquad (3.2)$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{(1+m+m^2)}{A^2} = \rho(8\pi + 3\mu) - 5\mu\bar{p} + \lambda\mu \qquad (3.3)$$

$$(1+m)\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - m\frac{\dot{C}}{C} = 0 \qquad (3.4)$$

Integrating equation (3.4) reduce to

$$A^{1+m} = c_1 BC^m, \qquad (3.5)$$

where c_1 is constants of integration. The constant c_1 without loss of generality, can be chosen as unity so that we have, from equation (3.5)

$$A^{1+m} = BC^m \qquad (3.6)$$

Now equations (3.1)-(3.3) & (3.6) are a system of four independent equations in six unknowns A, B, C, p, ρ and λ. Also the equations are non-linear. Hence to find a determinate solution we use the following physically plausible conditions.

In order to solve the system completely, we assume that

$$C = V^{b}, \tag{3.7}$$

where b is any constant.

Variation of Hubble's parameter proposed by Berman (1983) that yields constant deceleration parameter models of the universe defined by

$$q = -a\frac{\ddot{a}}{\dot{a}} = constant. \tag{3.8}$$

Which gives the solution

$$a = (\alpha t + \beta)^{\frac{1}{1+q}}, \tag{3.9}$$

where $\alpha \neq 0$ and β are constants of integration. This equation implies that the condition for acceleration expansion of the universe is $1+q>0$.

For a barotropic fluid, the combined effect of the proper pressure and the bulk viscous pressure can be expressed as

$$\bar{p} = p - 3\varsigma H = \varepsilon\rho \tag{3.10}$$

where

$$\varepsilon = \varepsilon_0 - \gamma\ (0 \le \varepsilon_0 \le 1),\ p = \varepsilon_0\rho, \tag{3.11}$$

and ε_0 and γ are constants.

Now from equations (2.12), (3.6), (3.7) and (3.9), we obtain

$$A = (\alpha t + \beta)^{\frac{3+3mb-3b}{(m+2)(1+q)}} \tag{3.12}$$

$$B = (\alpha t + \beta)^{\frac{3m+3-6mb-3b}{(m+2)(1+q)}} \tag{3.13}$$

$$C = (\alpha t + \beta)^{\frac{3b}{(1+q)}} \tag{3.14}$$

By taking $\alpha = 1$ *and* $\beta = 0$, the metric (2.1) can be written as

$$ds^2 = -dt^2 + t^{\frac{6+6mb-6b}{(m+2)(1+q)}}\left\{dx^2 + t^{\frac{6m-18mb}{(m+2)(1+q)}}e^{-2x}dy^2\right\} + t^{\frac{6b}{(1+q)}}e^{-2mx}dz^2 \tag{3.15}$$

4. Some physical properties of the model

Equations (3.15) represents the new class of Bianchi type bulk viscous string cosmological model in f(R,T) modified theory of gravitation with the following physical kinematical parameter which play a vital role in the discussion of cosmology:

Spatial volume

$$V^3 = (\alpha t + \beta)^{\frac{3}{1+q}} \tag{4.1}$$

Scalar of expansion

$$\theta = \frac{3}{(1+q)t} \tag{4.2}$$

The mean Hubble parameter

$$H = \frac{1}{(1+q)t} \tag{4.3}$$

The mean anisotropy parameter

$$A_h = \frac{2(1-3b)^2(m^2+m+1)}{(m+2)^2} \tag{4.4}$$

The shear scalar

$$\sigma^2 = \frac{3(1-3b)^2(m^2+m+1)}{(m+2)^2(1+q)^2t^2} \tag{4.5}$$

The string tension density

$$\lambda = \frac{-m}{(8\pi+2\mu)}\left\{\frac{3(3b-1)(2-q)}{(m+2)(1+q)^2t^2} + \frac{1-m}{t^{\frac{6+6mb-6b}{(m+2)(1+q)}}}\right\} \tag{4.6}$$

The energy density

$$\rho = \frac{1}{[8\pi+\mu(3-5\varepsilon)]}\{A_1 - \mu\lambda\} \tag{4.7}$$

where

$$A_1 = \frac{9(m+1)(1+b+2bm) - 3b(9m^2b+9mb+3m-3+9b)}{(m+2)^2(1+q)^2t^2} - \frac{(1+m+m^2)}{t^{\frac{6+6mb-6b}{(m+2)(1+q)}}} \tag{4.8}$$

The pressure

$$p = \frac{\varepsilon_0}{[8\pi + \mu(3-5\varepsilon)]}\{A_1 - \mu\lambda\} \qquad (4.9)$$

The coefficient of bulk viscosity

$$\varsigma = \frac{(\varepsilon_0 - \varepsilon)(1+q)t}{3[8\pi + \mu(3-5\varepsilon)]}\{A_1 - \mu\lambda\} \qquad (4.10)$$

Using above results, we now discuss the behavior of $f(R,T)$ gravity cosmological models given by (3.15). The results (4.1) shows that the models are expanding with time since $1+q>0$. It can be observed that the models have no initial singularity, i.e. at $t=0$. Also equation (4.6) gives that strings in this model do not survive, for $m=0$ and for $m=1$, $1-3b=0$. Since $A_h \neq 0$ and $\frac{\sigma^2}{\theta^2} \neq 0$, the universe remains anisotropic throughout the evolution of the universe. From equations (4.4) and (4.5), it is noted that $A_h = 0$ and $\sigma^2 = 0$ for $1-3b=0$ and hence the universe is isotropic and shear free. It can also be observed that $\theta, H, \rho, p, \lambda$ and ς decrease with time and approaches to zero as $t \to \infty$ and all diverges at $t=0$, if $b \neq \frac{1}{3}$.

Conclusion:

In this paper, we have additional classes of Bianchi models from the general class of Bianchi model for different values of m as follows : Bianchi type III corresponds to $m=0$, Bianchi type V corresponds to $m=1$, Bianchi type VI_0 corresponds to $m=-1$ and Bianchi type VI corresponds to all other values of m.Also, we have presented the general class of Bianchi type cosmological models of universe with bulk viscous fluid with one dimensional cosmic strings in modified theory gravity formulated by Harko et al. (2011) by modifying general relativity to explain the challenging problem of late time acceleration of the universe. To obtain a determinate solution of non-linear field equations, we have taken the help of spatial law of variation of Hubble's parameter proposed by Berman (1983).We have also used a barotropic equation of state for pressure and energy density and the bulk viscous pressure is assumed to be proportional to the energy density. It is observed that the average anisotropic parameter does not vanishes so that the model remain anisotropic throughout the evolution of the universe expect at $1-3b=0$. Also for $1-3b=0$ and $m=1$, our result match with the result of R.L.Naidu etc al. [30].

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